An Equilibrium Analysis of Daily Deal Strategies: When Should a Daily Deal Website Display Deal Sales?

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Abstract

Daily deal websites help small local merchants attract new consumers. One strategy adopted by some deal websites is displaying real-time deal sales information. We investigate a deal website’s strategic motive to display deal sales in a model where the merchant is privately informed of its type (probability of meeting consumer needs). We obtain three main results. First, displaying sales can help the website attain its maximum profits by enabling the high-type merchant to credibly signal through its deal price. Second, in some situations however, the website prefers to suppress signaling by not displaying sales even if the high-type merchant prefers to signal. Crucial to both results is the role of observational learning from deal sales by new consumers. Third, it can be optimal for the website to provide the merchant an upfront subsidy if deal sales are displayed. Our analysis leads to managerial insights for deal websites.

(Keywords: Daily Deals, Observational Learning, Perfect Bayesian Equilibrium, Signaling)

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As customers, we like the [deal sales] counter because it indicates how popular deals are.
- Director of Communications, Groupon (Groupon 2011)

We were concerned that the counter was having a negative impact on the consumers’ perception of the deal.
- VP of Research, TroopSwap

1 Introduction

Daily deal websites have emerged as popular means for small merchants to conduct online promotions. A consumer visiting a daily deal website can not only see the details of the deal but may also be able to see how many other consumers have bought the deal. Examples of such daily deal websites include Groupon, LivingSocial, and Amazon Local. Why might a daily deal website display the number of deals sold? As the opening quote suggests, this information may benefit consumers. In fact, Groupon provided this as the reason for continuing to display deal sales despite concerns that the information might be used by stock market analysts to predict its financial performance (Groupon 2011). One also observes however that not all daily deal websites display deal sales (e.g., AP Daily Deals, Restaurant.com, ValPak) and some websites that previously displayed deal sales no longer do (e.g., Dealsaver, KGB Deals, Tippr). In this paper we analyze a model of strategic interaction between a merchant, consumers, and the daily deal website to answer the question of under what conditions the strategy of displaying deal sales is an equilibrium outcome. Specifically, we seek to understand the strategic impact that displaying deal sales can have on website profits. Our analysis also leads to managerial insights and recommendations.

The daily deal website is one among the many innovative business models that have recently emerged on the Internet. Daily deals are so called because new deals from different merchants are announced on the website every day. Each deal is available on the website for a specified period of time ranging from a few days to a week or two. Most daily deals target subscribers of the website in a given city and are offered by merchants in that city, such as restaurants, spas and gyms. In a relatively short period of time, daily deal websites have become popular in many countries across the world. In the U.S., consumer spending on daily deals is estimated to have grown from $873 million in 2010 to $3.6 billion in 2012 and is expected to exceed $5 billion by 2015 (BIA/Kelsey 2011, 2012). Thus the study of daily deal website strategies is both relevant and important.

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1 Authors’ e-mail communication with the company.
2 Groupon made the statement in the opening quote when it changed how it displays deal sales. Since Nov 2011, Groupon displays an approximate figure for deal sales to discourage analysts from using this information to predict its financial performance. For instance, if 143 deals were sold, it might display “Over 125 bought”.
3 These daily deal websites are not publicly listed companies. Therefore, the concern about stock market analysts does not apply.
Daily deals serve to increase awareness of a merchant amongst potential consumers and entice new consumers through discounts. Indeed, in a recent survey of small businesses, a majority identified daily deals as the most effective online tool to attract new consumers (Clancy 2013). Similarly, other small business surveys report that a majority of daily deal consumers are new to the business (Dholakia 2011, 2012; Edison Research 2012; Pletz 2013). What immediately comes to mind is whether daily deal websites are simply online counterparts of traditional coupon mailer companies that distribute coupons from local merchants to consumers by mail. A closer look brings forth important differences between the two.

Unlike coupon mailer companies, a daily deal website can monitor consumer purchases linked to the deal with great ease because of the reduced cost of interaction on the Internet. How can the website use this capability? As noted earlier, an interesting strategy used by some daily deal websites is to display the number of deals sold in real time. Who benefits from this? Can this be an equilibrium outcome? If this strategy is profitable to the site, are there ways to make it more effective? We seek to address these questions in this paper.

Another way a daily deal website differs from coupon mailer companies is that it also enables transactions between the merchant and consumers. To avail a daily deal, a consumer must buy the deal through the website and redeem it later at the merchant. The website then receives a share of the deal sales revenues for facilitating the transaction. Hence, the website is paid only if a sale occurs. That is to say, it is paid for performance. Again this is possible by the ability to monitor transactions. In this important way, the business model of a daily deal website differs from that of a coupon mailer, which cannot monitor transactions between the merchant and consumers and is therefore paid an upfront fixed fee. In this paper, we will also analyze the equilibrium revenue-sharing contract between the merchant and the website.

We analyze a model in which the merchant is privately informed about its type (probability of meeting consumer needs). We obtain three main results in this setting. First, displaying sales can help the website attain its maximum profits by enabling the high-type merchant to credibly signal through its deal price. Therefore, the website can benefit from displaying sales. Second, in some situations, however, the website prefers to suppress signaling by not displaying sales even if the high-type merchant prefers to signal. Crucial to both results is the role of observational learning: displaying deal sales enables new consumers to infer their utility of buying the deal by observing the response of other consumers to the deal (i.e., the number of deals sold) which reflect their private information. We find that observational learning can be a double-edged sword.
In particular, a “little” observational learning hurts the website even if it benefits the high-type merchant. Consequently, displaying deal sales is not a dominant strategy. This could explain why some daily deal websites display deal sales and others do not.

Lastly, we show that it can be optimal for the website to provide the merchant an upfront subsidy if deal sales are displayed. This is interesting because daily deal websites such as Groupon and LivingSocial are known to employ a sizable team of copywriters to help the merchant develop the promotional material for the deal. They offer this service to merchants free of charge even though it is costly to provide. Hence, it is a subsidy. Our analysis also leads to offer managerial insights for daily deal websites that we discuss in §5.

1.1 Related Literature

There is growing research on daily deals. Much of the work has focused on situations in which there is a minimum number of deals that must be sold before the deal is valid. Anand and Aron (2003) show that a minimum limit may function as a quantity-discount schedule in markets where there is uncertainty about the level of demand. Jing and Xie (2011) show that a minimum limit may motivate consumers to act as “sales agents” to induce other consumers to buy the deal to ensure that the minimum limit is reached. Only Hu, Shi, and Wu (2013) have found a strategic role for displaying sales. They show that displaying deal sales plays a role in informing consumers whether the minimum limit will be reached, thereby co-ordinating their buying decisions. They find that this always benefits the seller. We, on the other hand, find a strategic role for displaying deal sales even in the absence of minimum limits. This is relevant because daily deal websites that do not use minimum limits also display deal sales (e.g. LivingSocial, Amazon Local) and those that previously employed minimum limits no longer do (e.g., Groupon). Also, to our knowledge, previous work has not explicitly examined the role of the daily deal website in an equilibrium framework.

Researchers have studied firm strategies when consumers can infer product quality by observing past sales. Caminal and Vives (1996) show that firms may compete more aggressively for market share in order to signal-jam consumer inferences. Bose, Orosel, Ottaviani, and Vesterlund (2006) show that the firm may distort its price to current buyers to facilitate information revelation to future buyers. Taylor (1999) shows that in housing markets, an individual house seller may distort its price in order to minimize the negative inferences associated with her house remaining unsold. Miklós-Thal and Zhang (2013) show that a monopolist may visibly de-market its product to early adopters in order to improve the product’s quality image amongst late adopters. We examine whether an intermediary, namely the daily deal website, should enable consumers to observe deal
sales by displaying it. We show that displaying deal sales can allow the merchant to signal through its deal price but this may not be in the website’s interests.

Starting with the seminal works of Nelson (1974), Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986), past research has examined how a firm can signal its private information about quality to consumers. In our model, this occurs in the presence of an intermediary who is also strategic. Thus, the website plays a crucial role in determining whether and how a merchant can convey private information to consumers. Indeed we find that under some conditions the high-type merchant wants to signal through price but is unable to do so because it is not in the interest of the website. In this manner, we add to the extant literature. Turning specifically to the role of price in revealing private information, prior research has also focused on whether price alone can signal a firm’s privately known quality. Milgrom and Roberts (1986) show that a firm may use price alone or price and non-informative advertising to signal its product quality in a setting with repeat purchases. Desai (2000) shows that a manufacturer may use a combination of wholesale price, slotting allowance, and advertising to signal demand for its product to a retailer. Moorthy and Srinivasan (1995) show that a combination of price and money-back guarantee may be necessary to signal product quality. Simester (1995) and Shin (2005) show that advertising prices of selected products can credibly signal the price image of a low-cost retailer. Our work has some similarity to Bagwell and Riordan (1991) who have examined the role of informed consumers in enabling the high-quality firm to signal through price. We study situations in which only if the website displays deal sales do informed consumers play an indirect role in enabling signaling. Stock and Balachander (2005) also find that a firm may not be able to signal its privately known quality unless consumers are aware about the product’s scarcity. We add to this literature by showing that reporting deal sales and observational learning may be necessary to support signaling through deal price.

