Solution

Spring 08 EE 6360 Quiz 3 Name:

1. Determine the autocorrelation function of x (n) = u(n) – u(n-4) using z-transform. (30 points)

$$x(n) = \{\underset{\uparrow}{1}\ 1\ 1\ 1\}$$

$$X(z) = 1 + z^{-1} + z^{-2} + z^{-3}, \quad |z| > 0,$$

$$\therefore x(-n) = \{1\ 1\ 1\ \underset{\uparrow}{1}\} \longleftrightarrow X(z^{-1}) = z^3 + z^2 + z + 1, \quad |z| < \infty.$$

$$r_{xx}(l) = x(l) * x(-l) \longleftrightarrow X(z) \cdot X(z^{-1}), \quad \text{ROC: } 0 < |z| < \infty$$

$$X(z)X(z^{-1}) = (1 + z^{-1} + z^{-2} + z^{-3})(1 + z + z^2 + z^3)$$

$$\begin{aligned} = \; & z^3 + z^2 + z + 1 \\ & z^2 + z + 1 + z^{-1} \\ & z + 1 + z^{-1} + z^{-2} \\ & 1 + z^{-1} + z^{-2} + z^{-3} \quad \text{Add} \end{aligned}$$

$$z^3 + 2z^2 + 3z + 4 + 3z^{-1} + 2z^{-2} + z^{-3}$$

$$r_{xx}(l) = Z^{-1}[X(z)X(z^{-1})]$$

$$= \boxed{\{1\ 2\ 3\ \underset{\uparrow}{4}\ 3\ 2\ 1\}}$$

2, **(20 points)**
Without explicitly solving for X(z), find the region of convergence of the z – transform of each of the following sequences, and determine whether a discrete time Fourier Transform is guaranteed.

1) $x(n) = \{(1/2)^n + (3/4))^n\} u(n-10)$

$|z| > 3/4$.

DTFT is guaranteed because the unit circle is in the ROC.

2) $x(n) = 1, \ -10 \le n \le 5$; $x(n) = 0$ otherwise.

$0 < |z| < \infty$

ROC is the entire z-plane, except $z=0$ & $|z|=\infty$

DTFT is guaranteed. because the unit circle is in the ROC.

3) $x(n) = u(n+10) - u(n+5)$

$|z| < \infty$

ROC is the entire z-plane except $|z| = \infty$.

DTFT is guaranteed because the unit circle is in the ROC

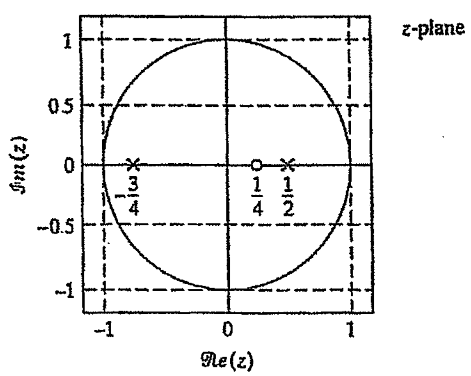
4) $x(n) = \{(2)^n u(-n)$

$|z| < 2$

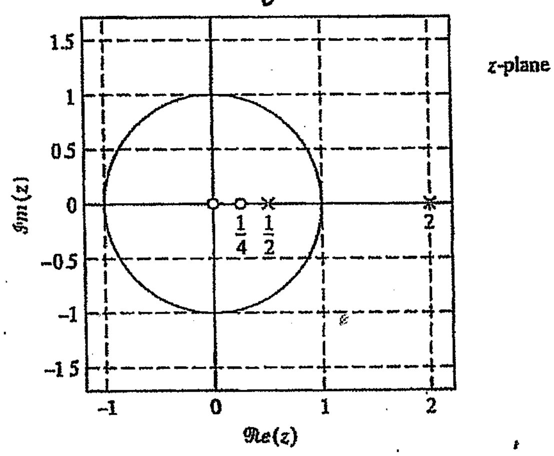
DTFT is guaranteed. because the unit circle is in the ROC.

3. The signal y (n) is the output of an LTI system with impulse response h (n) for a given stable, input sequence x(n). Throughout the problem, assume that y (n) is stable. The pole-zero configurations of X (z) and Y (z) are shown below. (a) What is the ROC of Y (z)? (b) Is y (n) right-sided, left-sided or two-sided? (c) What is the ROC of X (z)? (d) Is x (n) a causal sequence? (e) Draw the pole-zero plot of H(z) and specify its ROC. (f) Is h (n) causal, anti-causal or two-sided? [Note: Label your answers.] (50 pts.

X(z)



Y(z)



Stability implies that ROC includes unit circle.

(a) ROC of Y(z) : $\boxed{\frac{1}{2} < |z| < 2}$

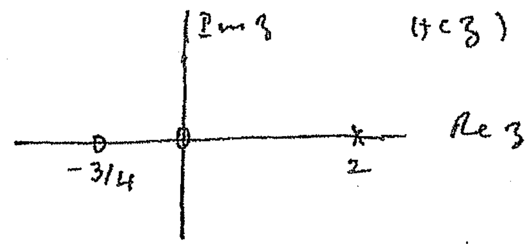
(b) y(n) is $\boxed{\text{two-sided.}}$

(c) ROC of X(z) : $\boxed{|z| > \frac{3}{4}}$

(d) x(n) is $\boxed{\text{Causal}}$.

(e) $$H(z) = \frac{Y(z)}{X(z)} = \left[\frac{\cancel{(1-\frac{1}{4}z^{-1})}\, z}{\cancel{(1-\frac{1}{2}z^{-1})}(1-2z^{-1})}\right]\left[\frac{(1+\frac{3}{4}z^{-1})\cancel{(1-\frac{1}{2}z^{-1})}}{\cancel{(1-\frac{1}{4}z^{-1})}}\right]$$

$$= \frac{(1+\frac{3}{4}z^{-1})\, z}{(1-2z^{-1})}$$



Valid ROC : $\boxed{|z| < 2}$

(f) h(n) is $\boxed{\text{anti Causal}}$