UNINTENDED CONSEQUENCES OF PROMOTIONS: SHOULD MANAGERS WORRY ABOUT CONSUMER STOCKPILING?

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ABSTRACT

Increase in sales due to promotions could come at the expense of competitors; such sales come from consumers who have relatively weak brand preferences. Increased sales from consumers with strong brand preferences are likely to be at the expense of the promoted brand. In other words, brand loyal consumers can take advantage of promotions to stockpile for future consumption. Thus, loyal consumers who would be otherwise willing to buy at high prices can strategically stockpile at low prices. What is its impact on firms’ profits? How should firms adapt to consumer stockpiling? To answer these questions we model a duopoly competing for loyal and switching consumers.

In contrast to extant finding that stockpiling by switching consumers does not affect firms’ profitability, we find that stockpiling by loyal consumers indeed reduces firms’ long-run profits. We also find that even when stockpiling may induce higher consumption, it reduces but does not eliminate losses. Furthermore, we establish an upper bound on the loss due to loyal consumers’ stockpiling. Surprisingly, we find that it amounts to a relatively small percentage of profits. We also obtain a novel finding on mixed strategies that firms’ equilibrium pricing distributions can have mass points in the interior of the support. Our results also offer several counter-intuitive insights of relevance to managers.

Keywords: Game Theory, Consumer Stockpiling, Promotional Strategies, Loyal Consumers, Interior Mass Point, Pricing Distribution, Pricing
1. Introduction

One goal of promotions by packaged goods manufacturers is to appeal to price sensitive consumers who might buy a competing brand absent promotions. But promotions may have an unintended consequence that could have negative fallout. Consumers may stockpile and purchase for future consumption when brands are promoted. If loyal consumers who shun competing brands stockpile their favorite brand opportunistically for later consumption, then future sales, potentially at higher prices, are cannibalized. Thus, when promoting to attract price sensitive (switching) consumers firms should internalize this possible downside. Gangwar, Kumar and Rao (2014) explored the effect of stockpiling by price sensitive consumers on firms’ profits and found that it does not reduce firms’ profits. Would that result hold if loyal consumers stockpile?

We could ask: is it important to consider stockpiling by loyal consumers? Recent empirical evidence suggests that brand loyal consumers are more likely to engage in stockpiling in response to promotions than switching consumers. The intuition behind this finding is that loyal consumers derive relatively more benefit from stockpiling during a promotion because they do not intend to avail of future promotions by competing brands. In contrast, switchers are not committed to any particular brand and hence can take advantage of promotions offered by any brand making stockpiling less attractive. For example, Coke lovers are likely to stockpile if Coke offers a lower price but cola lovers can take advantage of promotions not only by Coke but also those offered by Pepsi and other competing brands of cola. Chan, Narasimhan and Zhang (2008) found that brand loyal consumers respond to promotions by stockpiling while switchers merely change brands and do not stockpile. Sun, Neslin and Srinivasan (2003) have also found similar results. These recent studies, emphasizing modeling innovations, have findings that are consistent with the earlier evidence in Krishnamurthi and Raj (1991) that loyal consumers, though less price sensitive in the brand choice decision, are more sensitive to price in the quantity decision. Neslin, Henderson and Quelch (1985) found “loyal purchasers in the coffee market have learned to change their purchase patterns in order to take advantage of promotions for their preferred brand.” In summary, empirical researchers have found loyal consumers to be more likely than switchers to stockpile due to promotions.

If it is conceded that a model of stockpiling by loyal consumers is worth exploring we can then ask what is the effect of such behavior on firms’ pricing decisions and profits? On the one hand, stockpiling by loyal consumers in a given period leaves firms with a smaller stock of loyal consumers in the following period. This in turn makes firms compete for switchers without needlessly subsidizing too
many loyal consumers. On the other hand, this reduced leakage results only after incurring the expense of inducing loyal consumers to stockpile at reduced prices. Therefore, it is not apparent whether stockpiling by loyal consumers on promotions result in lower or higher profits compared to a situation of no stockpiling. It could also turn out that in a competitive setting, firms’ promotional strategies balance out these two effects so that there is no net effect on firms’ profits just as in Gangwar, Kumar and Rao (2014). So, the first question we address in this study is: does stockpiling by loyal consumers lower, raise or leave unchanged firms’ equilibrium profits? Second, if stockpiling by loyal consumers does affect firms’ profitability what is the nature and extent of it? Third, how are firms’ equilibrium strategies affected by stockpiling? Fourth, how should managers address practical issues arising from stockpiling by loyal consumers? Why is it that certain promotional prices appear more frequently in equilibrium? Finally, how would our results change if the very act of stockpiling leads to increase in consumption? Thus, our work contributes to extant understanding of the effects of competitive promotions on firms’ strategies and profits, and in doing so also offers managerial insights.

### 1.1 Literature Review

Price promotion is a well-studied topic in economics and marketing (Shilony 1977, Varian 1980, Narasimhan 1988, Raju, Srinivasan and Lal 1990, Rao 1991). This body of theoretical work assumes that some consumers respond to promotions by switching brands, and the firm that offers the lowest price sells to all switching consumers. The presence of such switching consumers introduces discontinuity in demand and that in turn leads to equilibrium prices in mixed strategies with realizations of low prices interpreted as promotions. Early empirical research (Chiang 1991, Chintagunta 1993, Bucklin, Gupta and Siddarth 1998, and Bell, Chiang and Padmanabhan 1999) has found substantial brand switching among consumers. In particular, researchers have found that a large fraction (about 75%) of the demand expansion due to promotions can be attributed to brand switching and the rest to purchase acceleration and increased purchase quantity. Recent work (Pauwels, Hanssens and Siddarth 2002, Heerde, Gupta and Wittink 2003, and Steenburgh 2007) that focuses on sales volume finds that in many instances only a third of the incremental unit sales in the promotional period can be attributed to brand switching. This suggests that there is substantial purchase acceleration and increase in purchase quantity, which cannot be ignored in deriving equilibrium pricing strategies. Therefore, in this paper we develop a model that explicitly accounts for both brand switching and consumer stockpiling behavior in characterizing firms’ equilibrium pricing strategies.

Our work should be seen in the context of prior exploration of consumer stockpiling. Salop and Stiglitz (1982) have shown that consumers’ forward looking behavior by itself may lead to promotions in
the form of mixed strategy even when consumer preferences are homogeneous. Bell, Iyer and Padmanabhan (2002) innovate on Salop and Stiglitz’s model of homogeneous consumers by making consumption state dependent, and so firms may induce stockpiling to increase consumption to get higher sales. Dudine, Hendel and Lizzeri (2006) provide an analysis of the role of commitment in a monopoly market with storable goods. They show that in contrast to the literature on Coase conjecture, absence of commitment leads to higher prices for storable goods. Hendel, Lizzeri and Roketskiy (2014) provide an alternative explanation for cyclical patterns in sales and prices and show that storability imposes a constraint on a monopolist's ability to extract surplus under non-linear pricing that leads to cyclical patterns. In our paper the focus is on competition for price sensitive brand switching consumers that leads to opportunistic buying by brand loyal consumers thus forcing the firm to forego profits. The question then is what a firm’s equilibrium pricing strategy should be to balance the forces of competing for switching consumers and taking into account opportunistic buying by loyal consumers. In this way we can offer managerial insights and guidelines.

Our work is closer to Hong, McAfee and Nayyar (2002) who use Varian's (1980) model to study consumer stockpiling by assuming that all switching (price sensitive) consumers stockpile at an exogenously specified threshold. Our work differs from this in two important ways. First, we explore stockpiling by loyal consumers. Second, we treat consumers’ stockpiling rule as endogenous. This is important since consumers’ stockpiling rule and firms’ pricing strategies are mutually dependent. For example, if firms’ equilibrium prices do not contain frequent deep promotions, consumers may find it profitable to stockpile at relatively higher prices. On the other hand, if deep promotions were frequent, incidence of stockpiling at relatively higher prices would be less. Firms’ equilibrium pricing strategies will depend on consumers’ stockpiling rule, as conceptualized in our work. Although there is extant work that makes consumer stockpiling rules endogenous our work focuses on different issues and so adds to existing research. For example, Guo and Villas-Boas (2007) consider a differentiated market in which consumer preferences for products are distributed over a Hotelling line. In their model consumers with strong brand preferences are more likely to stockpile. They establish an important result that when consumers have constant preferences over time they do not stockpile in equilibrium because of the strategic pricing behavior of firms. However, if consumer preferences can change over time firms have an incentive to induce stockpiling by consumers who prefer their product in the first period to offset potential switching by them due to changed preferences in period 2. The equilibrium pricing strategies in Guo and Vilas-Boas are in pure strategies given that they have employed the Hotelling model structure. Our model, on the other hand, leads to promotions in the form of mixed strategies because we focus on
consumer heterogeneity in price sensitivity. Some consumers are not price sensitive but are brand loyal while others are price sensitive and switch brands.

Gangwar, Kumar and Rao (2014) analyze a model where some price sensitive consumers stockpile by endogenously determining the stockpiling rule, and show why firms’ equilibrium profits do not suffer as a result of consumer stockpiling behavior relative to the case of no stockpiling as long as firms recognize it in determining their equilibrium promotional strategies. Intuitively this happens because all potential profits from switching consumers are competed away in equilibrium. Hence, stockpiling by switching consumers does not reduce firms’ profits in equilibrium.