Empirical researchers have found that a website can facilitate observational learning and influence consumer decisions by displaying popularity information (Chen and Xie 2008; Tucker and Zhang 2011; Zhang and Liu 2012; Luo, Andrews, Song, and Aspara 2014). In particular, Luo, Andrews, Song, and Aspara (2014) provide evidence for observational learning facilitated by deal sales information on a daily deal website. In a similar vein, Zhang and Liu (2012) find evidence of observational learning on a crowdfunding website, wherein lenders infer the creditworthiness of the borrower from the funding level. The question remains whether providing consumers such information is beneficial for the website. We address this question using a theoretical framework.

Grossman (1981) and Milgrom (1981) have shown that a firm will disclose its private quality
information if it is verifiable. Subsequent research has examined whether this “unraveling” result holds under different conditions (e.g., Okuno-Fujiwara, Postlewaite, and Suzumura 1990; Anderson and Renault 2009; Guo and Zhao 2009; Kuksov and Lin 2010; Sun 2011). Guo (2009) compares disclosure in a channel by a manufacturer vs. by a retailer when the product quality is known to them. In contrast to this literature stream, we study situations in which the merchant cannot disclose its type credibly and the website does not know the merchant’s type. We show that the website may either facilitate or suppress signaling by the merchant.

Our work is also broadly related to research on firm-level marketing strategies to leverage different forms of social interactions (e.g., Biyalogorsky, Gerstner, and Libai 2001; Amaldoss and Jain 2005; Godes et al. 2005; Mayzlin 2006; Chen and Xie 2008; Joshi et al. 2009; Kornish and Li 2010; Kuksov and Xie 2010; Jing 2011; Godes 2012). We study a deal website’s strategy of whether to enable one form of social interaction, namely observational learning from knowing the response of other consumers to the deal. Lastly, we should note that researchers have also examined a website’s incentives in helping consumers make more informed decisions in other contexts. Wu, Zhang, and Padmanabhan (2013) show that a match-making website may have an incentive to deliberately reduce the effectiveness of its matching technology. Liu and Dukes (2014) show that an online shopping intermediary may design a search environment that limits search by consumers.

In what follows, we describe our model of merchant, consumers and the daily deal website in §2. In §3, we establish two important results in this setting. First, we show that only if deal sales are displayed can the merchant credibly signal its type through its deal price. Further, in such a case, the website may be able to attain its maximum profits as the signaling may not involve any distortion. We also establish the second result that displaying deal sales is not a dominant strategy for the website. Then, in §4, we analyze the revenue-sharing contract between the website and the merchant and show that it can be optimal for the website to provide the merchant an upfront subsidy. Finally, we investigate extensions in which deal sales is a “noisy” indicator of the merchant’s type, and the merchant types differ in more than one dimension to generate additional insights and identify boundary conditions for some of our results. In §5, we discuss the managerial and practical implications of our results and offer directions for future work.

2 Model

We consider a market in which a merchant offers consumers a product (or service). The merchant’s product can meet the needs of some but not all consumers. In our model, the merchant can be one of two types - $H$ or $L$. A type $t \in \{H, L\}$ merchant’s product meets the needs of a proportion
$\alpha_t \in (0, 1)$ of consumers, where $\alpha_H > \alpha_L$. For a randomly chosen consumer, the product will meet the needs with probability $\alpha_t$. We therefore refer to $\alpha_t$ as the merchant’s probability of fit, or simply fit. The product delivers a positive utility $r > 0$ if it meets a consumer’s needs and zero utility otherwise. We will refer to $r$ as the merchant’s value. A merchant’s fit can be understood as the merchant’s capability to cater to the disparate needs of different consumers. Thus, it captures one dimension of how well the merchant can meet consumer needs. The merchant’s value captures another dimension. Later, in §4, we examine a setting in which the merchant types also differ in their value.

The merchant can reach new consumers by offering a deal through a daily deal website. But not all consumers who visit the website are new to the merchant. We assume that some consumers are already aware of the merchant, know the merchant’s type, and whether its product meets their needs. For instance, these could be consumers who have tried the merchant’s product in the past. We therefore refer to these consumers as experienced consumers. We assume that the remaining consumers who visit the website are not aware of the merchant, and hence neither know the merchant’s type nor whether its product will meet their needs. We refer to these consumers as new consumers. Let $N$ denote the size of new consumers. Without loss of generality, we normalize the size of experienced consumers to 1. Thus $N$ captures the relative proportion of new consumers.

We can now write down the utility from buying the deal for experienced consumers and new consumers. We assume that a consumer may buy at most one unit of the product and derives zero utility if she does not buy. Let $d_t > 0$ denote the deal price at which a type $t$ merchant offers the product. An experienced consumer’s utility from buying the deal is given by,

$$u_{EC} = i \cdot r - d_t,$$

where $i \in \{0, 1\}$ is an indicator variable that equals 1 if the product meets this consumer’s need. It should be noted that $i$ and $t$ are both known to an experienced consumer. Unlike experienced consumers, a new consumer is uncertain about the merchant’s type and whether its product will meet her needs. Conditional on the merchant’s type being $t$, her expected utility from the product is $r\alpha_t$. Therefore, a new consumer’s expected utility from buying the deal is given by

$$u_{NC} = \theta r\alpha_H + (1 - \theta) r\alpha_L - d_t,$$

where $\theta$ denotes her belief that the merchant’s type is $H$. In general, $\theta$ may depend on all observables including the deal price and deal sales if it is displayed.

We assume that a deal is available on the website for two periods, namely periods 1 and 2. We
assume that some consumers visit the website in both periods and refer to them as *frequent visitors*. Frequent visitors can buy the deal either in period 1 or in period 2. Other consumers are not able to visit the website frequently. They visit the website either only in period 1 or only in period 2 and we refer to them as *early visitors* and *late visitors*, respectively. The frequency of visits is an exogenous feature of our model. We note that consumers typically do not know a priori whether or when a particular merchant will offer a deal. Therefore, we model the visits of early and late visitors as being random relative to the deal timing (i.e., period 1) and assume that there is an equal number of either of them.\(^4\) Let \(\beta \in [0, 1]\) denote the proportion of consumers who visit the website only once. Thus, a proportion \(\frac{1}{2}\beta\) are early visitors, a proportion \(\frac{1}{2}\beta\) are late visitors and a proportion \(1 - \beta\) are frequent visitors. These proportions are the same for experienced consumers and new consumers. Thus, each consumer can be characterized along two dimensions - experienced vs. new, and frequent vs. early vs. late. For conciseness, we simply say “early-new consumers” to refer to new consumers who are early visitors and so on.

In our model, offering a deal on the website can generate new sales for the merchant from new consumers but can also lead to cannibalization. This cannibalization results from experienced consumers on the website who would have still bought from the merchant if a deal were not offered. Let \(p > 0\) denote the merchant’s *regular price*. If a deal is not offered, we assume that the experienced consumers can buy at the regular price. We take the regular price \(p\) to be exogenous noting that it will only influence merchant revenues if a deal is not offered. Let \(R_t^O\) denote the revenues for a type \(t\) merchant in this case. We have \(R_t^O = p\alpha_t\), where \(\alpha_t\) is the number of experienced consumers who are willing to pay a positive price \(p < r\). We should note that for the analysis to be meaningful, the regular price cannot be equal to or lower than the deal price. In particular, since the deal is meant to attract new consumers and because new consumers are uncertain about the utility from the product, it is natural to think about the deal price as being at a discount relative to the regular price.\(^5\) A sufficient condition to ensure that this occurs in equilibrium is

\[
p > r\alpha_H,
\]

where \(r\alpha_H\) is the upper bound for the maximum price that uninformed consumers will pay (as seen from equation (2)).

Consistent with practice, we assume that deal revenues are shared by the merchant and the

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\(^4\)If, for a given deal, some early visitors are able to return in period 2, then this will be equivalent in our model to assuming that there are more frequent visitors.

\(^5\)In particular, from equations (1) and (2), we note that new consumers will be willing to pay less than experienced consumers whose needs are met.
website. Let $R_t^D$ denote the deal revenues for a type $t$ merchant if it offers a deal. Let $\Pi^W$ denote the website’s expected profits. Let $\lambda \in (0, 1)$ denote the merchant’s share of revenues. This sharing can be profitable for both the merchant and the website as long as offering the deal generates incremental revenues, i.e., $R_t^D > R_t^O$. To begin with, we will assume that $\lambda$ is exogenous and sufficiently high such that if a deal can generate incremental revenues, then it will also be profitable for the merchant to offer a deal. We make this more precise as we proceed with the analysis. Assuming $\lambda$ to be exogenous in this manner allows us to bring out the essential effects of displaying deal sales on consumers and the merchant, and thereby the website’s incentives to display deal sales. Having established these effects, we later examine the website’s optimal equilibrium contract in §4.

