SOLUTION
MASTER

EE 6360
Exam 2
November 6, 2007

Please Print.

Last Name **First Name**

Instructions

1. Examination Duration: 1 hour 15 minutes. If you arrive 15 minutes early, you can have that extra time.
2. ONE 8.5" x 11"sheet, with both sides of hand-written material, is allowed. No solved problems of any kind are allowed. Violation will result in a grade of zero for this exam, and other disciplinary actions may be taken.
3. One calculator is allowed.
4. Answer in the space/sheets provided.
5. **Answer all 5 questions. There is a bonus question at the end for 10 points.**
6. **DO NOT RELY ON PARTIAL CREDIT, WHICH IS SOLELY AT THE DISCRETION OF THE INSTRUCTOR.**
7. Any copying or cheating will result in appropriate action as per university regulations.

1. (20 pts)

The transfer function of two causal FIR filters with identical magnitude responses are given below:

$H_1(z) = 1.56 + 2.89\ z^{-1} + 3.39\ z^{-2} + 2.19\,z^{-3} + 0.81\ z^{-4}$

$H_2(z) = 1.26 + 2.51\,z^{-1} + 3.50\ z^{-2} + 2.58\,z^{-3} + z^{-4}$

One of the above is a minimum phase filter; identify that filter giving appropriate reasoning. How many other causal FIR filters (nontrivial) filters exist with the same magnitude response? Identify them. For each of the case, state whether it is minimum phase, maximum phase or mixed phase filter.

[Note: you must determine the transfer functions].

* $H_1(z)$ is the minimum phase filter. Note that $|h_1(0)| > |h_2(0)|$ (i.e. quickest energy delivery).

* $H_2(z)$ is mixed-phase

* We can generate $H_3(z) = z^{-4}H_1(z^{-1})$ that will be maximum phase (slowest energy delivery).

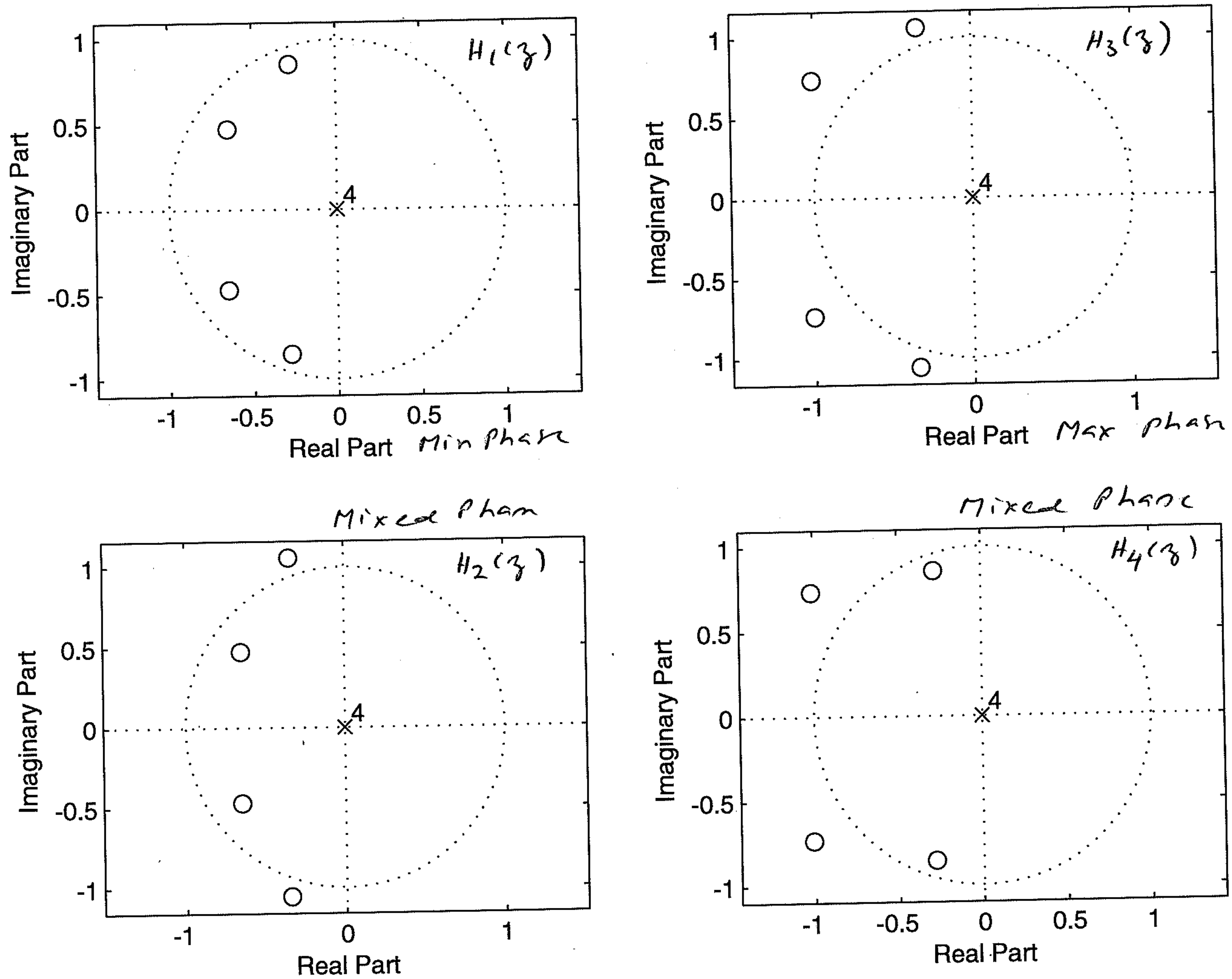
$$H_3(z) = 0.81 + 2.19\,z^{-1} + 3.39\,z^{-2} + 2.89\,z^{-3} + 1.56\,z^{-4}$$