Ailawadi et. al. (2007) found through simulations that stockpiling benefits would be substantial if stockpiling also increases consumption that offsets the negative aspects of stockpiling by loyal consumers. In an extension, we develop a theoretical model of promotions that allows for consumer stockpiling and stockpiling induced flexible consumption to examine these issues.

In summary, our paper adds to prior work in several ways. First, we incorporate stockpiling behavior by brand loyal consumers in deriving firms’ equilibrium pricing strategies while keeping the consumer stockpiling rules endogenous; this poses technical challenges, which we address successfully. In particular, not only do we find that equilibrium pricing strategies can have a hole in the interior of the support as seen in past work, but we also obtain a novel result of the existence of a mass point in the interior of the support. Second, we find that unlike in past work, long run stationary equilibrium profits in our model are reduced because of consumer stockpiling. What is important is that we obtain an upper bound on the losses due to stockpiling by loyal consumers. Finally, we show that even increased consumption induced by stockpiling does not improve profits for the firms compared to no stockpiling case.

The rest of the paper is organized as follows. In Section 2, we describe the consumer model and firms’ technology and then define the equilibrium we seek. In Section 3, we state consumer and firm’s decision problems and characterize consumers’ stockpiling behavior. In section 4, we analyze firms’ equilibrium pricing strategies. We then derive closed form expressions for firms’ profits conditional on consumer inventory and obtain both lower and upper bounds on the profits. We compare this with a benchmark case where consumers do not engage in stockpiling. We then extend our analysis to allow flexible consumption by stockpiling consumers. In Section 5, we discuss the relation of our work to managerial concerns and strategies. Finally, we conclude in Section 6.
2. Model

We study a market with two firms serving consumers over an infinite horizon. We first describe our model of consumers followed by our model of firms.

2.1 Consumers

In our model, there is a continuum of consumers. They each consume one unit of the product in every period. As in Narasimhan (1988) there are two types of consumers in our model, brand loyal and switchers: a proportion $\alpha$ of consumers in the market is loyal to each of the two brands. The remaining $(1-2\alpha)$ proportion of consumers consists of switching consumers who examine the prices of both brands and purchase the lower priced brand in each period. All consumers are assumed to have a reservation price of $r$. In other words, consumers’ maximum willingness to pay is $r$ per unit of the product.

Motivated by empirical findings that consumers who are brand-loyal are more likely (than switchers) to stockpile in response to price promotions we innovate with respect to Narasimhan (1988) by letting a fraction $\lambda$ of loyal consumers to stockpile for future consumption. Said differently, even though loyal consumers in our model need one unit of the product per period, they may buy more than one unit, or none, in some periods. Thus, in our model a fraction $\lambda$ of the loyal consumers of each brand may stockpile for the next period, if the price of their preferred brand is sufficiently low. We also assume that these consumers can stockpile for at most one period.\footnote{Niraj, Padmanabhan and Seetharaman (2008) and Ailawadi et. al. (2007) found that most of the purchases are concentrated at zero, one or two units. Moreover, allowing stockpiling for only one period keeps our exposition and analysis simple.} So inventory held by consumers can be 0 or 1. Finally, utility maximizing consumers discount the future using a discount factor $\delta_c$.

2.2 Firms

Firms in our model maximize expected profits discounted over an infinite horizon. Firms are assumed to discount the future using a discount factor $\delta_f$. They choose prices simultaneously in each period. Without loss of generality, we normalize marginal cost of both firms to zero, so per period profits are simply the product of demand and price. The switching consumers and the non-stockpiling loyal consumers demand one unit each period. However, the stockpiling loyal consumers demand can vary. In particular, if stockpiling loyal consumers have inventory, they demand zero unit at high prices and one unit at low prices and if they do not have inventory, they demand one unit at high prices and two units at low prices. Therefore, firms’ pricing strategies would depend on the state of stockpiling consumers’ inventory.
We should note that all pay-off relevant past information for firms in our model is captured by the inventory held by stockpiling consumers of each brand. Hence, we define a state variable 
\[ s = \{I_1, I_2\} \]
where \( I_i \) for \( i = 1, 2 \), is inventory of stockpiling consumers of brand \( i \). Since consumers in our model can stockpile for at most one period, \( I_i = 0 \) or 1. This leads to four possible states: \( s \in \{00, 01, 10, 11\} \). The state \( s=00 \) corresponds to the case in which stockpiling loyal consumers of neither firm have inventory. The state \( s=01 \) (\( s=10 \)) corresponds to the case where stockpiling loyal consumers of the focal (competing) firm do not have inventory but that of the competing (focal) firm have inventory of 1 unit. Finally, \( s=11 \) corresponds to the case where stockpiling loyal consumers of both firms have inventory.

2.3 Equilibrium

We invoke Markov Perfect Equilibrium (MPE) to characterize the firms’ optimal pricing strategies that maximize expected profits of the firms given consumers’ stockpiling decision. We focus on the expected profits in the stationary state. In other words, we derive firms’ equilibrium pricing strategies conditional on state recognizing that continuation profits of firms depend on the equilibrium strategies in steady state. Our equilibrium is sub-game perfect, ruling out non-markovian deviations by firms (Maskin and Tirole 1998 and 2001). Maskin and Tirole offered several reasons for invoking MPE as a desirable equilibrium concept. Moreover, Markov strategies often have more predictive power compared to strategies that rely on punishment because punishment strategies can have multiple equilibria. MPE has also been used in analyzing marketing strategies, for example, Villas-Boas (1993), Anderson and Kumar (2007), Gangwar, Kumar and Rao (2014). Finally, in our model the payoff-relevant state variable is low dimensional and so the model remains tractable.

3. Consumer Rule and Firm Decisions

There are four agents in our model: firms, switching consumers, non-stockpiling loyal consumers and stockpiling loyal consumers. Firms’ MPE strategies are defined as follows: firms take the current state as given and set prices simultaneously \( \sigma : s \in \{00, 01, 10, 11\} \rightarrow \mathbb{R}^+ \). We know from prior research that the presence of switching consumers results in a mixed strategy equilibrium, and firms

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1 In this paper, we use the state space notation using the focal firm's perspective. In other words, we follow the convention that first digit of the state space corresponds to current inventory position of stockpiling loyal consumers of the focal firm and second digit corresponds to that of the competing firm’s stockpiling loyal consumers. We use superscripts to denote the state and subscript to denote the firm.
charge prices over an interval such that expected profits are equal at all un-dominated prices. Therefore when we refer to firms’ pricing strategies we envisage mixed strategies. Switchers use the prices charged by the two firms to decide whether or not to purchase and which firm to purchase from, $\sigma_1 : \mathbb{R}^2 \rightarrow \{0,1\} \times \{i,3-i\}$; non-stockpiling loyal consumers use the price of the preferred firm to decide whether or not to purchase a unit $\sigma_2 : \mathbb{R} \rightarrow \{0,1\}$. The stockpiling loyal consumers’ purchase quantity decision in general may use the prices of both firms and their own inventory state to decide whether to buy 0, 1 or 2 units. It is helpful to consider the stockpiling decision rather than the quantity decision of the stockpiling loyal consumers. The consumer may buy 0 or 1 unit when her inventory is 1, and buy 0, 1 or 2 units when her inventory is zero. Thus, the quantity decision depends on her current inventory. The decision to buy for the future (stockpiling), on the other hand, does not depend on the current inventory position but on realized current prices and expectations of future prices. We can therefore define the strategy of stockpiling loyal consumers as stockpile or not $\sigma_3 : \mathbb{R}^3 \rightarrow \{\text{Don't stockpile, Stockpile}\}$.

3.1 Consumer Stockpiling Rule

How do stockpiling loyal consumers decide to stockpile? There are many ways to model the consumer stockpiling rule. Recall that firms need to know the inventory state among other things to enable them to formulate their strategy. One approach is to assume that like the firms in our model, consumers too have all information relevant to their stockpiling decision. For them the relevant information is prices of both brands and the equilibrium strategies of both firms. This can be thought of as a case of perfect information. We explicitly develop and analyze the perfect information model in the technical appendix. But first we take a different route. Why? We think it is useful, and important, to consider other approaches that endow consumers with less information. This can be thought of as invoking bounded rationality of consumers. For example, loyal consumers in our model derive no direct benefit from monitoring the price of the competing brand, which they never buy. Thus, an approach that assumes they know the price of only the brand they buy may make sense under some conditions, depending on the cost of monitoring prices. Marketing Scholars have long recognized that consumer knowledge of prices is far from perfect (Dickson and Sawyer, 1990) but nevertheless consumers can “recognize” prices, meaning they can tell whether an observed price is the one they have in mind (Vanhuele and Dreze, 2002). Moreover, consumers seem to have better price information when an item

4 The spirit of the need to think of carefully modeling the consumer is reflected in the following: "There are two kinds of people in the world: Johnny Von Neumann and the rest of us." Attributed to Eugene Wigner, a Nobel Prize winning physicist.
is on promotion meaning they can recognize low prices. (Le Boutillier, Le Boutillier and Neslin, 1994). Given the likelihood that consumers would have less precise information on prices of brands that they don’t buy, assuming that consumers know the pricing strategy of their brand but not that of the other brand has some appeal. In fact, because firms’ strategies are state dependent the cognitive cost of computing or inferring the strategies of firms could be high. Any model of consumer decision making should take into account these considerations. The challenge for us is to incorporate bounded rationality in an appropriate way (Simon 1972, Ellison 2006). We approach this challenge in two steps. We use a parsimonious representation of consumer decision making that does not assume consumers to act as powerful computers. Rather, we model them as recognizing the benefit of stockpiling when they encounter a low price on the brand they are loyal to. Accordingly, we simplify the loyal consumers stockpiling decision based only on the price of the brand they are loyal to: \( \sigma_j: \mathbb{R}_+ \rightarrow \{\text{Don't stockpile, Stockpile}\} \). We further model the information available to them to evaluate the benefit of stockpiling as statistics that they could develop based only on observing prices of the brand they buy (For a discussion of human inference using heuristics, see Geigerenzer and Goldstein, 1996). We use this consumer model to analyze MPE for firms’ pricing strategies. In the technical appendix we analyze the perfect information case and verify that our results remain robust even to that view of consumer decision making.

