SOLUTION KEY

EE 6360 - 501
Exam 2
November 17, 2009

Please Print.

Last Name **First Name**

Instructions

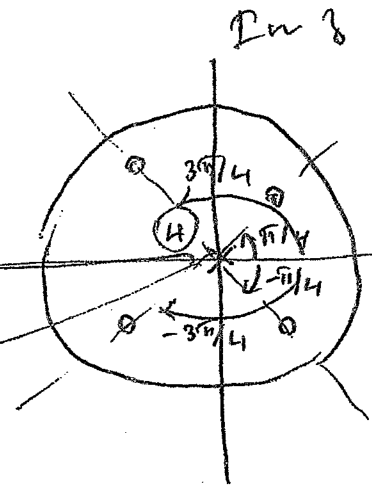
1. Examination Duration: 1 hour 15 minutes. If you arrive 15 minutes early, you can have that extra time.
2. ONE 8.5" x 11"sheet, with both sides of hand-written material, is allowed. No solved problems of any kind are allowed. Violation will result in a grade of zero for this exam, and other disciplinary actions may be taken.
3. One calculator is allowed.
4. Answer in the space/sheets provided.
5. **Two parts: Part A (answers only), 50 points; Part B (2 questions), 50 points. There is a bonus question at the end for 10 points.**
6. **<u>IN PART A, DO NOT WRITE ANYTHING OTHER THAN THE REQUIRED ANSWERS; IF YOU VIOLATE THIS INSTRUCTION, YOU WILL GET ZERO CREDIT FOR THAT PROBLEM.</u>**
7. **DO NOT RELY ON PARTIAL CREDIT, WHICH IS SOLELY AT THE DISCRETION OF THE INSTRUCTOR.**
8. Any copying or cheating will result in appropriate action as per university regulations.

Part A: (50 points, Answers only. Do your work on separate sheets, and do not submit them.)

1. (Counts as two problems)

Sketch the pole-zero diagrams, including the ROC, of the following systems with the given characteristics, which may be partial information. (The pole and zero location coordinates should be clearly given.). All the systems are causal LTI systems with real impulse responses.

a. $H_1(z)$ is minimum phase, and the impulse response is zero outside $0 \le n \le 4$; there are zeros at $z = \frac{1}{2} e^{j\pi/4}$, and at $z = \frac{1}{2} e^{j3\pi/4}$



zeros

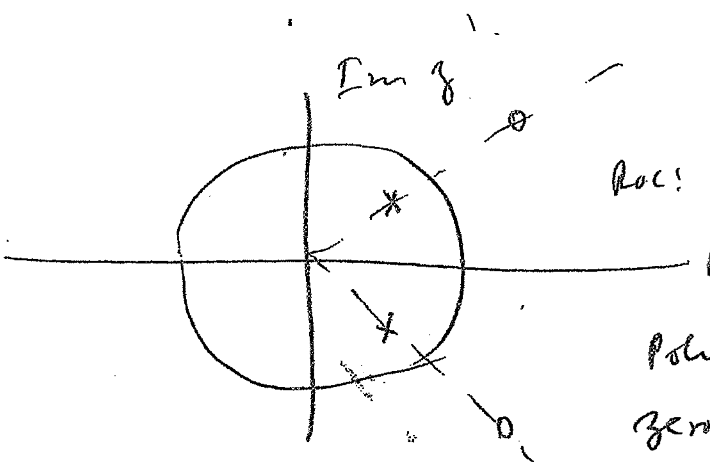
zeros at: $z = \frac{1}{2} e^{\pm j\frac{\pi}{4}}$

$z = \frac{1}{2} e^{\pm j 3\pi/4}$

These poles make H(z) causal.

ROC: Entire z-plane except z = 0.

b. $H_2(z)$ has a pole at $z = 0.8\, e^{j\pi/4}$ and $|H_2(\omega)| = 1$ for all ω.



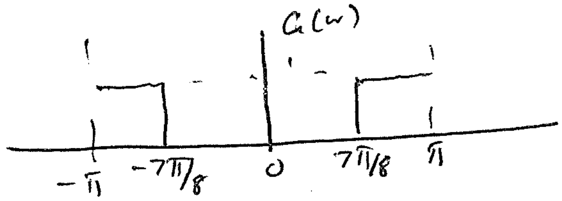
ROC: $|z| > 0.8$

Poles at $z = \frac{4}{5} e^{\pm j\pi/4}$

Zeros at : $z = \frac{5}{4} e^{\pm j\pi/4}$.