* We can generate $H_4(z) = z^{-4}H_2(z^{-1})$ that will be mixed phase.

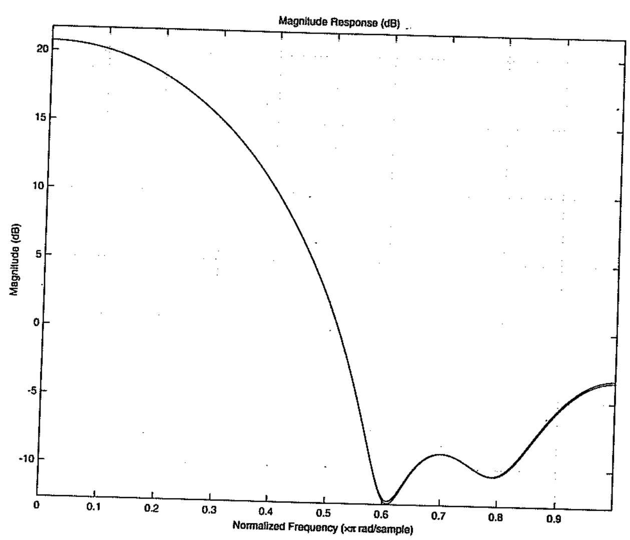
$$H_4(z) = 1 + 2.58\,z^{-1} + 3.50\,z^{-2} + 2.51\,z^{-3} + 1.26\,z^{-4}$$

Note: other "trivial" solutions include "delayed" versions of the above 4 FIR systems.

Pole-Zero Plots



For your Information only.



Mag. Response

2. (20 pts)
(a) Design a causal FIR filter of length 3 samples, which notches out the continuous time frequency of ¼th the sampling frequency, and has a unity gain at half sampling frequency. Sketch the pole-zero diagram of the filter. (12 points) (b) A continuous time signal x (t) = 4 + 2cos (2π 100t) + 2 cos(2π 200t) is C/D converted at the rate of 800 samples per second, and the resulting discrete time signal, x(n), is input to the above FIR filter. The output, y (n), is D/C converted to the continuous time signal y(t) at the same rate as C/D conversion. Determine the signal y (t). (8 points)

(a)