Recall that consumers know the price of their brand before making their purchase. We assume that stockpiling loyal consumers with bounded rationality have a threshold in mind such that they stockpile whenever the brand price is below the threshold and do not stockpile when it is above the threshold. Where does the threshold come from? We ensure that our model is closed under rational expectations thus providing a condition to solve for the threshold. Specifically, we assume consumers’ stockpiling rule is a best response to the pricing strategy of their preferred brand. We now turn to characterizing the threshold.

Imagine a brand loyal stockpiling consumer who encounters brand prices when she has inventory, \( I=1 \) (or does not have inventory, \( I=0 \)). Figure 1 depicts this pictorially, for either case. The prices allow the consumer to make inferences about two (equilibrium) price distributions. And so, in our model we assume that loyal stockpiling consumers know the price distributions conditional on their own inventory state \( I=0 \) or \( I=1 \). Denote the cumulative distribution function of prices conditional on loyal consumers inventory by \( G^0(p) \) and \( G^1(p) \); likewise denote the corresponding probability density
functions by $g^0(p)$ and $g^1(p)$. How can consumers use the knowledge of prices conditional on their inventory to characterize stockpiling threshold, $t$? In proposition 1, we derive the stockpiling rule that is the consumer’s best response under bounded rationality. Since consumers in general may use different thresholds depending on their level of inventory, we do not restrict them to a single threshold across states. However, it turns out that for consumers with bounded rationality in the way we have modeled the optimal rule uses a single threshold that is independent of their inventory state.

**Figure 1: Prices observed by consumer and their stockpiling decision**

**Proposition 1:** Stockpiling loyal consumers’ stockpile only if $p \leq t$ and do not stockpile if $p > t$, where the stockpiling threshold, $t$, is independent of the state and must satisfy:

$$t = \delta \left( G^0(t) \left( 2 \int_0^t pg^0(p) dp - \int_0^t pg^1(p) dp \right) + \int_0^t pg^1(p) dp - G^1(t) \right) + \frac{\int_0^t pg^1(p) dp}{1 + \delta G^0(t) - G^1(t)}$$

**Proof:** See Appendix.

Note that the threshold $t$ is independent of inventory state. So, regardless of state, the stockpiling decision only depends on the current price of the brand she is loyal to and $t$. The equation in proposition 1 contains an implicit solution for $t$. If consumers use threshold $t$, and they expect firms’ pricing strategies related to firms’ pricing strategies? Since firms’ strategies depend on the state characterized by the inventories of consumers of both firms, we can integrate out the firms’ mixed strategies suitably to obtain $G^0(p)$ and $G^1(p)$.

This result is also due to the fact that consumers in our model can only be at one of two levels of inventory. In general, if they could be at one of $n$ levels of inventory, they could use $n-1$ thresholds.
strategies will be such as to render their choice to be optimal under bounded rationality, then their expectations will indeed be fulfilled. Thus, our model is closed under rational expectations.

Before analyzing firms’ pricing decisions, in the next section, we describe how the state space evolves based on firms’ current prices and consumer stockpiling decision.

Table 1: State Transition under Different Conditions (for all current states)

<table>
<thead>
<tr>
<th>Focal firm, i’s price</th>
<th>Focal firm, i’s loyal consumer’s current period inventory state</th>
<th>Focal firm, i’s loyal consumer’s buying decision</th>
<th>Focal firm, i’s loyal consumer’s next period inventory state</th>
<th>Competing firm, 3-i’s price</th>
<th>Next period state s=(I_i, I_{3-i})</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>I_i = 0</td>
<td>buy 1 unit for current period</td>
<td></td>
<td>High</td>
<td>s=00</td>
</tr>
<tr>
<td></td>
<td>I_i = 1</td>
<td>do not buy, consume from inventory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_i = 0</td>
<td>buy 1 unit for current period</td>
<td>I_i = 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_i = 1</td>
<td>do not buy, consume from inventory</td>
<td></td>
<td>Low</td>
<td>s=01</td>
</tr>
<tr>
<td>Low</td>
<td>I_i = 0</td>
<td>buy 2 units for current and next period</td>
<td></td>
<td>High</td>
<td>s=10</td>
</tr>
<tr>
<td></td>
<td>I_i = 1</td>
<td>buy 1 unit for next period</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_i = 0</td>
<td>buy 2 units for current and next period</td>
<td>I_i = 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_i = 1</td>
<td>buy 1 unit for next period</td>
<td></td>
<td>Low</td>
<td>s=11</td>
</tr>
</tbody>
</table>

3.2 Consumer Inventory / State Evolution

To see how current period prices and behavior of customers determine the next period state consider the case when focal firm i’s price is high. In this case they would not stockpile, and so in the next period they have no inventory, I_i=0. Note that this transition is independent of current inventory. Now consider the case when the focal firm’s price is low enough and so its loyal consumers decide to stockpile. They will end up with an inventory of one unit, I_i=1, in the next period. Again, note that, irrespective of their initial inventory position when the firm offered a low price its stockpiling loyal consumers ended up with inventory, I_i=1, in the next period. Similarly, competing firm’s prices will
dictate the inventory position of its loyal consumers. All other consumers, the non-stockpiling loyal and switching consumers buy only one unit every period because they do not engage in stockpiling. Table 1 summarizes how current period prices and behavior of customers determine the next period state.

In each state, firms have two strategies: offer a low price, \( \leq t \), that would induce stockpiling or offer a high price, \( > t \). Both firms can offer low prices, or both can offer high prices, or the focal firm can offer a low price and the competing firm can offer a high price or vice versa. For example, if firms are in state \( s=00 \) in the current period they could end up in one of four possible states in the next period. If both firms charge high prices then the stockpiling loyal consumers purchase only one unit and firms will end up again in state \( s=00 \) in the next period. If the focal firm charges a high price and the competing firm offers a low price, stockpiling loyal consumers of the focal firm will buy only one unit while the stockpiling loyal consumers of the competing firm will buy two units and the next period state will be \( s=01 \). In contrast, if the focal firm’s current period price is low and the competing firm’s price is high we end up in state \( s=10 \). Finally, if both the firms offer low prices, the next period state will be \( s=11 \).

3.3 Demand Characterization

First note that no consumer is willing to pay more than the reservation price, hence we only characterize the demand function for prices below the reservation price, \( r \). Consider, for example, focal firm's demand, \( D_i^{00} \), in states \( s = 00 \) when stockpiling loyal consumers of the both firms have no inventory.

\[
D_i^{00}(p_i, p_j, t) = \alpha + \lambda \alpha \cdot 1^*(t - p_i) + \beta \cdot 1^*(p_j - p_i), \text{ where } (\text{indicator function}) \cdot 1^*(z) = \begin{cases} 
1 & z > 0 \\
0 & z < 0 
\end{cases}
\]

Note that since the focal firm’s stockpiling consumers have no inventory, all loyal consumers of proportion \( \alpha \) will buy at least one unit. Further, if the price of the focal firm is \( \leq t \), all stockpiling loyal consumers (a fraction \( \lambda \alpha \)) will buy an additional unit. In addition, if the focal firm’s price is less than the competitor’s price, switchers of size \( \beta \) will also buy one unit from the focal firm.