The website can choose between two possible website regimes: (i) displaying deal sales, and (ii) not displaying deal sales. In the former case, deal sales are displayed at the start of period 2. We will compare the outcomes in these two subgames to understand the role of displaying deal sales. Before we proceed to the analysis, it is useful to make clear the sequence of the game:

**Stage 1 (Period 0):** Website decides whether to display deal sales. The decision is known to the merchant and consumers.

**Stage 2 (Period 0):** Merchant decides whether to offer a deal and deal price $d_t$ if it offers a deal.

**Period 1 (if deal is offered):** Early and frequent visitors visit the website and decide whether to buy the deal. Frequent visitors can also decide to wait till period 2.

**Period 2 (if deal is offered):** Frequent and late visitors visit the website and decide whether to buy the deal. They observe period 1 sales before they buy if it is displayed.

We assume that prior to offering its deal, the merchant knows its type and this is private information. The website and new consumers have a belief about the type of the merchant and this is common knowledge. This belief can be conditioned on the regular price $p$ and other information about the attributes of the merchant. Nevertheless, it will be useful to think of this belief as the initial or prior belief about the merchant’s type (at the start of period 0). Denote this belief by $\tilde{\theta} \in (0, 1)$. The prior belief $\tilde{\theta}$ can be contrasted with the belief $\theta$ used in equation (2), which is a new consumer’s posterior belief at the time of making her buying decision. This posterior belief will depend on the website regime, the merchant’s decision to offer a deal, and the deal price. For new consumers making their buying decisions in period 2, their posterior belief will also depend on the realized period 1 sales if it is displayed.
We assume that firms maximize their expected profits and consumers maximize their expected utility. We solve for a perfect Bayesian equilibrium (PBE). We restrict our attention to pure-strategy equilibria. It is well known that in games of incomplete information, multiple PBE can be supported by specifying sufficiently pessimistic off-equilibrium beliefs so as to make any deviation unattractive. In our context, this pertains to specifying pessimistic beliefs for new consumers (i.e., $\theta = 0$) at off-equilibrium deal prices leading to multiple equilibria for merchant strategies. As in Miklós-Thal and Zhang (2013), because the intuitive criterion refinement (Cho and Kreps 1987) is not sufficiently strong to rule out unreasonable off-equilibrium beliefs in our model, we use the strongly-undefeated equilibrium (SUE) refinement (Mailath, Okuno-Fujiwara, and Postlewaite 1993; Spiegel and Spulber 1997; Taylor 1999; Mezzetti and Tsoulohas 2000; Gomes 2000; Gill and Sgroi 2012) to obtain a unique equilibrium. As noted in Miklós-Thal and Zhang (2013), the SUE refinement is equivalent to selecting the PBE that yields the type $H$ merchant the highest profits (amongst all PBEs). This property has intuitive appeal because it is the type $L$ merchant that will have an incentive to mimic the type $H$ merchant and not vice-versa. Thus, the SUE essentially allows the type $H$ merchant to follow its sequentially optimal strategy given that the type $L$ merchant can mimic its strategy. It is important to note that, in our model, the SUE is also the unique PBE that survives the intuitive criterion and yields the highest profits for both merchant types. We provide a more formal description of the SUE refinement in Appendix A.

3 Strategic Role of Displaying Deal Sales

Let $\bar{\alpha}$ denote the reservation price of new consumers based on their prior belief $\bar{\theta}$. Without loss of generality, we can normalize $r = 1$. From equation (2), we have

$$\bar{\alpha} = \bar{\theta}\alpha_H + (1 - \bar{\theta})\alpha_L$$

(4)

The following observations are useful for our analysis. First, if a merchant offers a deal, then its profits are given by $\lambda R_t^D$. Therefore, its deal price is always set to maximize $R_t^D$. It follows that if $R_t^D > R_t^O$ then there always exists a suitable $\lambda$ such that $\lambda R_t^D > R_t^O$. Thus, our assumption that $\lambda$ is sufficiently high such that it is profitable for the merchant to offer a deal if the deal generates incremental revenues is not restrictive. It also follows that a necessary and sufficient condition for the merchant to offer a deal is $R_t^D > R_t^O$.6

Second, if a deal is offered in equilibrium, then it must be that at least some new consumers buy the deal. Otherwise, the deal cannot generate incremental revenues (i.e., $R_t^D \leq R_t^O$) and offering

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6Alternatively, and equivalently, we can rescale the regular price $p$ by $\frac{1}{\lambda}$ and all our results will remain unaffected.
a deal cannot be mutually profitable for the merchant and the website. It follows that it will be sufficient to verify whether it is profitable to offer a deal in the price range \( d_t \in [\alpha_L, \alpha_H] \) since the reservation price of a new consumer can only be in this range. Since \( d_t \in [\alpha_L, \alpha_H] \) is below the regular price, it also follows that the experienced consumers who are willing to pay a positive price will always buy the deal in any equilibrium where a deal is offered. With some abuse of terminology, from here on, we will use the term “experienced consumers” to refer only to these \( \alpha_t \) experienced consumers who buy the deal.

Lastly, how will displaying deal sales affect consumer utility? As should be clear from equation (1), displaying deal sales will have no effect on the utility of experienced consumers. Thus, it will not influence their buying decisions. Amongst new consumers, displaying deal sales will also have no effect on the utility of early visitors because they visit only in period 1 and cannot observe deal sales. Displaying deal sales will however influence the utilities of both frequent- and late-new consumers in period 2. Before deciding whether to buy, these consumers can observe deal sales and revise their beliefs \( \theta \) in equation (2) about the merchant’s type. We further note that the opportunity to update their beliefs can influence frequent visitors to wait till period 2 to make their buying decision.

Our analysis proceeds as follows. We first show that website profits cannot be higher than in a benchmark setting in which new consumers are assumed to know the merchant’s type \( t \). Since the source of asymmetric information in our model is about the merchant’s type, we will refer to this benchmark setting as the symmetric information benchmark. We then show that the website can attain the benchmark profit level only if it displays deal sales, thus establishing that the website can benefit from displaying deal sales. We then show that displaying deal sales is however not a dominant strategy and characterize the situations in which it is counterproductive.

3.1 Symmetric Information Benchmark

If new consumers know the merchant’s type \( t \), then they will be willing to pay \( \alpha_t \). Recall that offering a deal can be profitable only if at least some new consumers buy the deal. Therefore, conditional on offering a deal, the optimal deal price is \( d_t = \alpha_t \). Deal revenues are then given by,

\[
R^D_t = \alpha_t (\alpha_t + N).
\]  

Recall our assumption that \( \lambda R^D_t > R^O_t \) whenever \( R^D_t > R^O_t \). Therefore, it will be profitable for the merchant to offer a deal iff \( R^D_t > R^O_t \), which yields,

\[
\alpha_t (\alpha_t + N) > p \alpha_t. 
\]
The foregoing inequality will hold if $N$ is sufficiently large. In other words, offering a deal is profitable if there are sufficient number of new consumers on the website (relative to the number of experienced consumers). To keep our analysis straightforward, we will assume that $N > \bar{p}$, which is sufficient to ensure that both merchant types will offer a deal in the symmetric information benchmark. In other words, without asymmetric information, it is efficient for both merchant types to offer a deal.

Thus it can be seen that under symmetric information the deal price reflects the merchant’s fit, i.e., $d_t = \alpha_t$. Owing to its higher fit, the type $H$ merchant charges a higher deal price than the type $L$ merchant. All experienced consumers buy the deal and obtain a surplus of $p - \alpha_t$ (relative to buying at the regular price). All new consumers also buy the deal and their ex-ante surplus is zero. Website profits in the symmetric information benchmark are given by,

$$\Pi^W_{SI} = (1 - \lambda) \left( \bar{\theta} \alpha_H (\alpha_H + N) + (1 - \bar{\theta}) \alpha_L (\alpha_L + N) \right)$$

Clearly, if new consumers did not know the merchant’s type, then the type $H$ merchant’s profits cannot be higher than in this benchmark. It is not immediately obvious however whether website profits can be higher than $\Pi^W_{SI}$ because its profits also depend on type $L$ merchant revenues. We show in Lemma 1 that website profits cannot in fact exceed $\Pi^W_{SI}$ whether or not deal sales are displayed. Moreover, the website can attain benchmark profits only in a separating equilibrium that resembles the benchmark outcome, i.e., $d_t = \alpha_t$ and all experienced consumers and new consumers buy the deal. We establish this result without explicitly solving for the equilibrium by constructing the upper bound for website profits in any potential PBE.

**Lemma 1.** Website profits cannot be higher than that in the symmetric information benchmark, and can be equal to it only in a separating equilibrium in which both merchant types offer a deal at a price $d_t = \alpha_t$ and all experienced consumers and new consumers buy the deal.