Digital notch frequency, $w_0 = 2\pi \cdot \frac{f_{notch}}{f_s} = \frac{2\pi}{4} = \boxed{\pi/2}$ rad/sample

Causal 3-pt FIR notch filter:

$$H(z) = G\,(1 - 2\cos w_0\, z^{-1} + z^{-2})$$

$$H(z) = G\,(1 + z^{-2})$$

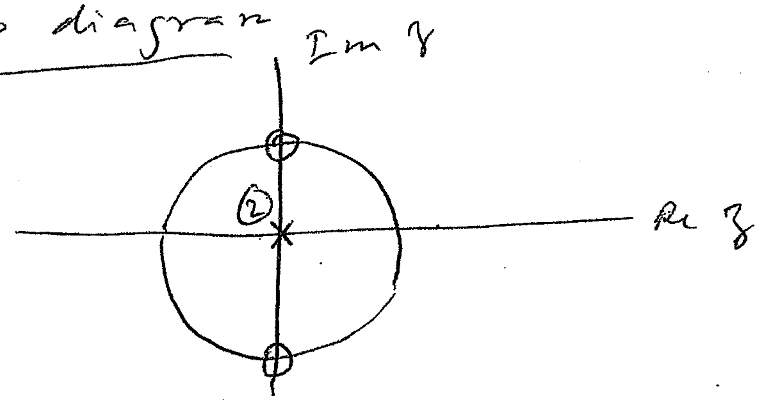
$$H(w) = G\,(1 + e^{-j2w})$$

$$|H(w)|_{w=\pi} = \left|G\,(1 + e^{-j2\pi})\right| = 1 \Rightarrow \boxed{G = \pm\tfrac{1}{2}}$$

We choose $\boxed{G = \tfrac{1}{2}}$

Thus $$\boxed{H(z) = \tfrac{1}{2}\,(1 + z^{-2})}$$

Pole-zero diagram



(b)

$$x(n) = 4 + 2\cos\left(2\pi \cdot \frac{100}{800} n\right) + 2\cos\left(2\pi \frac{200}{800}\right)$$
$$= 4 + 2\cos\left(\frac{\pi}{4} n\right) + 2\cos\left(\frac{\pi}{2} n\right)$$

$$y(n) = H(0)\cdot 4 + \left|H\left(\tfrac{\pi}{4}\right)\right|\cdot 2\cos\left(\tfrac{\pi}{4}n + \theta_H\left(\tfrac{\pi}{4}\right)\right)$$
$$+ \cancel{\left|H\left(\tfrac{\pi}{2}\right)\right|}^{\,0}\cdot 2\cos\left(\tfrac{\pi}{2}n + \theta_H\left(\tfrac{\pi}{2}\right)\right)$$
(by design).

$$H\left(\tfrac{\pi}{4}\right) = \tfrac{1}{2}\left(1 + e^{-j\pi/2}\right) = \tfrac{1}{2}\cdot(1-j) = \tfrac{1}{2}\cdot\sqrt{2}\,e^{-j\frac{\pi}{4}} = \tfrac{1}{\sqrt{2}}\,e^{-j\pi/4}$$

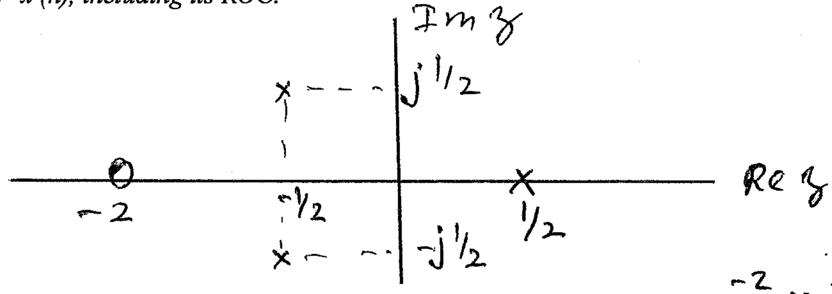
$$H(0) = \tfrac{1}{2}\left(1 + e^{-j\,2\cdot 0}\right) = \tfrac{1}{2}\cdot 2 = \boxed{1}.$$