There are two cases, which require special consideration: (a) when both firms charge the same price, and (b) when the focal firm charges price equal to the stockpiling threshold. The first case arises only if both firms charge a certain price with positive probability. In that case, we assume that the switching segment buys from the focal firm with probability \( \phi_i \) and they buy from the competing firm...
with probability $1 - \phi_i.$ In the second case, $(p_i = t)$, stockpiling consumers of brand $i$ are indifferent between stockpiling and not stockpiling. When that happens we suppose that stockpiling loyal segment of firm $i$ decide to stockpile with probability $\eta_i$, $0 \leq \eta_i \leq 1$. Note that we treat $\eta_i$ as endogenous. As we will see in proposition 3, this turns out to be important, and it has force, in characterizing the equilibrium strategies. Equation (1) below represents the demand taking into account all possibilities including $p_i = p_j$ and $p_i = t$.

$$D_i^t(p_i, p_j, t) = \begin{cases} \kappa + \lambda \alpha_1 (t - p_i) + \beta_1 (p_j - p_i) & p_i \neq p_j, p_i \neq t \\ \kappa + \lambda \alpha (t - p_i) + \beta \phi & p_i = p_j, p_i \neq t \\ \kappa + \lambda \alpha \eta_i + \beta_1 (p_j - p_i) & p_i \neq p_j, p_i = t \\ \kappa + \lambda \alpha \eta_i + \beta \phi & p_i = p_j = t \end{cases}$$

(1)

where $\kappa = \begin{cases} \alpha & \text{if state } s \in \{00, 01\} \\ (1 - \lambda) \alpha & \text{if state } s \in \{10, 11\} \end{cases}$ and (indicator function) $1^t(z) = \begin{cases} 1 & z > 0 \\ 0 & z < 0 \end{cases}$

Notice that in equation (1) the first term, $\kappa$, represents a guaranteed demand from loyal consumers at any price in the support. In state $s \in \{10, 11\}$ stockpiling loyal consumers of the focal firm already have stock and do not necessarily have to buy for the current period, hence guaranteed demand is lower, note $\kappa = (1 - \lambda) \alpha$. Loyal consumers may buy one unit for future if the price is below the stockpiling threshold; the second term, $\lambda \alpha_1 (t - p_i)$ in (1) represents that at any price $p_i < t$, the focal firm will get one unit of additional demand from its stockpiling loyal consumers. Terms $\beta_1 (p_j - p_i)$ and $\beta \phi$ in (1) account for the fact that switching consumers buy the lower priced brand and in case of a tie $(p_i = p_j)$ buy from the focal firm with probability $\phi_i$. Finally $\lambda \alpha \eta_i$ term describes that when focal firm offers a price $(p_i)$ equal to the stockpiling threshold, $t$ its stockpiling loyal consumers are indifferent between buying and not buying and buy with probability $\eta_i$.

### 3.4 Firm’s Decision Problem

As noted earlier, we seek a Markov Perfect Equilibrium (MPE) in firms’ pricing strategies in each state $s \in \{00, 01, 10, 11\}$ that is symmetric across firms and periods. However, we want to highlight that symmetry is across firms and periods but not states. In fact, in state $s=01$ for example, the two firms

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7 In other words, we assume that when prices charged by the two firms are identical, switchers employ an exogenous cue to break ties and purchase from one of the two firms. This tie-breaking rule is assumed in the spirit that all switching consumers are homogenous and make similar decisions. It is in the class of sunspot equilibria.
will have different pricing strategies because of the different inventory positions of their stockpiling consumers. Firms need to identify their optimal pricing strategy given consumers’ stockpiling rule. Let \( F_i^s(p) \) denote the firm \( i \)'s equilibrium cumulative distribution of prices corresponding to state \( s \) and let \( V_i^* \) denote the infinite period expected payoff in state \( s \in \{00, 01, 10, 11\} \).

One important consideration from equation (1) that is relevant for our analysis is that in our model, discontinuities in a firm's demand function arise from two different sources. The first discontinuity is in the firm's demand at its competitor's price, resulting from switching consumers. When the focal firm's price is below the competitor's price, the firm obtains the demand of switching consumers while when the focal firm's price is above the competitor's price it does not obtain that demand. The second source of discontinuity in demand arises because of stockpiling by consumers of the firm; this discontinuity occurs at the stockpiling threshold \( t \).\(^8\) Keeping this in mind, let us first consider the symmetric state \( s=00 \). In a mixed strategy Nash equilibrium, the competing firm \( 3-i \)'s mixed strategy should make focal firm \( i \) indifferent over all un-dominated prices. Recognizing that at high prices consumers do not stockpile and that leads to a different future state compared to when the firm offers a low price, we explicitly write the focal firm's equi-profit conditions at prices \( p_i \) below and above the consumer stockpiling threshold. First, consider the case \( p_i > t \). The expected profit \( V_i^{00} | p_i > t \) consists of the current period profit and the discounted continuation profit. The continuation profit depends on the state in the next period, which is affected by whether \( p_{3-i} \leq t \) or \( p_{3-i} > t \). Keeping that in mind and invoking equation (1) we can write \( V_i^{00} \) by conditioning it on \( p_i \) relative to \( t \).

\[
(V_i^{00} | p_i > t) = p_i[\alpha + (1-F_{3-i}^{00}(p_i))\beta] + \delta_i[(1-F_{3-i}^{00}(t))V_i^{00} + F_{3-i}^{00}(t)V_i^{01}]
\]  

(2)

The first term in equation (2) is the current period profit when it charges price \( p_i \). The second term denotes the discounted expected continuation profit after taking expectation over competitor’s current period prices. Thus, the relevant continuation profit is \( V_i^{00} \) with probability \( 1-F_{3-i}^{00}(t) \) when competitor’s price is above the stockpiling threshold, and likewise, \( V_i^{01} \) with probability \( F_{3-i}^{00}(t) \), when the competitor’s price is below the threshold.

---

\(^8\) Recall that the first kind of discontinuity also occurs in Varian (1980) and Narasimhan (1988) while the latter does not. From their analysis, we know that the presence of switching consumers results in a mixed strategy equilibrium over an interval such that expected profits are equal at all un-dominated prices. The mixing distribution in their model is strictly increasing and continuous everywhere over the support except possibly at \( r \). The discontinuity in the mixing distribution (mass point) at \( r \) arises only when firms are asymmetric. In our model another kind of discontinuity in the mixing distribution can occur due to stockpiling. Specifically, stockpiling could lead to a mass point in the interior of the support at \( t \), a feature that does not occur in extant models. A further novel feature of our model is that both firms’ mixing distributions could have a mass point at \( t \).
In the same way, expected profit $V_i^{00} | p_i < t$ can be computed as shown in equation (3). In this case, there are two differences. First, the current period profit accounts for stockpiling in the demand. Second, the continuation profit is now defined for future states $s = \{10,11\}$.

\[
\left( V_i^{00} | p_i < t \right) = p_i \left[ (1 + \lambda) \alpha + \left( 1 - F_{x_i}^{00}(p_i) \right) \beta \right] + \delta_j \left[ \left( 1 - F_{x_i}^{00}(t) \right) V_i^{10} + F_{x_i}^{00}(t) V_i^{11} \right] \tag{3}
\]

Similarly, we can write the equi-profit conditions for the focal firm in all other states. Equi-profit conditions in all states $s \in \{00,01,10,11\}$ are as follows:

\[
\left( V_i^{00} \right) = \frac{p_i \left[ \alpha + \left( 1 - F_{x_i}^{00}(p_i) \right) \beta \right]}{1 + \lambda} + \frac{\delta_j \left[ \left( 1 - F_{x_i}^{00}(t) \right) V_i^{00} + F_{x_i}^{00}(t) V_i^{01} \right]}{1 + \lambda} \quad t < p_i
\]

\[
\left( V_i^{01} \right) = \frac{p_i \left[ \alpha + \left( 1 - F_{x_i}^{01}(p_i) \right) \beta \right]}{1 + \lambda} + \frac{\delta_j \left[ \left( 1 - F_{x_i}^{01}(t) \right) V_i^{00} + F_{x_i}^{01}(t) V_i^{01} \right]}{1 + \lambda} \quad t < p_i \tag{4}
\]

\[
\left( V_i^{10} \right) = \frac{p_i \left[ \alpha + \left( 1 - F_{x_i}^{10}(p_i) \right) \beta \right]}{1 + \lambda} + \frac{\delta_j \left[ \left( 1 - F_{x_i}^{10}(t) \right) V_i^{10} + F_{x_i}^{10}(t) V_i^{11} \right]}{1 + \lambda} \quad t < p_i \tag{5}
\]

\[
\left( V_i^{11} \right) = \frac{p_i \left[ \alpha + \left( 1 - F_{x_i}^{11}(p_i) \right) \beta \right]}{1 + \lambda} + \frac{\delta_j \left[ \left( 1 - F_{x_i}^{11}(t) \right) V_i^{10} + F_{x_i}^{11}(t) V_i^{11} \right]}{1 + \lambda} \quad t < p_i \tag{6}
\]

Given the equi-profit conditions in (4)-(7), we are in a position to determine the firm’s equilibrium strategy conditional on the stockpiling threshold $t$.

4. Equilibrium

We wish to explore an equilibrium in which consumers stockpile. To that end, we derive the sufficient condition in proposition 2 for consumers to stockpile. We also wish to focus on a multi-state equilibrium in which all states are visited with positive probability. A sufficient condition for this to occur is that the reservation price $r$ should be un-dominated in all states. We will derive the sufficient condition for this in proposition 4 after characterizing the equilibrium and the value functions.

