Web Appendix to Brandt and Freeman's (2009) "Modeling Macro-political Dynamics" *Political Analysis*

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Bayesian Structural VAR specification and estimation

Posterior for the model

The structural model can be transformed into a multivariate regression by defining A_0 as the contemporaneous correlations of the series and A_+ as a matrix of the coefficients on the lagged variables by

$$YA_0 + XA_+ = E, \tag{1}$$

where Y is $T \times m$, A_0 is $m \times m$, X is $T \times (mp+1)$, A_+ is $(mp+1) \times m$ and E is $T \times m$. Here we have placed the constant as the last element in the respective matrices. Note that the columns of the coefficient matrices correspond to the equations.

To derive the Bayesian estimator for this structural VAR model, we need the conditional likelihood function for its normally distributed residuals:

$$\begin{aligned} L(Y|A) &\propto |A_0|^T \exp\left[-0.5tr(ZA_+)'(ZA_+)\right] & (2)\\ &\propto |A_0|^T \exp\left[-0.5a_+'(I \otimes Z'Z)a_+\right] & (3) \end{aligned}$$

where $tr()$ is the trace operator. This is a standard multivariate normal likelihood equation.

The posterior for the model coefficients is formed by combining the likelihood function with the prior:

$$\begin{aligned} q(A) &\propto L(Y|A)\pi(a_0)\phi(\widetilde{a_+}, \Psi) && (4) \\ &\propto \pi(a_0)|A_0|^T|\Psi|^{-0.5} \times \exp[-0.5(a_0'(I \otimes Y'Y)a_0 \\ &\quad -2a_+'(I \otimes X'Y)a_0 + a_+'(I \otimes X'X)a_+ + \widetilde{a_+}'\Psi\widetilde{a_+})]. && (5) \end{aligned}$$

Scaling the prior covariance of the parameters

To see how the hyperparameters in Tables 2 and 4 of the paper work to set the prior scale of A_+, remember that $V(A_+|A_0) = \Psi$ is the prior covariance matrix for $\widetilde{a_+}$. Each element of Ψ then corresponds to the variance of the VAR parameters. The variance of each of these coefficients has the form

$$\psi_{\ell,j,i} = \left(\frac{\lambda_0\lambda_1}{\sigma_j \ell^{\lambda_3}}\right)^2, \tag{6}$$

for the element of Ψ corresponding to the ℓ^{th} lag of variable j in equation i. The overall coefficient covariances are scaled by the value of the error variances from m univariate AR(p) OLS regressions of each variable on its own lag values, σ_j^2 for $j = 1, 2, \ldots, m$.[1] The parameter λ_0 sets an overall tightness across elements of the prior on $\Sigma = A_0^{-1'} A_0^{-1}$, which relates the reduced form error covariance Σ to the contemporaneous structural relationships in A_0. As λ_0 approaches 1, the conditional prior variance of the parameters is the same as in the sample residual covariance matrix, while smaller values imply a tighter overall prior. The hyperparameter λ_1 controls the tightness of the beliefs about the random walk prior or the standard deviation of the coefficients on first lags (since $\ell^{\lambda_3} = 1$ in this case). The ℓ^{λ_3} term allows the variance of the coefficients on higher order lags to shrink as the lag length increases. The constant in the model has a separate prior variance of $(\lambda_0\lambda_4)^2$. Any exogenous variables can be given a separate prior variance proportionate to a parameter λ_5 so that the prior variance on the coefficients of any exogenous variable is $(\lambda_0\lambda_5)^2$. deterministic variable coefficients should be set tighter than the prior for the intercept or, $\lambda_5 < \lambda_4$. Otherwise, the exogenous variables will dominate the variation in the endogenous variables.

[1]This is the only use of the sample data in the specification of the prior. The only reason the data are used in this way is so the scale of the prior covariance is proportionate to the sample data.