Thus
$$y(n) = 4 + \sqrt{2}\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{4}\right)$$

$$y(t) = 4 + \sqrt{2}\cos\left(2\pi\cdot\cancel{\tfrac{1}{8}}\cdot\cancel{800}^{\,100}t - \pi/4\right)$$

$$\Rightarrow \boxed{y(t) = 4 + \sqrt{2}\cos\left(2\pi 100t - \tfrac{\pi}{4}\right)}$$

3. (20 pts)
The pole-zero plot of a causal signal x(n) is shown below. (A) Determine the pole-zero plot of the signal $x_1(n) = x(-n+2)$, including its ROC. (B) Determine the pole-zero plot of the signal $x_2(n) = (1/2)^n x(n)$, including its ROC.



(A) $x_1(n) = x(-(n-2)) \Longrightarrow X_1(z) = z^{-2} X(z^{-1})$.

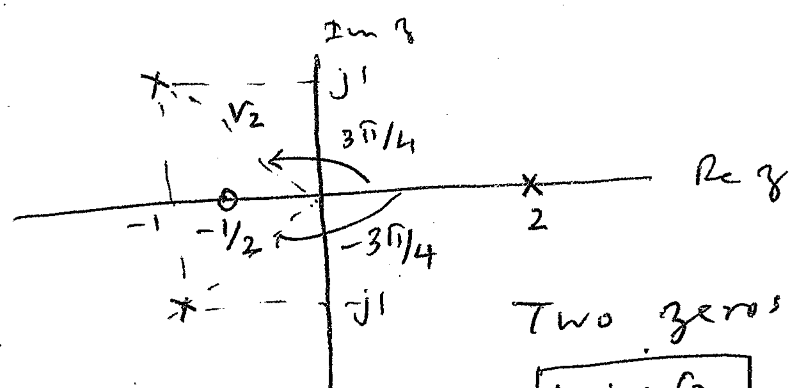
$$X(z) = \frac{(z+2)}{(z-1/2)(z-re^{j\theta})(z+re^{j\theta})} \quad \text{--- (1)} \quad \boxed{\text{ROC: } |z| > \frac{1}{\sqrt{2}}}$$

where $r = \frac{1}{\sqrt{2}}$ and, $\theta = \frac{3\pi}{4}$.

$$X_1(z) = z^{-2} X(z^{-1}) = \frac{z^{-2}(z^{-1}+2)}{(z^{-1}-1/2)(z^{-1}-re^{j\theta})(z^{-1}-re^{-j\theta})}$$

$$= \frac{z^{-3}(1+2z)}{(z^{-1}-1/2)(z^{-1}-re^{j\theta})(z^{-1}-re^{-j\theta})}$$

$$= \frac{(1+2z)}{(1-\frac{1}{2}z)(1-re^{j\theta}z)(1-re^{-j\theta}z)}$$



Two zeros at $z = \infty$..

ROC: $\boxed{|z| < \sqrt{2}}$

(why?)

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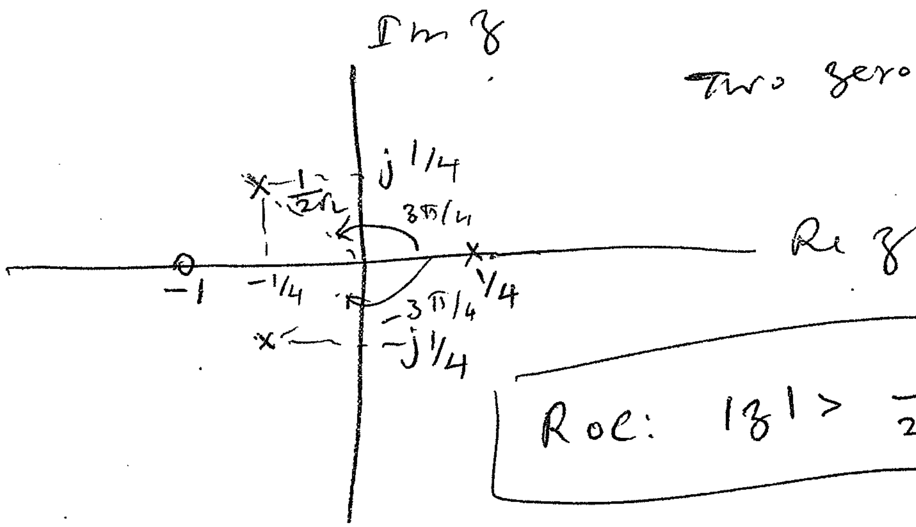
$x_2(n) = \left(\frac{1}{2}\right)^n x(n)$