**Proposition 2:** If $\delta^* < \delta_i$ stockpiling consumers stockpile with positive probability, where

\[
\delta^* = \frac{1 - 2\alpha}{(1 - \alpha) \ln \left( \frac{1 - \alpha}{\alpha} \right)}.
\]
**Proof:** Please see Appendix

Proposition 2 gives the conditions to rule out an equilibrium in which the state is always 00. In particular, when consumers’ discount factor is sufficiently high then in $s=00$ state, firms will charge prices below the stockpiling threshold with positive probability so that consumers will find it in their best interest to stockpile. For the remainder of the paper we assume that $\delta_s$ is high enough so that the condition in Proposition 2 is satisfied.

In 4.1 we characterize the mixed strategies and highlight the interesting aspects of the same with focus on the mass points at $t$ and $r$. We evaluate the value functions and derive the lower and upper bounds for profits in 4.2 and investigate the effects of flexible consumption in 4.4.

### 4.1 Mixed Strategy Prices in Multi-state Equilibrium

Wherever applicable we invoke prior results due to Varian (1980), Narasimhan (1988), and Maskin and Dasgupta (1986) to characterize the equilibrium mixed strategies. In our model, each firm has to formulate a mixed strategy corresponding to the four states $s \in \{00, 01, 10, 11\}$ conditional on $t$. Note that because consumers stockpile below $t$ but not above $t$, the cdf $F^s(p)$ consists of two functions: $F^s_{\text{above}}(p)$ and $F^s_{\text{below}}(p)$. In general, $F^s_{\text{below}}(t) \neq F^s_{\text{above}}(t)$. How can this be resolved? Recall that cdf, $F^s(p)$ should be a non-decreasing function in $p$. Therefore, if $F^s_{\text{below}}(t) > F^s_{\text{above}}(t)$, then price just above $t$ is dominated and $F^s(p)$ is constant over an interval above $t$. We will refer to this as a hole in $F^s(p)$. Alternatively, the if $F^s_{\text{below}}(t) < F^s_{\text{above}}(t)$ then, $F^s(p)$ will contain a mass point, $M^s(t)$, at $t$. Figure 2 shows this graphically.

![Figure 2: Behavior of Cumulative Distribution Function around the Stockpiling Threshold](image-url)
It is useful to note that in extant work, for instance in Varian (1980) and Narasimhan (1988), both firms cannot have a mass point at same price in the support of their mixing distribution. That is because when both firms have a mass at the same price they share the switching segment at that price. Hence, for each firm it becomes a dominant strategy to shift its mass point to a price slightly below the competitor’s mass point to capture the entire switching segment. However, in our model, for both firms $F^*(p)$ can have a mass point at $t$. This poses a unique challenge in characterizing $F^*(p)$. We show in proposition 3 how the mass point at $t$ is sustained in equilibrium. Denote the mass point at $t$ by $M^*(t)$, in $s \in \{00, 01, 10, 11\}$.

**Proposition 3:** When both firms have a mass point at $t$, it can be sustained iff $\eta_i = \phi_i$, i.e., probability that firm $i$’s loyal segment stockpiles at $t$ is equal to the probability that the switching segment buys brand $i$ when both brands are priced equally.

**Proof:** Please see Appendix

When both firms charge the same price the switching segment buys brand $i$ (or 3-$i$) with probability $\phi_i$ (or $1-\phi_i$). Carefully considering the options for stockpiling consumers when they are indifferent between stockpiling and not stockpiling resolves the mass point issue and $p = t$ is rendered un-dominated even in the presence of a mass point at $t$ for both firms. Given our focus on a symmetric equilibrium we set $\phi_i = \eta_i = 1/2, i = 1, 2$.

Turning to the case when $F^*(t) > F^*(t)$, as in Gangwar, Kumar and Rao (2014), we show in lemma 1 in the Appendix that the mixing distribution will have a hole above the stockpiling threshold. Next, we contemplate the possibility of mass point at other prices in the mixing distribution. It is easy to rule out the possibility of mass point at any other price except $r$ and $t$. Note that in the symmetric states $s=00$ and $s=11$ there can be no mass points at $r$. This leaves us with the asymmetric cases $s=01$ and $s=10$. In Narasimhan’s (1988) model, a mass point at $r$ occurs in the mixed strategy of the firm with the larger loyal segment. We cannot make direct use of this result because of the presence of continuation profits. How then can we identify which firm has the mass point? We do that next in lemmas 2A and 2B.

---

9 Narasimhan(1988) and Raju, Srinivasan and Lal(1990) have shown that in some cases one firm can have a mass point but only at the reservation price.

10 This is because if both firms charge $r$, then one can deviate to $r-e$ and capture the switching segment without affecting current profits and future states. This result occurs also in Varian (1980) and Narasimhan (1988) where future states do not matter.
and show that mixed strategy of the firm in state $s=01$ will have a mass point at the reservation price $r$. We denote, $M_i^s(p)$ as the mass point at price $p$ in the mixing distribution of the focal firm in state $s$.

**Lemma 2A:** At $p \in (\bar{t}^1_i, t)$, $0 < \frac{d}{dp} \left( F_{s=01}^i(p) - F_{s=01}^i(p) \right)$; moreover, $F_{s=01}^i(r^-) - F_{s=01}^i(r^-) < \alpha \lambda / \beta$.

**Proof:** Please see Appendix

**Lemma 2B:** At $p \in (t, r)$, $0 < \frac{d}{dp} \left( F_{s=01}^i(p) - F_{s=01}^i(p) \right)$; moreover, $0 < F_{s=01}^i(r^-) - F_{s=01}^i(r^-) < \alpha \lambda / \beta$.

**Proof:** Please see Appendix

Note that $0 < F_{s=01}^i(r^-) - F_{s=01}^i(r^-)$ implies that the focal firm (i) will have a mass point at the reservation price $(r)$ $0 < M_i^{01}(r)$ and since both firms can not have a mass point at $(r)$ $M_i^{01}(r) = 0$.

It turns out that in our case also (even with continuation profits) the intuition is similar to the asymmetric case in Narasimhan (1988).

### 4.2 Continuation profits under multi-state equilibrium

We now derive the firm’s value function, the sum of expected profits over the infinite horizon, in each state, to evaluate the profit implications of stockpiling by loyal consumers. Towards this end, we solve for the expected profit of the firms in each state in the multi-state equilibrium in lemmas 3-5. Specifically, we use (4) – (7) to derive closed form expressions for the value functions by evaluating the equi-profit conditions in (4) – (7) at $p = r$.

**Lemma 3:** $V_{i=00}^{i0} = V_{i=01}^{i0} = \frac{r \alpha}{1 - \delta_f}$

**Proof:** Please see Appendix

**Lemma 4:** $V_{i=11}^{i1} = \frac{r \alpha}{1 - \delta_f} - r \alpha \lambda$

**Proof:** Please see Appendix

**Lemma 5:** $\frac{r \alpha}{1 - \delta_f} - r \alpha \lambda = V_{i=11}^{i1} \leq V_{i=10}^{i1} \leq V_{i=10}^{i1}$ and $V_{i=10}^{i1} \leq V_{i=00}^{i0}$

---

11 By symmetry $M_i^{10}(r) = 0$ and $0 < M_i^{10}(r)$.
Proof: Please see Appendix

Note that the continuation profits are equal in states $s = \{00, 01\}$. It is also interesting to note that in these states $V_i^{00}$ and $V_i^{01}$ do not depend on $\lambda$. This is because loyal consumers of the focal firm do not have inventory in either state. Therefore, all loyal consumers, irrespective of $\lambda$, demand one unit at $r$. In contrast, in states $s = \{10, 11\}$ $V_i^{10}$ and $V_i^{11}$ do depend on $\lambda$. This occurs because only the loyal consumers who do not have inventory purchase at $r$. The size segment is $(1 - \lambda)\alpha$, which is a function of $\lambda$. This in turn makes both $V_i^{10}$ and $V_i^{11}$ less than $V_i^{00}$. Furthermore, we can see from lemma 5 the profits are lower in $s = 11$ relative to $s = 10$. The intuition for this is that in $s = 11$ switching consumers comprise a larger fraction of demand at $r$ relative to $s = 10$.

4.2.1 Unintended consequence of promotion

Denote the probability of state $s$ under steady state as $\psi^s$. Then the expected value over all states, $\bar{V} = \sum_s \psi^s V^s$. We now state our main result in proposition 4.

**Proposition 4:** In a multi-state equilibrium firm’s expected profit, $\bar{V}$ satisfies $r\alpha(1-\delta_f) - r\alpha \lambda \leq \bar{V}$.

**Proof:** Follows directly from lemmas 3-5.

Given that $\bar{V}$ is a convex combination of the state dependent value functions the statement in proposition 3 follows directly from lemmas 3-5. To understand the bounds on the profit presented in proposition 3, it is helpful to compare it with the profit firms would obtain when consumers do not stockpile. We denote the no stockpiling case as the benchmark case. In that case firms’ profit, $V^{\text{bench}}$, is obtained by considering a symmetric, infinite period version of Narasimhan (1988), and so the firms’ expected profit in each period is $r\alpha$ and the discounted sum of these expected profits over an infinite horizon yields: $V^{\text{bench}} = \frac{r\alpha}{1-\delta_f}$. Note that in our model the profit is bounded from above by $\frac{r\alpha}{1-\delta_f}$, which is equal to $V^{\text{bench}}$.