**Proof.** See Appendix B.

In the benchmark, each merchant type charges the reservation price $\alpha_t$ of new consumers and all consumers buy the deal. Under asymmetric information, neither merchant type can do any better in any potential separating equilibrium, since their equilibrium deal price cannot be higher than $\alpha_t$. Therefore, website profits cannot be higher than in the benchmark. In any potential pooling equilibrium, if the deal price is higher than $\bar{\alpha}$, then the type $L$ merchant always has an incentive to deviate since it cannot sell to any new consumers at this price. Therefore, the deal price is at most $\bar{\alpha}$ in a pooling equilibrium and website profits cannot be higher than in the benchmark. Specifically, if all consumers buy at a deal price of $\bar{\alpha}$, then on average (across the two merchant types), the website
earns the same profits from new consumers as in the benchmark. This is because new consumers pay $\alpha$ on average in the benchmark. But profits from experienced consumers are on average lower than in the benchmark. This is because the type $H$ merchant, which has more experienced consumers than the type $L$ merchant, sets a lower price than in the benchmark. In other words, there is higher cannibalization of revenues from experienced customers in a pooling equilibrium. Consequently, website profits are strictly lower in a pooling equilibrium than in the benchmark. It follows that the website can obtain benchmark profits only in a separating equilibrium as described in Lemma 1, because this maximizes the revenues from new consumers and minimizes the cannibalization of revenues from experienced consumers.

3.2 Can Displaying Deal Sales Benefit the Website?

Lemma 1 provides us a convenient way to establish that displaying deal sales can benefit the website. We show that a separating equilibrium that resembles the benchmark can exist in certain situations, but only if the website displays deal sales. From Lemma 1 it follows then that, in these situations, website profits are strictly higher if deal sales are displayed.

**Lemma 2.** If deal sales are not displayed, then there is no separating equilibrium in which both merchant types offer a deal.

*Proof (by contradiction).* Suppose towards a contradiction there exists a separating equilibrium in which both merchant types offer a deal. As shown in the proof of Lemma 1, it must be that $d_L = \alpha_L$ and $d_H \leq \alpha_H$. Further, $d_H > \alpha_L$ since the reservation price of new consumers cannot be below $\alpha_L$ irrespective of their beliefs. In this equilibrium, new consumers must believe that the merchant’s type is $t$ if the deal price is $d_t$ and be willing to pay $\alpha_t$. Therefore, all experienced consumers and new consumers will buy the deal at either deal price. But then, the type $L$ merchant can earn a higher margin for the same demand by deviating to $d_H > \alpha_L$. New consumers will still buy the deal as they mistakenly believe that the type $L$ merchant is of type $H$. Experienced consumers will also buy the deal since the deal price is still below the regular price. Therefore, the type $L$ merchant can profitably mimic the type $H$ merchant, which is a contradiction.

Thus, if deal sales are not displayed, then the merchant types cannot signal through the deal price. This is because the type $L$ merchant always finds it profitable to mimic the type $H$ merchant. As a result, the type $H$ merchant cannot credibly charge a higher deal price. In particular, the buying decisions of experienced consumers, who are informed about the merchant’s type, cannot discipline the deal price because these consumers always find it attractive to buy the deal irrespective of the merchant’s type. We next show that mimicking can be prevented by displaying deal sales.
Proposition 1. The website can attain the same profits as in the symmetric information benchmark iff the following conditions hold:

(i) Deal sales are displayed;

(ii) At the deal price $\alpha_H$, at least some new consumers condition their buying decision on the realized period 1 deal sales. They buy if deal sales is $\tau_H$ and do not buy if deal sales is $\tau_L$, where $\tau_t = \alpha_t \left(1 - \frac{1}{2}\beta\right) + \frac{1}{2}N\beta$;

(iii) $\alpha_L \geq \alpha_1$, where $\alpha_1 = \frac{1}{2} \left[\sqrt{N^2 - 2N(1-\beta)\alpha_H + \alpha_H^2} - (N - \alpha_H)\right]$ and $\alpha_1 \in (0, \alpha_H)$.

Proof (by construction). We know from Lemma 2 that benchmark profits cannot be attained if deal sales are not displayed. If deal sales are displayed but new consumers do not condition their buying decisions on deal sales, then the situation is identical to that when deal sales are not displayed. It remains to be shown that if deal sales are displayed, then there is an equilibrium in which the type $t$ merchant sets a deal price $d_t = \alpha_t$, all experienced consumers and new consumers buy and some new consumers condition their buying decision on period 1 sales. Further, such an equilibrium can occur iff $\alpha_L \geq \alpha_1$. We prove this by construction. Suppose that the type $t$ merchant offers a deal at a price $d_t = \alpha_t$ and consumers adopt the following strategies:

- Experienced consumers buy the deal at either deal price regardless of the merchant’s type, with frequent visitors buying in period 1.

- If the deal price is $\alpha_L$, then all new consumers buy the deal, with frequent visitors buying in period 1.

- If the deal price is $\alpha_H$, then early-new consumers buy in period 1 and frequent-new consumers wait till period 2. In period 2, frequent- and late-new consumers buy if period 1 sales equals $\tau_H$ and do not buy if period 1 sales equals $\tau_L$, where $\tau_t = \alpha_t \left(1 - \frac{1}{2}\beta\right) + \frac{1}{2}N\beta$.

Given the above consumer strategies, we note that $\tau_t$ is the number of consumers who will buy in period 1 if a type $t$ merchant offered the deal at a price $\alpha_t$. Specifically, $\alpha_t \left(1 - \frac{1}{2}\beta\right)$, is the number of experienced consumers who will buy in period 1 and $\frac{1}{2}N\beta$ is the number of new consumers who buy in period 1. We have $\tau_H \geq \tau_L$, since more experienced consumers buy from the type $H$ merchant owing to its higher fit. Thus, by conditioning their buying decisions on the realized deal sales, frequent- and late-new consumers can ensure that they buy at a deal price $\alpha_H$ only from the type $H$ merchant. We now show that these merchant and consumer strategies constitute a PBE iff $\alpha_L \geq \alpha_1$ and refer to it as our candidate equilibrium.

Clearly, consumer strategies are optimal given the merchant strategies. All experienced con-
sumers buy the deal and obtain a positive surplus $p - d_t$. All new consumer also buy and their ex-ante surplus is zero. Consumers cannot obtain a higher utility by deviating from their strategies. Further, the type $H$ merchant can also not do any better. It cannot sell to new consumers at any price higher than $\alpha_H$ and offering a deal is profitable since condition (6) holds. Similarly, it is also profitable for the type $L$ merchant to offer a deal.

Can the type $L$ merchant do better by mimicking the type $H$ merchant? Suppose that the type $L$ merchant deviated to $d_L = \alpha_H$. Deal sales in period 1 is then $\tau_L$. Consequently, frequent- and late-new consumers will not buy in period 2. Therefore, the type $L$ merchant will realize total deal sales of $\alpha_L + \frac{1}{2}N\beta$. Let $R'_L$ denote the corresponding revenues, given by,

$$R'_L = \alpha_H \left( \alpha_L + \frac{1}{2}N\beta \right).$$

(8)

The type $L$ merchant’s revenues at the equilibrium deal price are given by,

$$R^D_L = \alpha_L (\alpha_L + N).$$

(9)

In equilibrium, we require that the following no-mimicking constraint holds:

$$R^D_L \geq R'_L \implies \alpha_L (\alpha_L + N) \geq \alpha_H \left( \alpha_L + \frac{1}{2}N\beta \right)$$

(10)

We note that the no-mimicking constraint will hold if $\alpha_L \to \alpha_H$ and will not hold if $\alpha_L \to 0$. Therefore, by continuity, there exists $\alpha_1 \in (0, \alpha_H)$ such that non-mimicking constraint holds iff $\alpha_L \geq \alpha_1$. It is straightforward to show that $\alpha_1$ is as defined in the statement of the proposition.

As shown in the proof of Lemma 1, $d_H \leq \alpha_H$ and $d_L = \alpha_L$ in any separating PBE and $d_t \leq \bar{\alpha}$ in any pooling PBE. This has three implications. First, the type $H$ merchant cannot derive higher profits in any other PBE than in the candidate equilibrium. Therefore, the candidate equilibrium is a SUE. Second, any other (separating) PBE that yields the same profits for the type $H$ merchant as the candidate equilibrium cannot lead to a different outcome.⁷ Hence, the equilibrium outcome is unique. Lastly, any such PBE that leads to the same outcome can only exist for a narrower range of parameters than the candidate equilibrium, because the no-mimicking constraint cannot be less restrictive than equation (10). Thus, it is sufficient to focus on the candidate equilibrium. □

We have thus identified a role for the website to display deal sales that comes into play under asymmetric information. Displaying deal sales can induce the merchant to price its deal such that website profits are maximized. Specifically, the type $H$ merchant can credibly signal to early-new consumers by deviating to $d_L = \alpha_H$. Deal sales in period 1 is then $\tau_L$. Consequently, frequent- and late-new consumers will not buy in period 2. Therefore, the type $L$ merchant will realize total deal sales of $\alpha_L + \frac{1}{2}N\beta$. Let $R'_L$ denote the corresponding revenues, given by,

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$$R^D_L \geq R'_L \implies \alpha_L (\alpha_L + N) \geq \alpha_H \left( \alpha_L + \frac{1}{2}N\beta \right)$$

(10)

We note that the no-mimicking constraint will hold if $\alpha_L \to \alpha_H$ and will not hold if $\alpha_L \to 0$. Therefore, by continuity, there exists $\alpha_1 \in (0, \alpha_H)$ such that non-mimicking constraint holds iff $\alpha_L \geq \alpha_1$. It is straightforward to show that $\alpha_1$ is as defined in the statement of the proposition.