$X_2(z) = X\left(\frac{z}{\lambda}\right), \quad \lambda = 1/2.$

Thus the poles & zeros of $X(z)$ will be the poles & zeros of $X_2(z)$ with the relationship $\quad \frac{z}{\lambda} = p_0$ or z_0 (pole or zero of $x(n)$)

	Pole or zero of $X(z)$	$X_2(z)$
zeros:	-2	$(-2)\cdot\frac{1}{2} = -1$
	∞	∞
poles	$1/2$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$
	$re^{j\theta}$	$\frac{1}{2}re^{j\theta}$
	$re^{-j\theta}$	$\frac{1}{2}re^{-j\theta}$

where $r = \frac{1}{\sqrt{2}} \quad \theta = \frac{3\pi}{4}$.



Two zeros at ∞

$$\boxed{\text{ROC: } |z| > \frac{1}{2\sqrt{2}}}$$

4. *(Two Parts)*

(a) Let X (ω) denote the DTFT of the sequence x(n) shown below. Express the DTFT of the sequence in terms of X (ω), without explicitly computing either of the DTFT.

$x(n) = \{\underset{\wedge}{\mathbf{1}}\ 2\ 3\}$

$y_1(n) = \{\underset{\wedge}{\mathbf{1}}\ 2\ 3\ 0\ -3\ -2\ -1\}$

$y_2(n) = \{-3\ -2\ -1\ \underset{\wedge}{\mathbf{0}}\ 1\ 2\ 3\}$

$y_1(n)$:

$y_1(n) = x(n) + x_1(n-6)$

where $x_1(n) = \{-3\ -2\ \underset{\uparrow}{-1}\} = -x(-n)$

$x(n) \longleftrightarrow X(\omega)$ $\qquad x_1(n) \longleftrightarrow -X(-\omega)$

$\Rightarrow \boxed{Y_1(\omega) = X(\omega) - e^{-j\omega 6} X(-\omega)}$

$y_2(n)$:

$y_2(n) = x_1(n+1) + x(n-1)$

$\Rightarrow Y_2(\omega) = -e^{j\omega} X_1(\omega) + e^{-j\omega} X(\omega)$

$\Rightarrow \boxed{Y_2(\omega) = -e^{j\omega} X(-\omega) + e^{-j\omega} X(\omega)}$

(b) *Determine the discrete Fourier series (DFS) coefficients of the periodic sequence given below.*

y (n) = 2 cos [(2π/3) (n-2)] + 2 cos [(2π/5)(n-2)]

$y(n) = x(n-2)$ where

$x(n) = 2\cos\left[\frac{2\pi}{3}n\right] + 2\cos\left[\frac{2\pi}{5}n\right]$.

$\boxed{N=15}$

$x(n) = \left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right] + \left[e^{j\frac{2\pi}{5}n} + e^{-j\frac{2\pi}{5}n}\right]$

But $e^{j\frac{2\pi}{3}n} = e^{j\frac{2\pi 5}{15}n}$, $e^{-j\frac{2\pi}{3}n} = e^{j\frac{2\pi 10}{15}n}$

$e^{j\frac{2\pi}{5}n} = e^{j\frac{2\pi 3}{15}n}$, $e^{-j\frac{2\pi}{5}n} = e^{j\frac{2\pi 12}{15}n}$