Let us compare the lower bound, $V^{11}$, of equilibrium profits in proposition 4 to $V^{\text{bench}}$. Firms’ maximum losses in a multi-state equilibrium can be expressed in percentage terms as $\text{MAXL}^{\text{multi}} = \frac{V^{\text{bench}} - V^{11}}{V^{\text{bench}}}$, do not exceed $\lambda(1-\delta_f)$. Note that $\text{MAXL}^{\text{multi}}$ is increasing in the size of the stockpiling segment $\lambda$ but decreasing in $\delta_f$. If firms are extremely patient, $\delta_f \rightarrow 1$ $\text{MAXL}^{\text{multi}}$
approaches zero. Suppose 50% of the loyal consumers stockpiles, the purchase frequency in the product category is once every month, and the firm’s discount factor is 0.99 per month. Then $MAXL^{mul}$ will be a mere 0.5%. Since the maximum absolute loss, $r\alpha\lambda$, represents just one period sales (at a maximum price of $r$) to the stockpiling consumers it becomes infinitesimally small relative to sum of the discounted profits. In other words, the loss in the absolute dollar terms may be significant, depending on the category but as a proportion it is relatively small.

To understand the losses better it is useful to compare the benchmark profits to the profits conditional on state. Note that the continuation profits $V^{00}$ and $V^{01}$ are identical to the profits in the benchmark case. Also note that the continuation profit $V^{11}$ is lowest and is less than the profits in the benchmark case by $r\alpha\lambda$. The $r\alpha\lambda$ represents the forgone sale to stockpiling consumers at the reservation price. Thus, if the initial state is $s=11$, it is as if stockpiling loyal consumers got a free sample of the product (opportunity cost of which is $r\alpha\lambda$ to the firm) and the firm makes $\frac{r\alpha\lambda}{1-\delta_f}$, equal to the benchmark profit thereafter.\textsuperscript{12} What is interesting here is that equilibrium pricing fully takes into account future stockpiling behavior and so we are left only with a one-time loss captured by the initial state.

### 4.3 Existence of Equilibrium

We show that the set of parameters where our proposed equilibrium in mixed strategies exists, is non-empty. In lemma 6 we combine our earlier result in proposition 2 to delineate the parameter region in which our proposed equilibrium exists. Then, in lemma 7 we verify that the firms’ mixed strategies in section 4.1 satisfy the necessary conditions for an MPE. Finally, we demonstrate the existence of equilibrium by construction with a numerical example.

**Lemma 6:** A sufficient condition for $r$ to be in the support of firms' mixed strategy is $\delta_r < 1-\lambda$.

**Proof:** Please see Appendix

If the condition in Lemma 6 is violated then the sufficient condition that ensures that $r$ is in the support of the mixed strategy will be violated. Recall from proposition 2 that $\delta^* = \frac{1-2\alpha}{(1-\alpha)\ln\left(\frac{1-\alpha}{\alpha}\right)} < \delta_r$.

\textsuperscript{12} Suppose we allow consumers to stockpile multiple units. Then, the maximum loss will be the one time opportunity cost of selling stockpiled units to stockpiling loyal consumers.
Therefore a sufficient condition for $r$ to be in the support requires that

$$\delta^* = \frac{1 - 2\alpha}{(1 - \alpha) \ln \left( \frac{1 - \alpha}{\alpha} \right)} < (1 - \lambda).$$

This condition guarantees that the parameter region, for the existence of equilibrium of the kind we are exploring, is non-empty. Since this is a sufficient condition, we should note that our equilibrium could exist for other parameter values as well.

**Lemma 7:** The mixed strategies satisfy the necessary conditions for an MPE.

**Proof:** Please see Appendix

To demonstrate existence let us turn to a numerical example. Suppose $\alpha = 0.25, \lambda = 0.30, r = 100, \delta_c = 0.95, \delta_f = 0.99$

Then recall from lemma 3-5 that the upper bound of profit is $V^{00} = V^{01} \frac{r\alpha}{1 - \delta_f} = 2500$ and the lower bound of profits is $V^{11} = \frac{r\alpha}{1 - \delta_f} - r\alpha\lambda = 2492.5$.

Our numerical analysis confirms this. We find following result

$V^{00} = 2500, V^{01} = 2500, V^{10} = 2497.5210, V^{11} = 2492.5$ and $\overline{V} = 2496.0180$.

Note that the upper bound is indeed $V^{00} = 2500$ and the lower bound is $V^{11} = 2492.5$.

We next focus on the mixed strategies. They are given by:

$$F^{00}(p) = \begin{cases} \frac{1.5p - 50}{p} & 59.9551 < p < 100 \\ \frac{1.65p - 61.3358}{p} & 37.1732 < p < 59.9551 \end{cases}, \quad M^{00}(t) = 0.0391,$$

$$ph^{00} = 75.6733, \quad pl^{00} = 47.1615, \quad F^{00}(t) = 0.6465$$
First, we can verify that the equi-profit condition holds at all prices with positive probability given the computed mixed strategies. Next, we note that at the lower limit \( F^{01}(l = 37.1732) = 0 \) and \( F^{10}(l = 37.1732) = 0 \). We also note that, as mentioned in lemma 2A, \( F^{01}(p) < F^{10}(p) \) since the slope of \( \frac{dF^{01}}{dp} = \frac{55.7598}{p^2} \) is less than the slope of \( \frac{dF^{10}}{dp} = \frac{61.3358}{p^2} \).

Similarly, we note that the slope of \( \frac{dF^{01}}{dp} = \frac{45.0419}{p^2} \) is less than the slope of \( \frac{dF^{10}}{dp} = \frac{50}{p^2} \). We also, note that \( M^{01}(r) = 0.1004 \) is less than \( \alpha \lambda / \beta = 0.15 \) as shown in lemma 2B. Moreover mass points at \( t \), \( M^{01}(t) = 0.0288 \) is less than \( M^{10}(t) = 0.0391 \).

Next, we see from \( \psi^{00} = 0.1083, \psi^{01} = 0.2161, \psi^{10} = 0.2161, \psi^{11} = 0.4595 \) that all states are visited with positive probability in equilibrium.

Other quantities of interest are noted below.
Consumers’ continuation utilities are $U^0 = 956.05578$ and $U^1 = 1019.16643$, which leads to $t = \delta (U^0 - U^1) = (0.95)(956.05578 - 1019.16643) = 59.9551$. Thus, we can see the existence of the proposed equilibrium. More examples can be found in the Appendix.

How different are the prices in a stockpiling equilibrium from those in a situation without stockpiling? To understand this, we computed the distribution of prices in a randomly chosen period by integrating out the mixed strategies over the states under stationary conditions, denoting it as the unconditional PDF with stockpiling. In figure 3 we display that and compare it to the distribution of prices without stockpiling. We can readily see that managers move away from frequently promoting below the threshold. Moreover, there is an increase in the probability of charging the reservation price. Thus, it is important to understand that the relatively low losses due to stockpiling accompany significant changes in pricing strategies. We will elaborate on this further in section 5.

![Figure 3: Equilibrium Probability Density Function of Prices without (with) Consumer Stockpiling](image)

### 4.4 Flexible Consumption Due to Stockpiling

Ailawadi, Gedenk, Lutzky and Neslin (2007) identified two major effects of consumer stockpiling which can have a positive effect on firms’ profits: preemptive switching and increased consumption. The idea that consumers' inventory can increase consumption also finds support in Wansik (1996). The interaction of increased or flexible consumption and stockpiling has been studied by Bell, Iyer and Padmanabhan (2002) and Heerde, Leeflang and Wittink (2004) while Jain (2012) accounts for increased consumption based on inventory in modeling the effect of package size on consumption. How might flexible consumption due to stockpiling affects firms’ profits in our model? Gangwar, Kumar and Rao (2014) show that when only switchers stockpile, even increased consumption due to stockpiling...
does not lead to higher profits. The question remains if this would hold if stockpiling by loyal consumers leads to increased consumption. We address that in this section.

We suppose that when loyal consumers stockpile their consumption is flexible. When consumers stockpile they have two units on hand. We assume that they either consume both units in the current period with probability \(1 - \theta\) or consume just one unit in the current period, with probability \(\theta\), where, \(\theta \in [0,1]\). Given that consumers may exhaust their inventory earlier than anticipated it can lead to a situation where \(\theta\) proportion of stockpiling consumers have inventory of \(I_i = 1\) while the remaining \((1 - \theta)\) proportion end up with no inventory, \(I_i = 0\), in the next period. From the firm's perspective, this means, if it charges a low price in the next period, \(\theta\) proportion of stockpiling consumers will buy only 1 unit and \((1 - \theta)\) proportion will buy 2 units. As before, this leads to four possible states \(s \in \{00, 0\theta, \theta 0, \theta\theta\}\) in our model. It is instructive to note that in the flexible consumption case, states correspond to the average inventory position of stockpiling consumers of the focal firm. So \(\theta = 1\) will correspond to our base model. The consumer decision problem is the same as in the base case. In Table 2, we display how consumer decision to stockpile (or not) and average inventory position for the focal firm’s stockpiling loyal consumers determines the state transitions in the case with flexible consumption.