As shown in the proof of Lemma 1, $d_H \leq \alpha_H$ and $d_L = \alpha_L$ in any separating PBE and $d_t \leq \bar{\alpha}$ in any pooling PBE. This has three implications. First, the type $H$ merchant cannot derive higher profits in any other PBE than in the candidate equilibrium. Therefore, the candidate equilibrium is a SUE. Second, any other (separating) PBE that yields the same profits for the type $H$ merchant as the candidate equilibrium cannot lead to a different outcome.⁷ Hence, the equilibrium outcome is unique. Lastly, any such PBE that leads to the same outcome can only exist for a narrower range of parameters than the candidate equilibrium, because the no-mimicking constraint cannot be less restrictive than equation (10). Thus, it is sufficient to focus on the candidate equilibrium □

⁷For instance, it may be possible to construct a separating PBE in which the no-mimicking constraint is enforced only by the late-new consumers conditioning their buying decisions on deal sales.
consumers through its deal price only if deal sales are displayed. This is different from the role that displaying deal sales can play in co-ordinating consumer choices in the presence of a minimum limit (Hu, Shi, and Wu, 2013). An interesting point is that the two merchant types separate on deal prices. But they can credibly do so only with the support of an intermediary, namely the website. Furthermore, there is no distortion in the deal prices or demand compared to the symmetric information benchmark. This differs from what we commonly encounter in signaling models, where signaling entails a distortion to avoid mimicking.

Displaying deal sales helps because it enables frequent- and late-new consumers to avoid buying from the type $L$ merchant if it mimics the type $H$ merchant. In turn, this allows the type $H$ merchant to credibly signal through its deal price to early-new consumers. Specifically, in equilibrium, at the deal price $\alpha_H$, frequent- and late-new consumers condition their buying decisions on the realized period 1 sales. They buy if the deal sales is $\tau_H$ and do not buy if the deal sales is $\tau_L$, where $\tau_t = \alpha_t (1 - \frac{1}{2} \beta) + \frac{1}{2} N \beta$. Period 1 sales is informative since the number of experienced consumers who buy the deal in period 1 ($= \alpha_t (1 - \frac{1}{2} \beta)$) depends on the merchant’s type. In this way, displaying deal sales enables observational learning. That is to say, frequent- and late-new consumers can infer their utility of buying the deal by observing the buying decisions of other consumers which reflects their private information. Experienced consumers are necessary for observational learning to occur. If there are no experienced consumers, then observational learning cannot occur.

It is important to note that the website cannot attain the benchmark profit level if frequent- and late-new consumers do not condition their buying decisions on period 1 sales. In other words, observational learning must occur and has force in equilibrium. While frequent- and late-new consumers always buy on the equilibrium path, they do not buy off the equilibrium path. This is necessary to ensure that if the type $L$ merchant mimics the type $H$ merchant, which is an off-equilibrium occurrence, then these consumers do not buy the deal. The reason that they do not buy is because they observe deal sales of $\tau_L$, which is possible only if the merchant is type $L$, and therefore update their belief to $\theta = 0$. In other words, they identify the merchant to be of type $L$ and their expected surplus from buying the deal is negative. In contrast, on the equilibrium path, they observe deal sales of $\tau_H$ ($> \tau_L$), which indicates that the merchant is type $H$ and their expected surplus is non-negative. We note that the off-equilibrium strategy of frequent- and late-new consumers is both sequentially rational and credible in that it is robust to a small tremble by the type $L$ merchant. Specifically, given any arbitrarily small probability that the type $L$ merchant deviates to $d_L = \alpha_H$, it is the strictly dominant strategy for frequent-new consumers to wait till
period 2 and for frequent- and late-new consumers to buy only if the realized deal sales are $\tau_H$.\(^8\)

Displaying deal sales can help attain the benchmark profits only if the no-mimicking constraint (10) holds. For the type $L$ merchant, mimicking involves a trade-off between gaining a higher margin and losing demand from frequent- and late-new consumers due to observational learning. All else equal, it is easier to prevent mimicking if the fit of the type $H$ merchant ($\alpha_H$) is lower or if the fit of the type $L$ merchant ($\alpha_L$) is higher, because the gain in margin from mimicking is then lower. It is also easier to prevent mimicking if there are fewer early visitors ($\beta$ is lower) or the number of new consumers ($N$) is higher, since more consumers then condition their purchase on realized deal sales and the loss in demand from mimicking is higher. Proposition 1 shows that the no-mimicking constraint holds iff $\alpha_L \geq \alpha_1$. In particular, we note that $\alpha_1$ is decreasing in $\beta$ and tends to zero if $\beta \to 0$. In other words, if more new consumers engage in observational learning, then there is more scope for the website to attain benchmark profits by displaying deal sales.

3.3 Is Displaying Deal Sales A Dominant Strategy for the Website?

If displaying deal sales does not lead to benchmark profits, is it still beneficial for the website? Interestingly, we find that it need not be beneficial even if it does not influence the merchant’s pricing strategy. We now show that if both merchant types set the same deal price, i.e., they pool on deal price, then displaying deal sales is counterproductive for the website. We start by deriving the merchant revenues and website profits in a pooling equilibrium under each website regime. Let $R^D_{t|\text{pooling}}$ and $R^D_{t|\text{pooling}}$ respectively denote the type $t$ merchant revenues in a pooling equilibrium if the website does not and does display deal sales. Let $\Pi^W_{t|\text{pooling}}$ and $\Pi^W_{t|\text{pooling}}$ respectively denote the corresponding website profits.

If the website does not display deal sales, then in a pooling equilibrium, new consumers will maintain their prior belief $\bar{\theta}$ at the equilibrium deal price. In Lemma 3, we show that the equilibrium deal price will be $d_t = \bar{\alpha}$ and that all experienced and new consumers buy the deal. We then obtain $R^D_{t|\text{pooling}}$ and $\Pi^W_{t|\text{pooling}}$ from the equilibrium strategies.

**Lemma 3.** If the website displays deal sales, then in a pooling equilibrium:

(i) The merchant’s pricing strategy is $d_t = \bar{\alpha}$;

(ii) All experienced consumers and new consumers always buy the deal;

(iii) Merchant revenues are given by $R^D_{t|\text{pooling}} = \bar{\alpha} (\alpha_t + N)$.

\(^8\)The alert reader will note that for the equilibrium to be trembling-hand perfect, the type $H$ merchant should set a deal price $\alpha_H - \epsilon$, where $\epsilon > 0$ is arbitrarily small. This ensures that, for a sufficiently small tremble by the type $L$ merchant, early-new consumers still buy from the type $H$ merchant in period 1. Thus, the equilibrium that we have constructed in Proposition 1 is the limit of the trembling-hand perfect equilibrium as $\epsilon \to 0$.\}
(iv) Website profits are given by \( \Pi^W \bigg|_{\text{pooling}}^0 = (1 - \lambda) \bar{\alpha} (\bar{\alpha} + N) \).

Proof. Let \( d_t = d \) denote the deal price in a pooling equilibrium. In the proof of Lemma 1, we showed that \( d \leq \bar{\alpha} \) in a pooling equilibrium as otherwise new consumers will not buy. We now show that a pooling PBE in which \( d_t = d < \bar{\alpha} \) cannot be a strongly-undefeated equilibrium (SUE). Consider a pooling PBE in which \( d_t = d \in [\alpha_L, \bar{\alpha}] \). In this equilibrium, all experienced consumers and new consumers will buy the deal as they obtain non-negative surplus. Merchant revenues are given by \( R^D_t = d (\bar{\alpha} + N) \). The pooling PBE exists iff \( R^D_t > R^O_t = \rho \alpha_t \) so that offering a deal is profitable. Therefore, if a pooling PBE in which \( d < \bar{\alpha} \) exists, then a pooling PBE in which \( d = \bar{\alpha} \) must also exist, since both merchant types derive higher revenues in the latter case. Further, the latter PBE leads to higher profits for the type \( H \) merchant. Hence, a pooling PBE in which \( d < \bar{\alpha} \) cannot be a SUE.

If deal sales are displayed, then period 1 sales will be informative about the merchant’s type. We show in Lemma 4 that the equilibrium deal price is again \( d_t = \bar{\alpha} \). In this case, frequent- and late-new consumers can update their beliefs through observational learning. If they determine that the merchant’s type is \( L \), which is the case if the realized deal sales is \( \tau_L \), then they will not buy the deal since \( \bar{\alpha} > \alpha_L \). We also note that, while frequent-new consumers will obtain non-negative surplus if they buy in period 1, it is a dominant strategy for them to wait till period 2. By waiting, they can observe deal sales and avoid buying from the type \( L \) merchant. Consequently, if deal sales are displayed, the type \( L \) merchant does not realize any sales from frequent- and late-new consumers at a deal price \( \bar{\alpha} \). Lemma 4 shows that nevertheless \( d_t = \bar{\alpha} \) in a pooling equilibrium, and then derives \( R^D_t \bigg|_{\text{pooling}}^1 \) and \( \Pi^W \bigg|_{\text{pooling}}^1 \) from the equilibrium strategies.