2. Let g(n) = - h(n) for <u>odd values of n</u>, and g(n) = h(n) for <u>even values of n</u>. Let

$H(\omega) = 1$, for $0 \le |\omega| \le \pi/8$, and zero otherwise. Sketch $G(\omega)$.



3. What is the impulse response, h (n), of the ideal multi-band filter with the frequency response (in the principal range)

$$H(\omega) = 1, \text{ for } \pi/8 \le |\omega| \le \pi/4 \text{ and } 3\pi/8 \le |\omega| \le \pi/2. \quad \text{It is zero otherwise.}$$

$$h(n) = 2\,\frac{\sin(\frac{\pi}{16}n)}{\pi n} \cdot \left[\cos\left(\frac{3\pi}{16}n\right) + \cos\left(\frac{7\pi}{16}n\right)\right]$$

$$= 4\,\frac{\sin(\frac{\pi}{16}n)}{\pi n} \cdot \cos\left(\frac{5\pi}{16}n\right)\cos\left(\frac{\pi}{8}n\right)$$

4. The periodic signal $x_p(n) = \sum_{k=-\infty}^{k=\infty} x(n-k4)$, with $x(n) = \{1\ 2\ 3\}$ is input to the LTI system given by

$$H(z) = [1 - z^{-12}].$$

Write the output sequence y (n).

$$y(n) = \{\ldots\ \underset{\uparrow}{0},\ 0,\ 0,\ 0,\ 0\ \ldots\}$$

5. A real-valued periodic sequence, x(n), of period 4 samples has the following property:

$$\sum_{k=-6}^{k=5} x(k) = 12.$$

What is the value of the Fourier series coefficient, c0?

$$c_0 = 1$$

6. The DTFT of a sequence is given by X (ω) = 3 cos ^{5}(3 ω). Evaluate the sum

$$\sum_{k=-\infty}^{k=\infty} (-1)^k x(k) = \boxed{-3}$$

7. What is the sequence x (n) that corresponds to the DTFT, X (ω) = cos (2 ω) - j sin (ω)?

$$\left\{ \frac{1}{2} \quad -\frac{1}{2} \quad \underset{\uparrow}{0} \quad \frac{1}{2} \quad \frac{1}{2} \right\}$$

8. A periodic signal, x (n) is created by repeating the prototype signal xp (n) = {**3** 2 1} every 5 samples. What is the z-transform of the signal x_c (n) = x (n) u (n)? What is its region of convergence? [Bold denotes n= 0.]

$$X(z) = \frac{(3 + 2z^{-1} + z^{-2})}{(1 - z^{-5})}$$

9. Could the z-transforms shown below be the z-transform of a causal sequence? Give reason ~~for eachrc~~

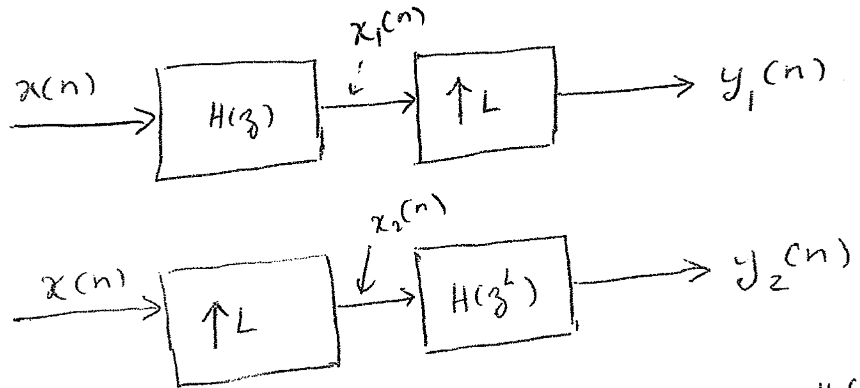
$$H_1(z) = \frac{(z - 1/4)^6}{(z - 1/2)^5}$$

Cannot be Causal. $H_1(z) = \infty$ at $z = \infty$ showing that $z = \infty$ is a pole. A causal system cannot have a pole at $|z| = \infty$.

Part B: (2 questions, 25 points each)

B1 (Two parts)

(1) For the two systems shown below, given the same input sequence, will the outputs be different or the same? You must prove your answer.



