SOLUTION

EE 6360 Fall 07 Quiz 3 **Name:**

1. (50 points)
   (a) Determine the Discrete Fourier Series coefficients of the periodic sequence

   x (n) = 2 cos [(2π/3) n] + 2 cos [(2π/5) n]

   (b) Determine the Discrete Fourier Series Coefficients of the periodic sequence

   y (n) = 2 cos [(2π/3) (n-2)] + 2 cos [(2π/5)(n-2)]

a $\boxed{N=15}$

$$x(n) = \left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right] + \left[e^{j\frac{2\pi}{5}n} + e^{-j\frac{2\pi}{5}n}\right]$$

But $e^{j\frac{2\pi}{3}n} = e^{j\frac{2\pi 5}{15}n}$, $e^{-j\frac{2\pi}{3}n} = e^{j\frac{2\pi 10}{15}n}$

$e^{j\frac{2\pi}{5}n} = e^{j\frac{2\pi 3}{15}n}$, $e^{-j\frac{2\pi}{5}n} = e^{j\frac{2\pi 12}{15}n}$

$$x(n) = e^{j\frac{2\pi(3)n}{15}} + e^{j\frac{2\pi(5)n}{15}} + e^{j\frac{2\pi(10)n}{15}} + e^{j\frac{2\pi}{15}(12)n} \quad — (1)$$

But $x(n) = \sum_{n=0}^{14} c_k e^{j\frac{2\pi k}{15}n}$ — (2)

Comparing (1) & (2), we get

$$\boxed{C_3 = C_5 = C_{10} = C_{12} = 1. \quad \text{other } C_k\text{'s are zero}}$$

b. $y(n) = x(n-2) \implies C_k^y = C_k^x e^{-j\frac{2\pi k n_0}{15}}$, $n_0 = 2$

$C_3^y = 1 \cdot e^{-j\frac{2\pi . 6}{15}} = e^{-j\frac{2\pi 2}{5}} = \boxed{e^{-j\frac{4\pi}{5}}}$

$C_{12}^y = (C_3^y)^* = \boxed{e^{j\frac{4\pi}{5}}}$ because $y(n)$ is real.

$C_5^y = 1 \cdot e^{-j\frac{2\pi}{\cancel{15}\,3}\cancel{10}\,2} = \boxed{e^{-j\frac{4\pi}{3}}}$

$C_{10}^y = (C_5^y)^* = \boxed{e^{j\frac{4\pi}{3}}}$

$\boxed{\text{other } C_k\text{'s are zero}}$

2. (50 points)
Let

$$x_1(n) = \{[\sin(\pi n/3)] / (\pi n)\} \quad \text{and} \quad x_2(n) = \{\sin[\pi(n-2)/2)] / [\pi(n-2)]\}$$

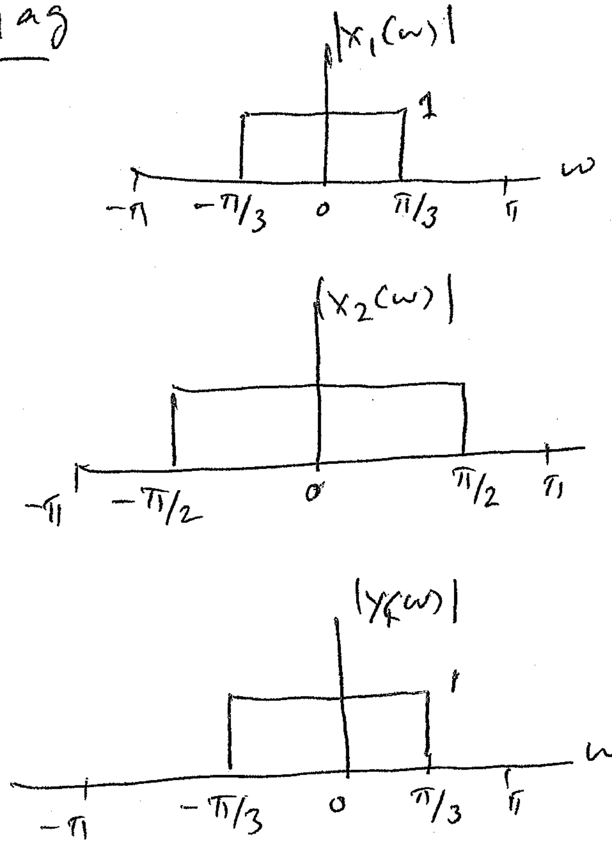
for $n \neq 0$ with

$$x_1(0) = 1/3 \text{ and } x_2(2) = ½.$$

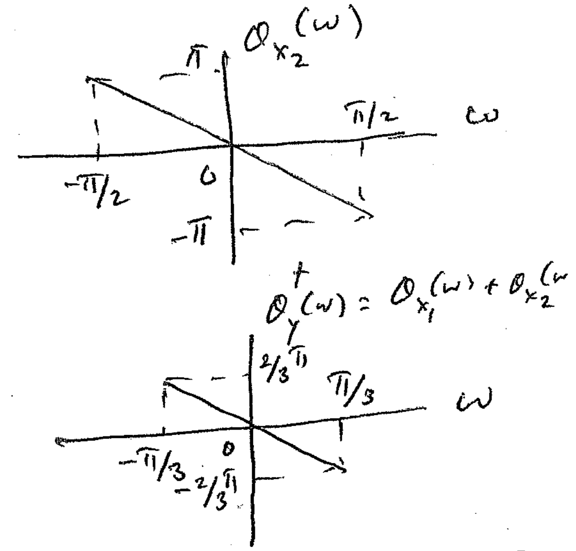
Determine $y(n) = x_1(n) * x_2(n)$ (* denotes convolution), and sketch $Y(\omega)$.

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega). \Rightarrow x(n) = \mathcal{F}^{-1}[Y(\omega)].$$

Mag



Phase



[† Wherever $Y(\omega) \neq 0$].

$$\boxed{y(n) = \frac{\sin\left[\frac{\pi}{3}(n-2)\right]}{\pi(n-2)}}$$