$x(n) = e^{j\frac{2\pi(3)n}{15}} + e^{j\frac{2\pi(5)n}{15}} + e^{j\frac{2\pi(10)n}{15}} + e^{j\frac{2\pi(12)n}{15}}$ —(1)

But $x(n) = \sum_{n=0}^{14} c_k e^{j\frac{2\pi k}{15}n}$ —(2)

Comparing (1) & (2), we get

$\boxed{C_3 = C_5 = C_{10} = C_{12} = 1.}$ other C_k's are zero

$y(n) = x(n-2) \Rightarrow c_k^y = c_k^x e^{-j\frac{2\pi k n_0}{15}}$, $n_0 = 2$

$c_3^y = 1 \cdot e^{-j\frac{2\pi 6}{15}} = e^{-j\frac{2\pi 2}{5}} = \boxed{e^{-j\frac{4\pi}{5}}}$

$c_{12}^y = (c_3^y)^* = \boxed{e^{j\frac{4\pi}{5}}}$ because $y(n)$ is real.

$c_5^y = 1 \cdot e^{-j\frac{2\pi \cancel{10}\,2}{\cancel{15}\,3}} = \boxed{e^{-j\frac{4\pi}{3}}}$

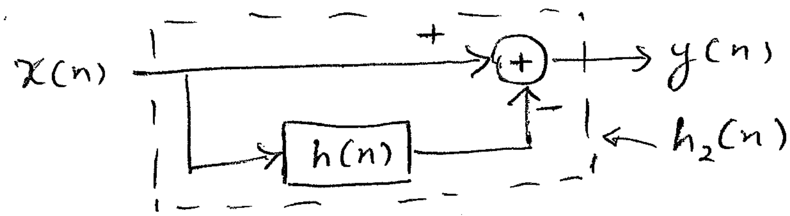
$\boxed{\text{other } C_k\text{'s are zero}}$

$c_{10}^y = (c_5^y)^* = \boxed{e^{j\frac{4\pi}{3}}}$

5. *Let h(n) be the impulse response of an ideal low pass filter with a cutoff frequency of $\pi/4$ radians/sample. Determine and sketch the spectrum of the overall system in each of the following cases. In each case, state whether it is low pass, band pass, band stop or high pass.*

(a) (10 pts) $h_1(n) = \begin{cases} h(n), & \text{for } n \text{ even} \\ 0, & \text{for } n \text{ odd} \end{cases}$

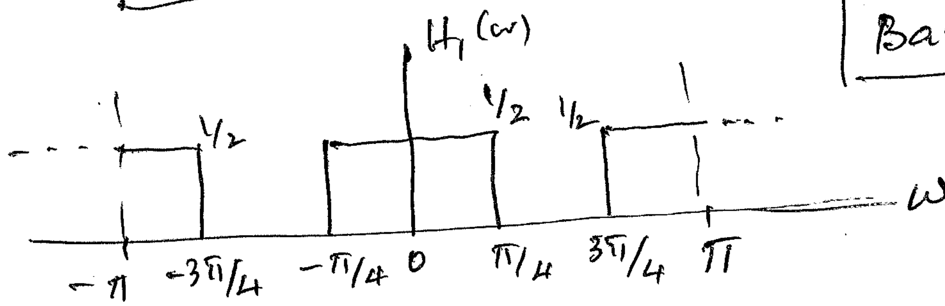
(b) (10 points) The system $h_2(n)$ as shown below.



a.

$h_1(n) = \frac{1}{2}\left[h(n) + (-1)^n h(n)\right]$ (why?)