$X_1(\omega) = H(\omega)\,X(\omega)$

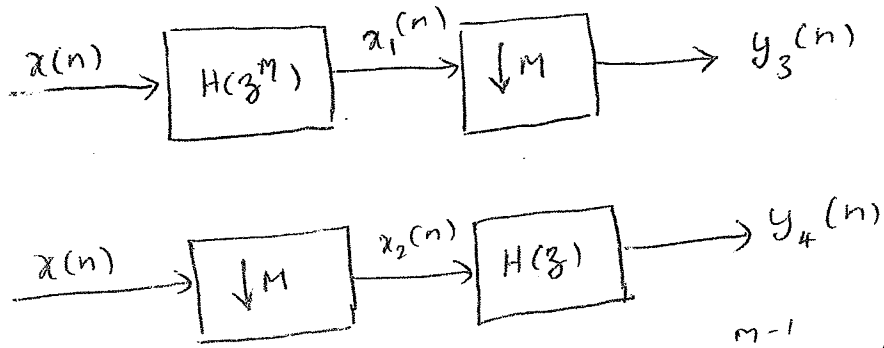
$X_2(\omega) = X(\omega L)$

$Y_1(\omega) = X_1(L\omega) = H(L\omega)\,X(L\omega)$ — (1)

$Y_2(\omega) = H(L\omega)\,X_2(\omega) = H(L\omega)\,X(L\omega)$ — (2)

$\boxed{y_1(n) = y_2(n) \text{ for the same input } x(n)}$

(2) For the two systems shown below, given the same input sequence, will the outputs be different or the same? You must prove your answer.



$$X_1(\omega) = X(\omega)\,H(M\omega), \quad Y_3(\omega) = \frac{1}{M}\sum_{k=0}^{M-1} X_1\left(\frac{\omega - k2\pi}{M}\right)$$

$$= \frac{1}{M}\sum_{k=0}^{M-1} X\left(\frac{\omega - k2\pi}{M}\right) . \overset{H(\omega)}{\cancel{H(\omega - k2\pi)}}$$

$$= \left[\frac{1}{M}\sum_{k=0}^{M-1} X\left(\frac{\omega - k2\pi}{M}\right)\right] H(\omega) \quad \text{— (1)}$$

$$X_2(\omega) = \frac{1}{M}\sum_{k=0}^{M-1} X\left(\frac{\omega - 2\pi k}{M}\right), \quad Y_4(\omega) = H(\omega)\cdot X_2(\omega)$$

$$= \left[\frac{1}{M} \cdot \sum_{k=0}^{M-1} X\left(\frac{\omega - 2\pi k}{M}\right)\right] H(\omega) \quad \text{— (2)}$$

$\Longrightarrow$ $\boxed{y_3(n) = y_4(n) \text{ for the same input } x(n)}$.

B2

A periodic signal $x_p(n) = \sum_{m=-\infty}^{\infty} x(n-4m)$ is input to a digital filter with the transfer function H(z).

Determine the output y (n) of the digital filter with the following x (n) and H (z).

$$x(n) = \{3 \;\; 1 \;\; -1 \;\; 1\} \qquad H(z) = G\,\frac{(1+z^{-1})}{(1+0.8\,z^{-2})}, \text{ with } G = 1/(5\sqrt{2})$$

$x_p(n)$ is periodic with $N = 4$ samples as fund. period

$\Rightarrow$ Frequencies in $x_p(n)$ are DC, $\frac{2\pi}{4}$, $\frac{2\pi \cdot 2}{4}$, $\frac{2\pi \cdot 3}{4}$ rad/sample

or $\omega = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ (harmonics)

$$x_p(n) \longrightarrow \boxed{H(z)} \longrightarrow y(n)$$

$$y(n) = \sum_{n=0}^{N-1} c_k \cdot H\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k}{N}n} \qquad \text{—— (1)}$$

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$$

$$c_0 = \frac{1}{4}\cdot\left[x(0) + x(1) + x(2) + x(3)\right] = \boxed{1}$$

$$c_1 = \frac{1}{4}\left[x(0) + x(1)\cdot e^{-j\frac{2\pi}{4}1\cdot 1} + x(2)\, e^{-j\frac{2\pi\cdot 1}{4}\cdot 2} + x(3)\, e^{-j\frac{2\pi\cdot 1}{4}\cdot 3}\right]$$

$$= \frac{1}{4}\left[3 + 1\cdot e^{-j\pi/2} - 1\, e^{-j\pi} + 1\cdot e^{-j\frac{3\pi}{2}}\right]$$