Table 2: Inventory Position Evolutions and State Space Transitions under Flexible Consumption

<table>
<thead>
<tr>
<th>firm (i)'s current period price</th>
<th>firm (i)'s stockpiling consumers’</th>
<th>firm (i)'s stockpiling loyal consumers inventory state in next period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Price</strong> (No Stockpiling)</td>
<td>(I_i = 0) buy 1  (I_i = 1) consume one unit  (I_i = 0)</td>
<td>All loyal consumers of brand (i) have zero inventory</td>
</tr>
<tr>
<td>(I_i = 1) not buy  (I_i = 1) consume one unit  (I_i = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Low Price</strong> (Stockpiling)</td>
<td>(I_i = 0) buy 2  (I_i = 2) ((1 - \theta)) proportion consume both units  (\theta) proportion of loyal consumers of brand (ij) have</td>
<td></td>
</tr>
</tbody>
</table>
Note that the average inventory position in the next period is independent of current inventory position of stockpiling consumers. As in our base model, the next period state will only depend on current period prices. In the flexible consumption case, the state \( s = 00 \) corresponds to the situation in which stockpiling consumers of neither firm have inventory while state \( s = \theta \theta \) corresponds to the situation where \( \theta \) proportion of stockpiling consumers of both firms have inventory and \( (1 - \theta) \) proportion of both firms have zero inventory. The state \( s = 0 \theta \) \((s = \theta 0)\) corresponds to the case where stockpiling consumers of the focal (competing) firm and \( (1 - \theta) \) proportion of competing (focal) firm have no inventory and a proportion \( \theta \) of the competing (focal) firm’s stockpiling loyal consumers have an inventory of one unit. The demand dynamics and the solution procedure are similar to that in the base model. Stockpiling threshold with flexible consumption is also independent of states.

Consider, for example, focal firm's demand, \( D_{i}^{\theta \theta} \), in states \( s = \theta \theta \) when \( \theta \) proportion of stockpiling loyal consumers of the both firms have inventory.

\[
D_{i}^{\theta \theta}(p_{i}, p_{j}, t) = (1 - \theta \lambda)\alpha + \lambda\alpha.1^{\prime}(t - p_{i}) + \beta.1^{\prime}(p_{j} - p_{i}),
\]

where (indicator function) \( .1^{\prime}(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z < 0 \end{cases} \)

First, note that since the \( \theta \) proportion of stockpiling consumers of focal firm have inventory, they do not buy when prices are high. In other words, only non-stockpiling consumers \((1 - \lambda)\alpha\) and stockpiling consumers with no inventory \((1 - \theta)\lambda\alpha\), buy 1 unit at high prices. Therefore, guaranteed demand at all prices in state \( s = \theta \theta \) is \((1 - \theta)\lambda\alpha + (1 - \lambda)\alpha = (1 - \theta \lambda)\alpha\). Further, if the price of the focal firm is \( \leq t \), all stockpiling loyal consumers (a fraction \( \lambda\alpha \)) will buy an additional unit. In addition, if the focal firm’s
price is less than the competitor’s price, switchers of size \( \beta \) will also buy one unit from the focal firm.

Then, for any state \( s \) demand can be written as

\[
D^i(p_i, p_j, t) = \begin{cases} 
\kappa + \lambda \alpha \beta (t - p_i) + \beta \phi_i (p_j - p_i) & p_i \neq p_j, p_i \neq t \smallskip \\
\kappa + \lambda \alpha \beta t - p_i + \beta \phi_i & p_i = p_j, p_i \neq t \smallskip \\
\kappa + \lambda \alpha \eta_i + \beta \phi_i (p_j - p_i) & p_i \neq p_j, p_i = t \smallskip \\
\kappa + \lambda \alpha \eta_i + \beta \phi_i & p_i = p_j = t
\end{cases}
\]  

(8)

where \( \kappa \) = \[\begin{cases} \alpha & \text{if state } s \in \{00, 0\theta\} \\
(1- \theta \lambda) \alpha & \text{if state } s \in \{00, \theta \theta\}
\end{cases}\]

and (indicator function) \( 1^+(z) = \begin{cases} 1 & z > 0 \\
0 & z < 0
\end{cases}\).

Compared to the base case, note that in (8) the focal firm gets an extra demand of \((1- \theta) \lambda \alpha \) from stockpiling consumers who end up consuming both units in states \( s \in \{\theta \theta, \theta \theta\} \). To characterize the equilibrium in the flexible consumption case, we follow the same procedure outlined earlier in Section 4.1. After determining the equilibrium mixing distributions in each of the four states, we evaluate the equi-profit conditions at \( p = r \) to compute profits of the firms in each state. In Proposition 5 below, we provide bounds on the firms’ profits in a multi-state equilibrium under flexible consumption.

**Proposition 5:** In a multi-state equilibrium with flexible consumption firm’s expected profit, \( \bar{V}_{\text{flex}} \), is bounded such that \( \frac{r \alpha}{1- \theta} - r \alpha \lambda \theta \leq \bar{V}_{\text{flex}} \).

**Proof:** Once we recognize that the guaranteed demand under flexible consumption is

\[\kappa = \begin{cases} \alpha & s \in \{00, \theta \theta\} \\
(1- \theta \lambda) \alpha & s \in \{00, 01\} \end{cases}, \quad \text{while } \kappa = \begin{cases} \alpha & s \in \{00, \theta \theta\} \\
(1- \lambda) \alpha & s \in \{00, 11\}\end{cases}\]

in our base model, then the proof is straightforward and is similar to that in Proposition 4.

When we compare the lower bound of profit in a multi-state equilibrium with and without flexible consumption (Propositions 5 and 4), it is easy to see that the lower bound of firm's profit with flexible consumption is higher. Recall that in Proposition 4, the maximum loss was \( r \alpha \lambda \), which is the forgone sale to stockpiling consumers that the firm could have made if consumers had no inventory. With flexible consumption this loss is \( r \alpha \lambda \theta < r \alpha \lambda \), \( \forall \theta \in (0,1) \). Thus, relative to our base model, the maximum loss with flexible consumption is lower. In particular, it is lower by \((1- \theta)r \alpha \lambda \) which is the maximum profit that the firm can make from stockpiling consumers who stockpile, consume both units and re-enter the market with no inventory.

What is interesting is that even with increased consumption, the multi-state equilibrium profits
with stockpiling do not exceed the benchmark profits with no stockpiling. This happens because with flexible consumption although firms end up in state $s=00$ more often, the value function (in state $s=00$) itself is unaffected by the increased consumption due to stockpiling. Our finding, that relative to our base model, the maximum loss with flexible consumption is lower, is consistent with the empirical findings in Ailawadi et. al. (2007) that shows that increased consumption after stockpiling can offset the negative aspects of stockpiling by loyal consumers.

5. Numerical Exploration and Managerial Implications

We have found in our discussion with consumer product managers that they are indeed concerned with consumer stockpiling. They are aware of the possibility of flexible and increased consumption resulting from stockpiling but they are also aware of, and unhappy with, what they see as current sales at low prices “borrowing” from future sales, presumably at higher prices. Thus, stockpiling by consumers during promotions is very much on their mind. Managers are less clear about what the appropriate pricing strategy should be to take into account stockpiling because of the potentially increased consumption but also because of competitive pressures on the need to have promotions. For instance, if managers offer a discount that triggers stockpiling in one period should they change their frequency and depth of discounts in the following period; if so how? Our analysis offers interesting and some counter-intuitive prescriptions for managers to these important questions. These prescriptions are based on the equilibrium strategies derived in our model. We explore different market scenarios by varying the size of the stockpiling consumer segment, $\lambda$ and the loyal segment, $\alpha$. In Table 3, we report the results of our analysis: the stockpiling threshold, the value in each state, the expected prices when the firms offer discounts that trigger stockpiling or that do not, and the frequency of such discounts conditional on each state. The steady state probability of each state’s realization in equilibrium is also reported.\(^\text{13}\)

\(^{13}\) We present the results by fixing the size of the loyal segment to 0.25 and for two different values of the size of the strategic consumer segment, $\lambda$. Given the far too many quantities of interest in Table 3, we display some elements of consumer stockpiling corresponding to other values of $\alpha$ and $\lambda$, in Table 4 in the Appendix.
Table 3: Stockpiling Threshold and Expected Prices in Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$r = 100, \delta_c = 0.95, \delta_f = 0.99$</th>
<th>$\lambda = 0.25$</th>
<th>$\lambda = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stockpiling Threshold</td>
<td>$t = 58.44$</td>
<td>$t = 78.39$</td>
<td></td>
</tr>
<tr>
<td>State</td>
<td>$s = 00$</td>
<td>$s = 01$</td>
<td>$s = 10$</td>
</tr>
<tr>
<td>Steady State</td>
<td>0.12</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Probabilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Profit</td>
<td>2500</td>
<td>2500</td>
<td>2498</td>
</tr>
<tr>
<td>Expected High Price</td>
<td>74.8</td>
<td>79.8</td>
<td>74.8</td>
</tr>
<tr>
<td>Expected Low Price</td>
<td>46.1</td>
<td>46.1</td>
<td>46.1</td>
</tr>
<tr>
<td>Frequency of</td>
<td>0.63</td>
<td>0.58</td>
<td>0.63</td>
</tr>
<tr>
<td>Stockpiling</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparing the expected prices when firms induce stockpiling or not across the two scenarios $\lambda = 0.25$ and $\lambda = 0.75$ reveals that the equilibrium price in each state of stockpiled units as well as the units bought without stockpiling increase in $\lambda$. What is interesting is that consumers realize that equilibrium prices are higher but their stockpiling threshold price also rises and as a result the probability of stockpiling increases, albeit at a higher price. From the manager's point of view, we can see this adjustment of higher equilibrium prices conditional on state not as a way to restrict stockpiling but rather as a way of taking into account stockpiling. This is interesting when we recall a manager’s reaction: “we try to cope by loping off deeper promotions”. This uncertain response can be better resolved by a closer examination of equilibrium strategies in our model.