$\Rightarrow \boxed{H_1(\omega) = \frac{1}{2}\left[H(\omega) + H(\omega - \pi)\right]}$

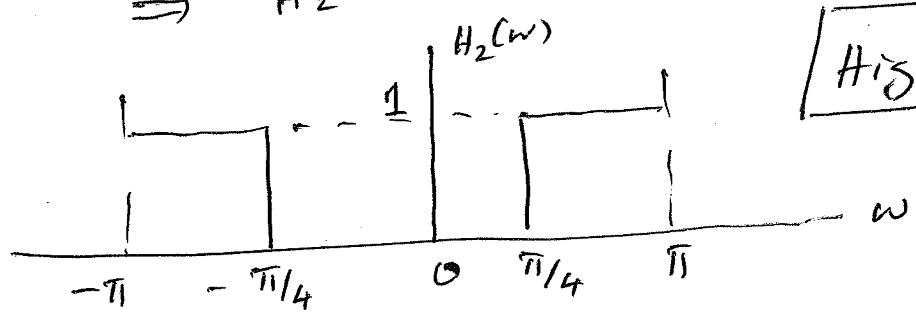


$\boxed{\text{Band Stop}}$

b.

$h_2(n) = \delta(n) - h(n)$

$\Rightarrow H_2(\omega) = 1 - H(\omega)$.



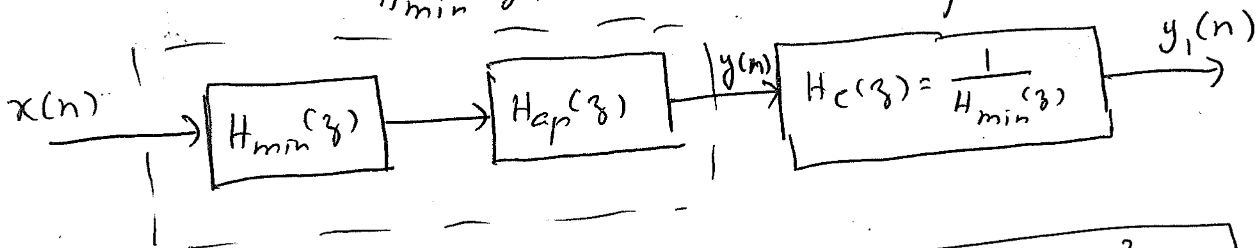
$\boxed{\text{High Pass}}$

Bonus Question (10 points)

Express the following causal, stable non-minimum phase system as a cascade of minimum phase and all pass systems (with a magnitude of unity for all frequencies), clearly identifying each. (5 pts) Determine the transfer function of the stable magnitude compensator; sketch its pole-zero diagram and state its ROC.(5 pts)

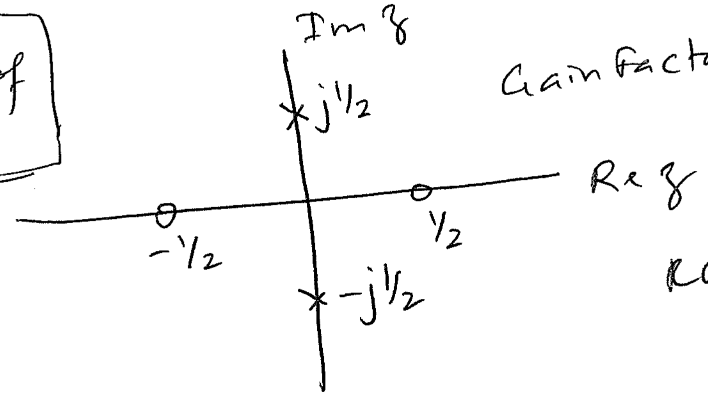
$$H(z) = \frac{(1+4z^{-2})}{(1-0.25z^{-2})}$$

$$H(z) = \underbrace{\left[\frac{(z^{-2}+4)}{(1-\frac{1}{4}z^{-2})}\right]}_{\text{Min. Phase } H_{min}(z)} \underbrace{\left[\frac{1+4z^{-2}}{z^{-2}+4}\right]}_{\text{All Pass } H_{ap}(z)}$$



$$\boxed{H_c(z) = \frac{1}{H_{min}(z)} = \frac{(1-\frac{1}{4}z^{-2})}{(z^{-2}+4)} = \frac{1}{4}\frac{(1-\frac{1}{4}z^{-2})}{(1+\frac{1}{4}z^{-2})}}$$

Pole-zero Plot of $H_c(z)$.



Gain factor = $\frac{1}{4}$.

ROC: $|z| > 1/2$.