$$= \frac{1}{4}\left[3 - \cancel{j} + 1 + \cancel{j}\right] = \boxed{1}$$

$$c_2 = \frac{1}{4}\left[x(0) + x(1)\, e^{-j\frac{2\pi\cdot 2}{4}1} + x(2)\, e^{-j\frac{2\pi\cdot 2}{4}2} + x(3)\, e^{-j\frac{2\pi\cdot 2}{4}\cdot 3}\right]$$

$$= \frac{1}{4}\left[3 + e^{-j\pi} - e^{-j2\pi} + e^{-j3\pi}\right] = \boxed{0}$$

$$c_3 = c_1^{*} = \boxed{1} \quad \text{(why?)}$$

$$H(z) = \left(\frac{1}{5\sqrt{2}}\right) \cdot \frac{(1+z^{-1})}{(1+0.8z^{-2})}$$

$$H(w) = \frac{1}{5\sqrt{2}} \frac{(1+e^{-jw})}{(1+0.8\,e^{-j2w})}$$

$$H(0) = \frac{1}{5\sqrt{2}} \frac{(1+1)}{(1+0.8)} = \frac{1}{\cancel{5}\sqrt{2}} \cdot \frac{2}{9/\cancel{5}} = \boxed{\frac{\sqrt{2}}{9}}$$

$$H\left(\frac{\pi}{2}\right) = \frac{1}{5\sqrt{2}} \frac{(1+e^{-j\pi/2})}{(1+0.8\,e^{-j\pi})} = \frac{1}{\cancel{5}\sqrt{2}} \cdot \frac{(1-j)}{(1/\cancel{5})} = \frac{1}{\sqrt{2}}(1-j)$$

$$= \boxed{1\,e^{-j\pi/4}}$$

$H(\pi)$ is not needed because $c_2 = 0$

$$H\left(\frac{3\pi}{2}\right) = H^*\left(\frac{\pi}{2}\right) = 1\,e^{j\pi/4}$$

Substituting the values in ①, we have

$$y(n) = 1 \cdot \frac{\sqrt{2}}{9} + 1 \cdot e^{-j\pi/4} \cdot e^{j\frac{\pi}{2}n} + 1 \cdot e^{-j\pi/4} e^{j\frac{3\pi}{2}n}$$

$$= \boxed{\frac{\sqrt{2}}{9} + 2\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)}$$

Bonus question (10 points)

A causal, LTI system is described by the following LCCDE:

$$y(n) - 0.5\, y(n-1) = x(n), \quad n \geq 0 \quad \text{with } y(-1) = 1, \text{ and } x(n) = (1/3)^n u(n). \qquad (1)$$

Determine the zero state and zero input responses. Identify the homogenous and particular solution.

1-sided z-transforming (1), we have

$$Y(z) - \tfrac{1}{2}\left[z^{-1}Y(z) + y(-1)\right] = X(z)$$

$$Y(z) = \underbrace{\frac{X(z)}{(1-\frac{1}{2}z^{-1})}}_{Y_{ZS}(z)} + \underbrace{\frac{\frac{1}{2}y(-1)}{(1-\frac{1}{2}z^{-1})}}_{Y_{ZI}(z)}$$

Zero input Response:

$$\boxed{y_{zi}(n) = Z^{-1}\left[\frac{\frac{1}{2}y(-1)}{(1-\frac{1}{2}z^{-1})}\right] = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^n u(n)} \quad -(2)$$

Zero state Response:

$$y_{zs}(n) = Z^{-1}\left[\frac{X(z)}{1-\frac{1}{2}z^{-1}}\right] = Z^{-1}\left\{\frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}\right\}$$

$$= Z^{-1}\left[\frac{3}{(1-\frac{1}{2}z^{-1})} + \frac{-2}{(1-\frac{1}{3}z^{-1})}\right]$$

$$\boxed{y_{zs}(n) = 3\left(\tfrac{1}{2}\right)^n u(n) - 2\cdot\left(\tfrac{1}{3}\right)^n u(n)} \quad -(3)$$

Total soln.:

$$y(n) = y_{zs}(n) + y_{zi}(n)$$

$$= \tfrac{7}{2}\cdot\left(\tfrac{1}{2}\right)^n u(n) - 2\left(\tfrac{1}{3}\right)^n u(n).$$

mimics the input

$$\Rightarrow \boxed{y_p(n) = -2\left(\tfrac{1}{3}\right)^n u(n).}$$

$$\boxed{y_h(n) = \tfrac{7}{2}\left(\tfrac{1}{2}\right)^n u(n)}.$$