Lemma 4. If the website displays deal sales, then in a pooling equilibrium:

(i) The merchant’s pricing strategy is \( d_t = \bar{\alpha} \);

(ii) All experienced consumers and early-new consumers always buy the deal. Frequent and late-new consumers buy iff period 1 sales are \( \tau_H \);

(iii) Merchant revenues are given by \( R^D_H \bigg|_{\text{pooling}}^1 = \bar{\alpha} (\alpha_H + N) \) and \( R^D_L \bigg|_{\text{pooling}}^1 = \bar{\alpha} (\alpha_L + \frac{1}{2} \beta N) \).

(iv) Website profits are given by \( \Pi^W \bigg|_{\text{pooling}}^1 = (1 - \lambda) \bar{\alpha} (\bar{\alpha} + N (\bar{\theta} + \frac{1}{2} \beta (1 - \bar{\theta}))) \).

Proof. See Appendix B. □

Comparing the outcomes under the two website regimes from Lemmas 3 and 4, we immediately see that website profits are higher in the regime where the website does not display deal sales. We
show in Proposition 2 that if pooling occurs if deal sales are displayed, then pooling must also occur if deal sales are not displayed. Specifically, we show that the conditions for pooling to occur are less stringent if deal sales are not displayed because in this regime the type \( L \) merchant’s revenues are higher and there is no separating equilibrium in which both merchant types offer a deal. It then follows that the the website would be better off not displaying deal sales in such situations.

**Proposition 2.** If the website displays deal sales and both merchant types set the same deal price, then displaying deal sales leads to lower website profits.

**Proof.** Suppose that the equilibrium is a pooling equilibrium if deal sales are displayed. In this equilibrium it must be profitable for both merchant types to offer a deal. We therefore have,

\[
R^D_1 |_{pooling} > R^Q_t. \tag{11}
\]

From Lemmas 3 and 4, we also have that,

\[
R^D_0 |_{pooling} \geq R^D_1 |_{pooling}. \tag{12}
\]

Conditions (11) and (12) imply that

\[
R^D_0 |_{pooling} > R^Q_t. \tag{13}
\]

Condition (13) is both necessary and sufficient for a pooling PBE to exist if deal sales are not displayed. Further, this pooling PBE is the unique SUE because we know from Lemma 2 that there is no separating PBE in which both merchant types offer a deal. Following the same arguments, there also does not exist a separating PBE in which only the type \( H \) merchant offers a deal, since the type \( L \) merchant will find it profitable to mimic the type \( H \) merchant by offering a deal. Thus, there is no other PBE in which the type \( H \) merchant earns higher profits than in the pooling PBE. Consequently, the equilibrium is also a pooling equilibrium if deal sales are not displayed. From Lemmas 3 and 4, it follows that website profits are lower if deal sales are displayed. \( \square \)

Thus, we find that observational learning can be a double-edged sword: if it does not prevent mimicking, then it causes a loss of demand and hurts the website. Specifically, observational learning allows frequent- and late-new consumers to avoid buying from the type \( L \) merchant in situations where it mimics the type \( H \) merchant. In contrast, if deal sales were not displayed, then these consumers would have always bought the deal since they expect to obtain a non-negative surplus. Consequently, the type \( L \) merchant’s revenues are lower if deal sales are displayed \(( R^D_L |_{pooling} < R^D_L |_{pooling})\), while the type \( H \) merchant’s revenues are the same.
Therefore, the website prefers not to display deal sales in such situations.

We next identify the conditions under which displaying deal sales does not prevent mimicking.

**Lemma 5.** If the website displays deal sales, then both merchant types set the same deal price iff \( \alpha_L < \alpha_1 \) and \( \theta > \theta_1 \), where \( \alpha_1 \) is defined in Proposition 1, \( \theta_1 < 1 \) is defined in the Appendix.

**Proof.** See Appendix B.

Displaying deal sales cannot prevent mimicking if the type \( L \) merchant prefers to forgo the demand from frequent- and late-new consumers at the deal price \( d_L = \bar{\alpha} \) instead of setting \( d_L = \alpha_L \) to sell to all new consumers (see condition 20 in Appendix B). We also require that it is profitable for the type \( H \) merchant to offer a deal even when it is forced to set a deal price \( \bar{\alpha} < \alpha_H \) (see condition 18 in Appendix B). Lemma 5 describes when both conditions hold. Along with Proposition 2, it establishes that displaying deal sales is not a dominant strategy for the website. In particular, we note that the scope for mimicking to occur is higher if fewer consumers can engage in observational learning, i.e., the thresholds \( \alpha_1 \) and \( \theta_1 \) are increasing in \( \beta \).

### 3.4 Should the Website Display Deal Sales if it Allows the Merchant to Signal?

Proposition 2 established that it is counterproductive for the website to display deal sales if doing so does not allow the type \( H \) merchant to signal. Proposition 1 showed that the website can attain its highest possible profits if the type \( H \) merchant can signal without distortion, which occurs if \( \alpha_L \geq \alpha_1 \). It remains for us to explore whether displaying deal sales is beneficial whenever it enables the type \( H \) merchant to signal its type. We now derive the separating equilibrium that can occur if \( \alpha_L < \alpha_1 \). Let \( R_{t}^{D} \) denote the type \( t \) merchant revenues and \( \Pi_{t}^{W} \) denote the website profits in this equilibrium. In Lemma 6, we show that the type \( H \) merchant sets the highest deal price \( d_{H}^{*} < \alpha_H \) such that the type \( L \) merchant will not mimic it. The equilibrium is again supported by observational learning: frequent- and late-new consumers buy the deal at the deal price \( d_{H}^{*} \) only if period 1 sales equals \( \tau_H \). We also derive the conditions under which the separating equilibrium occurs. In particular, if \( d_{H}^{*} < \bar{\alpha} \), then separation does not occur because the type \( H \) merchant is better off in the pooling equilibrium. That is to say, it prefers to set a deal price \( \bar{\alpha} \) and allowing the type \( L \) merchant to mimic it than to signal through a deal price \( d_{H}^{*} < \bar{\alpha} \). If instead \( d_{H}^{*} \geq \bar{\alpha} \) then the type \( H \) prefers to signal its type.

**Lemma 6.** If the website displays deal sales and \( \alpha_L < \alpha_1 \), then in a separating equilibrium:

(i) The merchant’s pricing strategy is \( d_L = \alpha_L \) and \( d_H = d_{H}^{*} = \frac{\alpha_L(\alpha_L + N)}{\alpha_L + \frac{1}{2} \beta N} < \alpha_H \);

(ii) Merchant revenues are given by \( R_{H}^{D} = d_{H}^{*} (\alpha_H + N) \) and \( R_{L}^{D} = \alpha_L (\alpha_L + N) \).
(iii) Website profits are given by \[ \Pi^W_{\text{separation}} = (1 - \lambda) (\bar{\theta}d^*_H (\alpha_H + N) + (1 - \bar{\theta}) \alpha_L (\alpha_L + N)) \]

The separating equilibrium occurs iff \( \alpha_L \in (\alpha_2, \alpha_1) \) and \( \bar{\theta} \in (0, \theta_2) \), where \( \theta_2 = \frac{N\alpha_L(1 - \frac{1}{2}\beta)}{(\alpha_L + \frac{1}{2}\beta N)(\alpha_H - \alpha_L)} < 1 \)

and \( \alpha_2 = \sqrt{(N^2 + N\alpha_H - p\alpha_H)^2 + 2\beta N p\alpha_H (N + \alpha_H) + p\alpha_H - N (N + \alpha_H)} < \alpha_H \).

Proof. See Appendix B. \( \square \)

There is lesser separation in this separating equilibrium compared to the symmetric information benchmark. This is because the type \( H \) merchant “distorts” its price downwards to \( d^*_H < \alpha_H \) to credibly signal its type. The level of \( d^*_H \) reflects the relative attractiveness of mimicking for the type \( L \) merchant. Higher the relative attractiveness of mimicking, the lower is \( d^*_H \) and lesser the extent of separation. We find that \( d^*_H \) is decreasing in the number of early visitors (\( \beta \)) and is increasing in the number of new consumers (\( N \)) and the fit of the type \( L \) merchant (\( \alpha_L \)). Moreover, lesser the extent of separation, lower the website profits. This is because \( \Pi^W_{\text{separation}} \) is increasing in \( d^*_H \) as can be seen from Lemma 6. In fact, if \( d^*_H \) is not sufficiently higher than \( \bar{\alpha} \) then the website is better off not displaying deal sales. Proposition 3 describes this finding.

**Proposition 3.** If displaying deal sales leads to a separating equilibrium in which \( d_H = d^*_H < \alpha_H \), then there exists \( \theta_3 \in (0, 1) \) and \( \delta > 0 \) such that \( d^*_H - \bar{\alpha} < \delta \) and website profits are lower than if deal sales are not displayed iff \( \bar{\theta} > \theta_3 \).

