DSP Quiz 3A Spring 04 Name:

1. The following LCCDE describes the input-output relationship of a causal, LTI system; find the impulse response of the h (n) of the system.

$$y(n) - y(n-1) + \frac{1}{2} y(n-2) = \frac{1}{2} x(n-1)$$

Taking z-transform on both sides, we get.

$$Y(z) - z^{-1} Y(z) + \frac{1}{2} z^{-2} Y(z) = \frac{1}{2} z^{-1} X(z)$$

or

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1/2\, z^{-1}}{(1 - z^{-1} + \frac{1}{2} z^{-2})} \quad — ①$$

From z-transform table, we have:

$$[r^n \sin w_0 n] u(n) \longleftrightarrow \frac{(r \sin w_0) z^{-1}}{1 - (2r\cos w_0) z^{-1} + r^2 z^{-2}} \quad — ②$$

from ① and ②, we have

$$r \sin w_0 = 1/2, \quad r\cos w_0 = 1/2 \quad \text{and } r^2 = \frac{1}{2}$$

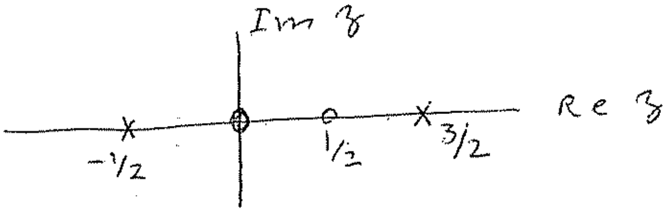
$$\Longrightarrow r = \frac{1}{\sqrt{2}}, \quad \sin w_0 = \cos w_0 = \frac{1}{\sqrt{2}} \Longrightarrow w_0 = \pi/4 .$$

Thus,

$$\boxed{h(n) = \left[\left(\frac{1}{\sqrt{2}}\right)^n \sin\left(\frac{\pi}{4} n\right)\right] u(n)}$$

Q 3A

2. Determine the impulse response h (n) of the stable LTI system, whose pole-zero plot is given below; the gain factor is 2



$$H(z) = \frac{2 \cdot z\,(z - 1/2)}{(z + 1/2)\,(z - 3/2)}$$

ROC: $\frac{1}{2} < |z| < 3/2$ (from stability)

$$\frac{H(z)}{z} = \frac{A}{(z + 1/2)} + \frac{B}{(z - 3/2)}$$

A: $$A = \left.\frac{2 \cdot (z - 1/2)}{(z - 3/2)}\right|_{z = -1/2} = 2 \cdot \frac{(-1/2 - 1/2)}{(-1/2 - 3/2)} = \boxed{1}$$

B: $$B = \left.\frac{2 \cdot (z - 1/2)}{(z + 1/2)}\right|_{z = 3/2} = \frac{2\,(3/2 - 1/2)}{(3/2 + 1/2)} = \boxed{1}$$

$$H(z) = \underset{\substack{\uparrow \\ \text{Causal}}}{\frac{z}{(z + 1/2)}} + \underset{\substack{\uparrow \\ \text{anti causal (from stability)}}}{\frac{z}{(z - 3/2)}}$$

$$\boxed{h(n) = \left(-\tfrac{1}{2}\right)^n u(n) - (3/2)^n\, u(-n-1)}$$

DSP Quiz 3 B Spring 04 Name:

1. The following LCCDE describes the input-output relationship of a causal, LTI system; find the impulse response of the h (n) of the system.

$$y(n) - \frac{1}{2} y(n-1) + \frac{1}{4} y(n-2) = x(n) - \frac{1}{4} x(n-1)$$

Taking z-transform on both sides, we have:

$$Y(z) - \frac{1}{2} z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) = X(z) - \frac{1}{4} z^{-1} X(z)$$

or

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(1 - \frac{1}{4} z^{-1})}{(1 - \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2})} \quad \text{—} ①$$

From z-transform tables, we have:

$$[r^n \cos w_0 n] u(n) \longleftrightarrow \frac{[1 - (r \cos w_0) z^{-1}]}{(1 - (2 r \cos w_0) z^{-1} + r^2 z^{-2})} \quad \text{—} ②$$

Comparing ① & ②, we have:

$r \cos w_0 = \frac{1}{4}$ and $r^2 = 1/4 \Rightarrow r = \frac{1}{2}$, $\cos w_0 = \frac{1}{2}$

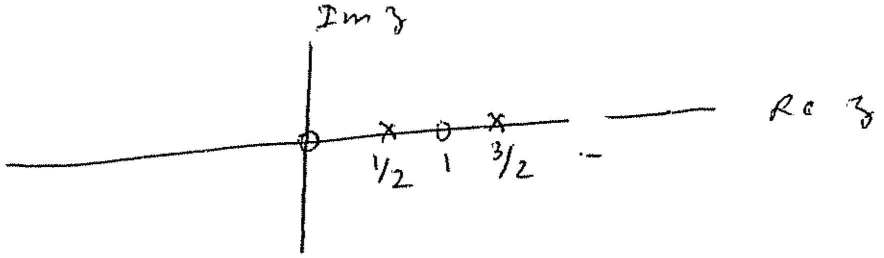
$\Rightarrow w_0 = \frac{\pi}{3}$.

Thus,

$$\boxed{h(n) = \left[\left(\frac{1}{2}\right)^n \cos \frac{\pi}{3} n\right] u(n)}$$

Q3B

2. Determine the impulse response h (n) of the stable LTI system, whose pole-zero plot is given below; the gain factor is 2.



$$H(z) = \frac{2z(z-1)}{(z-1/2)(z-3/2)} .$$ ROC: $\frac{1}{2} < |z| < 3/2$

$$\frac{H(z)}{z} = \frac{2(z-1)}{(z-1/2)(z-3/2)} = \frac{A}{(z-1/2)} + \frac{B}{(z-3/2)}$$

A :

$$A = \frac{2(z-1)}{z-3/2}\bigg|_{z=1/2} = \frac{2(1/2-1)}{(1/2-3/2)} = 2\times\frac{-1/2}{-1} = \boxed{1}.$$

B :

$$B = \frac{2(z-1)}{(z-1/2)}\bigg|_{z=3/2} = 2\cdot\frac{(3/2-1)}{(3/2-1/2)} = 2\cdot\frac{1/2}{1} = \boxed{1}$$

$$H(z) = \frac{z}{(z-1/2)} + \frac{z}{(z-3/2)} = \underset{\uparrow \text{causal}}{\frac{1}{(1-1/2z^{-1})}} + \underset{\uparrow \text{anti causal}}{\frac{1}{(1-3/2z^{-1})}}$$

(from stability)

$$\boxed{h(n) = \left(\tfrac{1}{2}\right)^n u(n) - (3/2)^n u(-n-1)}$$