What could be a good pricing strategy response to consumer stockpiling? Our equilibrium offers prescriptions on how strategies change. First, consider the scenario when a manager offers a promotion that induces stockpiling; should the manager offer a deep promotion again to induce stockpiling in the next period? Based on conventional wisdom managers might be inclined to increase price following periods of deep promotions. Our model prescribes quite the opposite. To see this consider the case when $s = 00$, $\lambda = 0.25$ and the manager offers a discount that induces stockpiling. The next period state will be $s = 10$ or $s = 11$ depending upon the competing firm’s price. In the former case, our model predicts that the manager should follow a very similar strategy in the following period (note that the expected price with shallow (74.75) and deep discounts (46.1) in $s = 10$ are identical to that in $s = 00$). However, in the latter case when firms transition to $s = 11$, our model suggests that managers should offer lower expected prices (42.86 vs. 46.10) with greater frequency (0.71 vs. 0.63). The intuition behind this prescription is that in periods following stockpiling, the stock of loyal consumers in the market reduces, leaving firms to
compete for a clientele mix skewed in favor of the switchers (more switchers to loyals) in $s=11$. As a result managers should promote more aggressively to compete for the switchers. We obtain similar results when $\lambda=0.75$ with the exception that the expected price with shallow (87.22 vs. 86.9) and deep discounts (59.02 vs. 58.67) in $s=10$ while similar are marginally higher relative to $s=00$ and the frequency of deep discounts is virtually the same.

Next, we consider the scenario when the manager offers a discount that does not induce stockpiling; should the manager offer a deep promotion to induce stockpiling in the next period? Once again depending upon whether the competing firm offers a shallow or deep discount the state in the next period will be $s=00$ or $s=01$ respectively. In the former case, our model predicts that the manager should adopt the same strategy. However, in the latter case if the competing firm offers a deep discount in the previous period, the knee jerk reaction for most managers, given the erosion in market share, would be to offer a deep discount. Our model prescribes that the prudent thing for managers to do is exactly the opposite: reduce both the depth of shallow discounts and frequency of deep promotions. Compare the expected prices in the $s=00$ and $s=01$ column in Table 3 when $\lambda=0.25$. The expected price with shallow discounts (79.82 vs. 74.75) are higher while the expected price with deep discounts are marginally lower (46.05 vs. 46.10) in $s=01$ relative to $s=00$. Note that the frequency of deep discounts is lower in $s=01$ relative to $s=00$ (0.58 vs. 0.63). These results are in the same direction and much more pronounced when $\lambda=0.75$. The rationale for this finding is that when the competing firm offers a deep discount in the previous period, the stock of loyals it can serve at high prices is lower in the current period. The competing firm is therefore in a much better position to compete for the switching consumers. Since prices are strategic complements it is prudent for the focal firm to increase price rather than engage in price competition with a firm that can compete more effectively.

6. Conclusions

In this paper, we have characterized the equilibrium promotional strategy for firms when they face stockpiling loyal consumers who are willing to stockpile if prices are sufficiently low. This provides us new insights into firms’ promotional strategies and the effect of consumer stockpiling behavior on firms' profits. We obtain three important results.

Our main result is that in categories in which consumption is constant, stockpiling by brand loyal consumers results in lower profits compared to a benchmark case of no stockpiling. This loss is a collateral effect due to firms competing for switching consumers. Guo and Villas-Baos (2007) show that stockpiling by loyal consumers could lead to lower profits due to shifting brand preferences, which in
turn causes firms to “lock in” customers by inducing them to stockpile. In our model, losses occur for a different reason. Firms induce switching by offering discounts to consumers who have weak brand preferences but the resulting promotion provides consumers with strong brand preferences an opportunity to stockpile. By modeling consumer heterogeneity in price sensitivity to purchase quantity and analyzing opportunistic stockpiling, we extend our understanding of the effect of consumer stockpiling on firms’ profits. This has important implications for managers who deal with categories where consumption is relatively inelastic and brand preferences are strong. Chasing higher market share through promotions may not work in a firm’s favor unless promotions are adjusted to account for stockpiling by loyal consumers.

Our next result is also in contrast with results derived in prior work. Extant models find that stockpiling has no effect on equilibrium profits when only switching consumers stockpile. However, when brand loyal consumers stockpile this result does not hold. What is more important is that we are able to provide an upper bound on the loss due to stockpiling by loyal consumers. The losses are confined to one period sales of stockpiling units. This bound allows us to claim that for frequently purchased product categories the losses due to stockpiling in percentage terms are minimal, provided firms re-calibrate their promotional strategies to opportunistic stockpiling by loyal consumers. This is important because managers concerned with consumer stockpiling often attempt to exogenously impose restrictions to curb consumer stockpiling. These kinds of restrictions will leave some consumers unhappy. A better approach is to re-optimize promotions in such a way that the adverse effects of stockpiling are either reduced or nullified.

Finally, we show that when stockpiling leads to increased (flexible) consumption, then the maximum losses that could result from loyal consumers' stockpiling is lower than that in our base model. However, it is important to understand that in several markets, firms may still end up incurring some losses due to consumer stockpiling despite increased consumption after stockpiling.

In our model we assume that only loyal consumers can stockpile. In reality switching consumers may also stockpile. Since past research has studied the case of stockpiling by switching consumers, we can now offer some insights into what might happen if some fraction of all consumers stockpile. First, with respect to profits, recall that stockpiling by switching consumers has no effect (Gangwar, Kumar and Rao 2014). Hence, when all consumers stockpile, equilibrium profits will be identical to what we find in our model. However, the equilibrium pricing strategies will differ. Second, stockpiling thresholds will differ across consumer types. Given that switching consumers buy units at lower prices compared to loyal consumers, switching consumers will have lower stockpiling threshold. Implying that in certain markets only loyal consumers may get the opportunity to stockpile and switching consumers do not.
Finally, in our model, after stockpiling, deep discounts are more likely to be followed by another deep discount that induces stockpiling. In contrast, in the model where only switchers stockpiling, firms raise prices after stockpiling. Given that very deep discounts may induce even switchers to stockpile, managers should modify the strategy prescribed in our model to accommodate switchers stockpiling. Deep discounts that induce switchers also to stockpile does not require follow up by another deep discount. However, deep discounts that do not induce switchers to stockpile should still be followed by another deep discount.

**References**


### Table A1: Notation

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Reservation price</td>
</tr>
<tr>
<td>$t$</td>
<td>Stockpiling threshold</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of consuming extra unit</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Proportion of loyal consumers for each firm</td>
</tr>
<tr>
<td>$\beta = 1 - 2\alpha$</td>
<td>Proportion of switchers</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Stockpiling fraction of loyal consumer</td>
</tr>
<tr>
<td>$s$</td>
<td>Superscript for state $s = {00, 01, 10, 11}$</td>
</tr>
<tr>
<td>$F^S_i(p_i), F^S(p)$</td>
<td>Cumulative distribution functions in state $s$</td>
</tr>
<tr>
<td>$F^S_i(t), F^S(t)$</td>
<td>Probability that firm will charge a price below the stockpiling threshold in state $s$</td>
</tr>
<tr>
<td>$V^S_i, V^S$</td>
<td>Firm’s net payoff in state $s$</td>
</tr>
<tr>
<td>$\psi^s$</td>
<td>Probability of state $s$ in equilibrium</td>
</tr>
<tr>
<td>$M^S_i(r), M^S(r)$</td>
<td>Mass point at ‘$r$’ in state $s$</td>
</tr>
<tr>
<td>$M^S_i(t), M^S(t)$</td>
<td>Mass point at ‘$t$’ in state $s$</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Firms’ discount factor</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>Consumer discount factor</td>
</tr>
<tr>
<td>$U^0_i, U^1_i$</td>
<td>Stockpiling consumer’s net utility from purchase when she has zero inventory or inventory of one respectively</td>
</tr>
<tr>
<td>$G^0(t), G^1(t)$</td>
<td>Probability of encountering a low price when she has zero inventory or inventory of one respectively</td>
</tr>
<tr>
<td>$\int_0^r p g^0(p) dp, \int_r^t p g^1(p) dp$</td>
<td>Expected price of her favorite brand conditioned on prices being below stockpiling threshold when she has zero inventory and inventory of one respectively</td>
</tr>
<tr>
<td>$\int_0^r p g^0(p) dp, \int_r^t p g^1(p) dp$</td>
<td>Expected price of her favorite brand conditioned on prices being above stockpiling threshold when she has zero inventory and inventory of one respectively</td>
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