Proof. See Appendix C. \( \square \)

Thus, even if the type \( H \) merchant prefers to signal its type, the website may prefer to suppress signaling by not displaying deal sales. This is because the website’s strategy maximizes its profits from both merchant types, and displaying deal sales reduces type \( L \) merchant revenues. One might then conjecture that the website should not display sales if \( \bar{\theta} \) is low since website profits are then more dependent on type \( L \) merchant revenues. Interestingly, we find that the opposite is true. On the one hand, if the website does not display deal sales, it does not attain its highest profits because of an adverse selection problem. That is to say, because of asymmetric information, the type \( H \) merchant is forced to either set the same deal price as the type \( L \) merchant or not offer a deal (if offering a deal at \( d_H = \bar{\alpha} \) is not profitable). On the other hand, if the website displays deal sales, it still cannot attain its highest profits because the type \( H \) merchant must distort its deal price to signal its type. Thus, the website faces a tradeoff between the costs from adverse selection and the costs from signaling. If \( \bar{\theta} \) is low, then the costs of adverse selection are relatively higher: the type \( H \) merchant faces a steeper drop in margin compared to its symmetric information benchmark.
price. Consequently, the website displays deal sales. Whereas if $\bar{\theta}$ is high, then the costs of adverse selection are relatively lower and the website prefers not display deal sales.

It is important to note that the website has a part in determining whether the merchant can signal in equilibrium and its incentives are distinct from that of the merchant. Our analysis of a strategic website brings this out clearly. While Proposition 1 showed how displaying deal sales and observational learning can benefit the website, Proposition 3 along with Proposition 2 showed that a “little” observational learning can be harmful for the website.

4 Extensions

To obtain further insights we examine three extensions to our main analysis. First, we endogenize the revenue-sharing contract and explore whether it is optimal for the website to provide the merchant an upfront subsidy. For instance, some daily deal websites such as Groupon and LivingSocial offer considerable support to a merchant in designing the promotional material for the deal and employ a substantial team of copywriters and editorial staff for this purpose (e.g., Streitfeld 2011, LivingSocial 2013). But they do not charge the merchant for this service. We show that providing a subsidy can be optimal only if the website displays deal sales.

Second, we investigate the implications when the number of consumers on the website is uncertain such that deal sales provides an “noisy” indication of merchant type. We show that if deal sales is “too noisy” an indicator of merchant type then displaying deal sales hurts the website. Interestingly, even the type $H$ merchant can be hurt in this case. Lastly, we study a setting in which the merchant types also differ in their value, i.e., the level of utility their product provides consumers. We show that in some situations the merchant types can separate in deal prices even if deal sales is not displayed and displaying deal sales may have no further impact. But in other situations, displaying deal sales is still necessary for separation to occur and can be beneficial for the website.

4.1 Should the Website Offer the Merchant an Upfront Subsidy?

To examine this question, we endogenize the revenue-sharing contract. We assume that the contract consists of the revenue-sharing rate $\lambda \in [0, 1]$, which is the merchant’s share of revenues, and a fixed-fee $F$ that the merchant must pay the website. Prior to period 1 (in period 0), after deciding the website regime, the website offers the merchant a revenue-sharing contract. The merchant must accept the contract to be able to offer a deal. The rest of the game proceeds as before.

Given the contract terms $(\lambda, F)$, the type $t$ merchant will accept the contract iff

$$\lambda R_t^D - F \geq R_t^O. \quad (14)$$
We refer to the incremental revenues $R_t^D - R_t^O$ as the surplus generated by the type $t$ merchant. If the merchant accepts the contract, then it retains a portion $\lambda R_t^D - F - R_t^O$ of the surplus, while the website captures a portion $(1 - \lambda) R_t^D + F$. We note that if only one of the merchant types accepts the contract in equilibrium, then the equilibrium contract is not uniquely determined as there are a range of $(\lambda, F)$ that will lead to the same outcome. Therefore, for our analysis to be meaningful, we focus on situations in which the website offers a contract that both merchant types accept in equilibrium. The equilibrium deal price and revenues is then the same as in our main analysis. We now examine the conditions under which the equilibrium contract will have a subsidy built in. In Lemma 7, we derive a necessary and sufficient condition for this to occur.

**Lemma 7.** The equilibrium revenue sharing contract will involve a subsidy iff $\frac{R_H^O}{R_H^D} > \frac{R_L^O}{R_L^D}$.

**Proof.** See Appendix B.

The revenue-sharing component allows the website to capture more surplus from the type $H$ merchant than from the type $L$ merchant. This is because $R_H^D > R_L^D$ since the type $H$ merchant sets a (weakly) higher deal price and realizes (strictly) higher deal sales in any equilibrium. Consequently, we find that in situations where the type $H$ merchant generates relatively more surplus than the type $L$ merchant, it is optimal for the website to offer a subsidy in conjunction with taking a larger share of deal revenues. Setting a low $\lambda$ allows the website to capture the higher surplus generated by the type $H$ merchant, while the subsidy ensures that that the type $L$ merchant find its attractive to offer a deal even though $\lambda$ is low. Lemma 7 establishes the necessary and sufficient condition for this to occur. It is useful to note that this condition cannot hold if the type $H$ merchant generates lower surplus than the type $L$ merchant, i.e., if $R_H^O - R_H^O < R_L^D - R_L^O$. Thus, only if the type $H$ merchant generates sufficiently higher surplus is it optimal to provide a subsidy.

We find that offering a subsidy is not optimal if deal sales are not displayed. This is because in a pooling equilibrium, both merchant types realize the same revenues from new consumers, but the type $H$ merchant faces higher cannibalization since it has a larger number of experienced consumers. Consequently, the type $H$ merchant generates lower surplus than the type $L$ merchant. In contrast, offering a subsidy can be optimal if deal sales are displayed. This occurs in situations where observational learning enables the type $H$ merchant to obtain sufficiently higher margin (in a separating equilibrium) or causes the type $L$ merchant to realize sufficiently lower demand from

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\[^9\text{We note in passing that it will be optimal for the website to offer a contract that both merchant types accept if } \alpha_L \geq \alpha_4 \in (0, \alpha_3), \text{ where } \alpha_3 \text{ is defined in Proposition 4. Otherwise, the website offers a contract that only the type } \text{L merchant accepts. Essentially, the type } H \text{ merchant is “driven” out of the market because of adverse selection. We do not include an analysis of this outcome as it does not provide any additional insights.}\]
new consumers (in a pooling equilibrium). In particular, a subsidy is always used in situations in which displaying deal sales allows the website to attain the upper bound on profits. Proposition 4 describes the condition under which the website should provide a subsidy.

**Proposition 4.** The equilibrium revenue sharing contract will involve a subsidy iff the website displays deal sales and $\alpha_L > \alpha_3$, where $\alpha_3 = \frac{1}{2} \beta \alpha_H$.

**Proof.** See Appendix C.

Some daily deal websites such as Groupon and LivingSocial are known to take a substantial share (40 - 50%) of deal revenues. At the same time, they are also known to employ creative writers and editorial staff who provide the merchant support in designing its promotional material for the deal (e.g., Streitfeld 2011, LivingSocial 2013). Interestingly, they offer this service free of charge to the merchant, effectively providing an upfront subsidy. Our results, offer a potential explanation why. The subsidy enables the website to fully capture the surplus of both merchant types in situations where displaying deal sales enables the type $H$ merchant generate considerably more surplus than the type $L$ merchant. In particular, the subsidy makes it attractive for the type $L$ merchant to offer a deal even when it has to share a high portion of its revenues with the website.

### 4.2 When the Number of Consumers on the Website is Uncertain

Let $M$ denote the number of consumers that visit the website. In our main analysis $M = N + 1$. To study the implications when deal sales is a “noisy” indicator of the merchant’s type, we introduce uncertainty in $M$. We assume that $M$ is distributed uniform over $[0, 2(N + 1)]$ and a proportion $\frac{N}{N+1}$ are new consumers. We assume that the distribution of the number of consumers is common knowledge to the merchant and to the consumers. On average, the number of experienced consumers and new consumers is the same as in the main analysis. Consequently, our earlier results in the symmetric information benchmark and in the regime where the website displays deal sales hold.

