SOLUTION

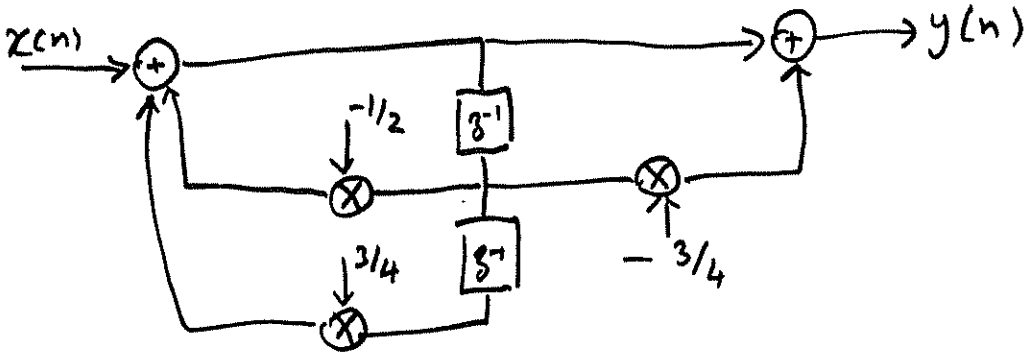
Name: 4-digit ID:

DSP Spring 04 Quiz 2A

1. **For the difference equation given below, draw the Direct Form II (minimum delay or canonical) structure representing the input-output relationship.**

$$y(n) + \tfrac{1}{2} y(n-1) - \tfrac{3}{4} y(n-2) = x(n) - \tfrac{3}{4} x(n-1)$$

$$y(n) = -\tfrac{1}{2} y(n-1) + \tfrac{3}{4} y(n-2) + x(n) - \tfrac{3}{4} x(n-1)$$



2. **The step response of a discrete LTI system is given by s(n) = {1, -1, 1}. What is the response, y(n), of the system due to an input x(n) = {1,-1}?**

$$h(n) = s(n) - s(n-1)$$

$$= \delta(n) - \delta(n-1) + \delta(n-2) - \delta(n-1) + \delta(n-2) - \delta(n-3)$$

$$= \delta(n) - 2\delta(n-1) + 2\delta(n-2) - \delta(n-3)$$

$$y(n) = h(n) * x(n) = h(n) * [\delta(n) - \delta(n-1)]$$

$$= h(n) - h(n-1)$$

$$= \delta(n) - 2\delta(n-1) + 2\delta(n-2) - \delta(n-3) - \delta(n-1) + 2\delta(n-2) - 2\delta(n-3) + \delta(n-4)$$

$$= \delta(n) - 3\delta(n-1) + 4\delta(n-2) - 3\delta(n-3) + \delta(n-4)$$

$$y(n) = \{1, -3, 4, -3, 1\}$$

Q 2 A

3. **The input output relationship of a discrete time linear system is as given below.**

	x(n)	y(n)	
$x_1(n) \longrightarrow$	{-1,2,1}	{1,-2,-1,0,1}	$\leftarrow y_1(n)$
$x_2(n) \longrightarrow$	{1,-1,-1}	{-1,1,0,2}	$\leftarrow y_2(n)$
$x_3(n) \longrightarrow$	{0,1,1}	{1,2,1}	$\leftarrow y_3(n)$

Could this be a time invariant system? Give reasons.

$x_1(n) + x_2(n) = \delta(n)$

For a linear system, then

$x_1(n) + x_2(n)$ → [Linear System] → $y_1(n) + y_2(n)$

$= \{0, -1, -1, 2, 1\} \leftarrow y_4(n)$

$x_3(n) = \delta(n) + \delta(n-1)$.

If $y_3(n)$ equals $y_4(n) + y_4(n-1)$ the system could be time invariant.

$y_4(n) + y_4(n-1) = \{0, -1, -1, 2, 1\} + \{0, 0, -1, -1, 2, 1\}$

$= \{0, -1, -2, 1, 3, 1\}$

which is not equal to $y_3(n)$

$\Longrightarrow$ System is NOT TIME INVARIANT

SOLUTION

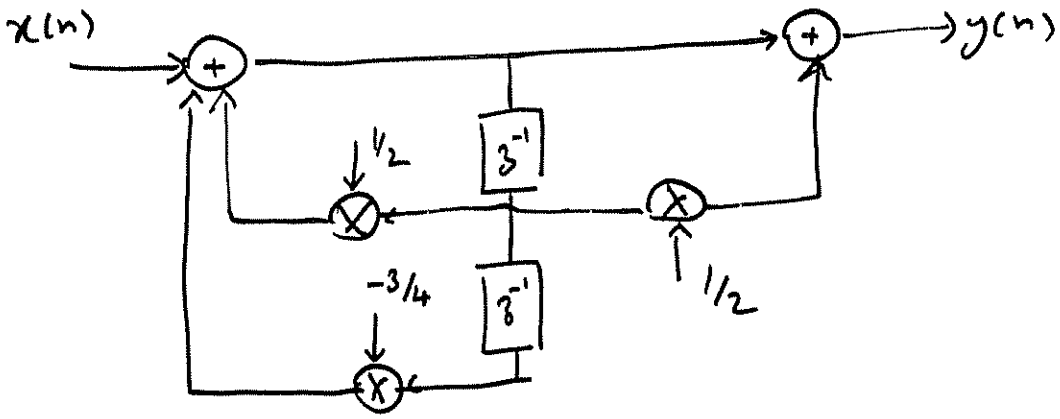
Name: 4-digit ID:

DSP Spring 04 Quiz 2B

1. **For the difference equation given below, draw the Direct Form II (i.e. minimum delay or canonical) structure representing the input-output relationship.**

$$y(n) - \tfrac{1}{2}\, y(n-1) + \tfrac{3}{4}\, y(n-2) = x(n) + \tfrac{1}{2}\, x(n-1)$$

$$y(n) = \tfrac{1}{2}\, y(n-1) - \tfrac{3}{4}\, y(n-2) + x(n) + \tfrac{1}{2}\, x(n-1)$$



2. **The step response of a discrete LTI system is given by s(n) = {1, -1, 1}. What is the response, y(n), of the system due to an input x(n) = {-1,1}**

$$h(n) = s(n) - s(n-1) = \delta(n) - \delta(n-1) + \delta(n-2) - \delta(n-1) + \delta(n-2) - \delta(n-3)$$

$$= \delta(n) - 2\delta(n-1) + 2\delta(n-2) - \delta(n-3)$$

$$y(n) = h(n) * x(n) = h(n) * [-\delta(n) + \delta(n-1)]$$

$$= -h(n) + h(n-1)$$

$$= -[\delta(n) - 2\delta(n-1) + 2\delta(n-2) - \delta(n-3)]$$

$$+ [\delta(n-1) - 2\delta(n-2) + 2\delta(n-3) - \delta(n-4)]$$

$$= -\delta(n) + 3\delta(n-1) - 4\delta(n-2) + 3\delta(n-3) - \delta(n-4)$$

$$\boxed{y(n) = \{-1, 3, -4, 3, -1\}}$$

Q2B

3. **The input output relationship of a discrete time linear system is as given below.**

x(n)	y(n)
{-1, 2, 1} ↑	{1,-2,-1, 0, 1}
{1,-1,-1} ↑	{-1, 1, 0, 2}
{0, 1, 1} ↑	{1, 2, 1}

Could this be a time invariant system? Give reasons.

See Q 2A # 3 Solution