Deal sales are now a noisy indicator of the merchant’s type. Specifically, consider a separating equilibrium in which $d_t = \alpha_t$ and at the deal price $\alpha_H$, experienced consumers and early-new consumers buy the deal in period 1. Period 1 sales for the type $H$ merchant is then distributed uniform over $[0, 2\tau_H]$. If the type $L$ merchant were to mimic the type $H$ merchant, then its period 1 sales will be distributed uniform over $[0, 2\tau_L]$. Thus, there is an overlap in the possible range of deal sales for the two merchant types. That is to say, realized deal sales in the range $[0, 2\tau_L]$ can be observed in the case of either merchant type. It follows from our earlier analysis that for mimicking not to be attractive, a necessary condition is that frequent- and late-new consumers condition their
buying decisions on the realized deal sales. In particular, they will buy the deal only if period 1 sales exceed $2\tau_L$ so as to avoid buying from the type $L$ merchant.\footnote{Clearly, the strategy of frequent- and late-new consumers is optimal on the equilibrium path. To see that it is also optimal and credible off the equilibrium path, consider an arbitrarily small probability that the type $L$ merchant trembles to set a deal price $\alpha_H$ in equilibrium. Then, the expected surplus of buying the deal if the realized deal sales is in $[0, 2\tau_L]$ is negative. Therefore, the optimal strategy of frequent- and late-new consumers is to buy the deal in period 2 iff the realized deal sales is higher than $2\tau_L$.} Consequently, these consumers will also not buy from the type $H$ merchant in instances where its realized deal sales is below $2\tau_L$. Thus, when deal sales is a noisy indicator of the merchant’s type, new consumers do not buy from the type $H$ merchant with positive probability in equilibrium. This is necessary to enforce the no-mimicking constraint and support separation through observational learning. In other words, the observational learning that is necessary for separation also causes a loss of demand for the type $H$ merchant. Consequently, while displaying deal sales can still benefit the website, it does not enable the website to attain the benchmark profit level. Moreover, displaying deal sales lowers website profits if the extent of overlap in the distribution of realized deal sales for the two merchant types, which is given by $\frac{\tau_L}{\tau_H}$, is sufficiently high. In fact, even the type $H$ merchant can be worse off. The following proposition describes this result.

**Proposition 5.** There exists $t^* \in (0, 1)$ such that if $\frac{\tau_L}{\tau_H} > t^*$ then displaying deal sales hurts website profits and results in lower revenues even for the type $H$ merchant.

**Proof.** See Appendix C.

We have thus identified an additional reason why displaying deal sales can be counterproductive for the website, namely, if deal sales is too “noisy” an indicator of merchant type. The analysis also provides an additional perspective on the importance of experienced consumers for observational learning. If the relative proportion of experienced consumers is higher (i.e., $N$ is lower), then deal sales is more informative (less noisy) because $\frac{\tau_L}{\tau_H}$ is lower. Surprisingly, even the type $H$ merchant can be hurt if consumers engage in observational learning. If deal sales are displayed, then in any PBE in which $d_H \geq \bar{\alpha}$, frequent- and late-new consumers do not buy from the type $H$ merchant with positive probability ($= \frac{\tau_L}{\tau_H}$) in period 2. Thus, displaying deal sales suppresses demand from new consumers even if it enables the type $H$ merchant to earn a higher margin. In particular, displaying deal sales suppresses demand even in a pooling PBE in which $d_L = \bar{\alpha}$. This is because realized period 1 sales less than $2\tau_L$ is more indicative of a type $L$ merchant, and the expected

\footnote{It is possible to construct equilibria in which new consumers buy the deal for some range of realized sales lower than $2\tau_L$. Our main insights will still hold qualitatively because new consumers do not buy with positive probability from the type $H$ merchant. We do not consider such equilibria as the off-equilibrium strategy is not robust to a tremble.}
surplus of buying the deal is then negative. Consequently, if $\frac{r_L}{r_H}$ is sufficiently high, then the type $H$ merchant is worse off than in the pooling equilibrium in which deal sales are not displayed.

4.3 When the Merchant Types Differ in their Value

We analyze a setting in which consumers obtain a utility $r_t > 0$ if the type $t$ merchant’s product fits their needs, where $r_H > r_L$. As before, we assume that $p > \alpha_H r_H$ so that the deal price will be lower than the regular price. We also assume that $N > 1$ such that it is attractive for either merchant type to sell to new consumers in the symmetric information benchmark. In the benchmark, new consumers are willing to pay $\alpha_t r_t$ for the type $t$ merchant, and the optimal deal price is $d_t = \alpha_t r_t$. As before, website profits cannot be higher than in the symmetric information benchmark, and can be attained only in a separating equilibrium in which $d_t = \alpha_t r_t$.

We find that in some situations the website can attain the benchmark profit level even if deal sales are not displayed. Specifically, if $\alpha_H r_H > r_L \geq \frac{N}{(N+\alpha_L)\alpha_L} \alpha_H r_H$, then a separating equilibrium in which $d_t = \alpha_t r_t$ can be supported without deal sales being displayed. In this case, if the type $L$ merchant mimics the type $H$ merchant then it cannot sell to experienced consumers since $\alpha_H r_H > r_L$. Further, this loss in demand makes mimicking unattractive if $r_L \geq \frac{N}{(N+\alpha_L)\alpha_L} \alpha_H r_H$. In other words, if the difference in merchant value is neither too high nor too low, the buying decisions of experienced consumers directly discipline the deal prices. Consequently, displaying deal sales will have no further impact. Proposition 6 describes this finding.

Proposition 6. The website can attain the same profits as in the symmetric information benchmark even if deal sales are not displayed iff $\alpha_L > \frac{1}{2} \left( \sqrt{N^2 + 4N} - N \right)$ and $\alpha_H r_H > r_L \geq \frac{N \alpha_H}{(N+\alpha_L)\alpha_L} r_H$.

Proof. See Appendix C.

In all other situations, the website can attain the benchmark profit level only if deal sales are displayed. In particular, the no-mimicking constraint can be enforced through observational learning if there are a sufficient number of frequent-new consumers.

5 Conclusion

In a relatively short period of time, daily deal websites have become a popular means for small merchants to attract new consumers. Unlike traditional coupon mailer companies, a daily deal website can track and report deal sales to consumers in real time. Moreover, a daily deal website functions as a marketplace enabling transactions between a merchant and consumers. Our work contributes to the understanding of this emerging business model. Our analysis provides three main insights.
First, displaying deal sales can play an important role in the functioning of this marketplace by helping the high-type merchant signal to new consumers through its deal price. Displaying deal sales facilitates observational learning by some new consumers, which enables the merchant to signal to other new consumers. Moreover, it may be possible to achieve this signaling without a distortion in price. Consequently, the website can attain its maximum profits if the merchant signals its type. Therefore, it can be beneficial for the website to display deal sales.

Second, displaying deal sales is however not a dominant strategy for the website. The website faces a trade-off between the costs of adverse selection by not displaying deal sales and the costs of signaling by displaying deal sales. If deal sales is not sufficiently informative about the merchant’s type or if sufficient number of consumers do not engage in observational learning, then displaying deal sales is counterproductive. This is because signaling can entail a distortion in price as well as a loss of demand from new consumers. Stated differently, a “little” observational learning is harmful for the website. In particular, the website prefers to suppress signaling by not displaying deal sales even if the high-type merchant prefers to signal. Interestingly, this occurs in situations in which it is more likely that the merchant is of high-type. It is important to note that the website has a role in determining whether the merchant can signal through deal price in equilibrium. Our model of a strategic website brings this out clearly.

Lastly, if deal sales are displayed, it may be necessary for the the website to offer the merchant a subsidy in order to better capture the profits generated by daily deals. This is because, if deal sales are displayed, the high-type merchant’s deal can generate higher surplus than the low-type merchant’s deal. Offering a subsidy in combination with retaining a high portion of deal revenues can then be optimal: the revenue-sharing component extracts the higher surplus generated by the high-type merchant while the subsidy ensures that the low-type merchant will still offer a deal.

Taken together, our results could help understand why some daily deal websites display deal sales while others do not, and why a daily deal website might offer costly services free of charge to merchants. More generally, prior empirical research has shown that websites can facilitate observational learning by displaying popularity information in various forms (Chen and Xie 2008; Tucker and Zhang 2011; Zhang and Liu 2012; Luo, Andrews, Song, and Aspara 2014). But little is known about how observational learning impacts the website. Our work sheds light on whether, when, and why observational learning can benefit or hurt the website. We also identify boundary conditions for our results. We show that there are situations in which displaying deal sales is not necessary for the website to attain its maximum profits (Proposition 6). We also show that if deal sales is a
“noisy” indicator of the merchant’s type, then while displaying deal sales can still be beneficial, the website cannot attain its maximum profits.

5.1 Managerial Implications

Our findings have implications for the management of daily deal websites. Daily deal websites have been criticized both for the high share of revenues that they take and the deep discounts that merchants offer (Mulpuru 2011; Bice 2012; Kumar and Rajan 2012). These criticisms essentially question the viability of the business model. Our results provide guidance to daily deal websites on how the depth of discounts offered on the website can be managed so as to maximize the profitability of daily deals. Essentially, this occurs in a separating equilibrium. We show that it can be necessary to display deal sales to obtain a separating equilibrium. One might conjecture that a daily deal website could instead sort the merchants by individually verifying their characteristics or by offering a menu of contracts. Given that daily deal websites typically market their services to a large number of small merchants through a relatively low-skilled sales force, these alternative approaches can be impractical as they can make the selling task more effortful and complex. In this context, reporting deal sales can play an important role in inducing the merchant to provide the right level of discount and increasing industry profitability.

A second implication is the important role of experienced consumers buying the deal. In the case of traditional promotions, there is no benefit if experienced consumers who would have bought at the regular price buy the deal because this only results in cannibalization and lowers profits. Based on this logic, industry experts recommend that when possible daily deal offers should include restrictions to ensure that they are availed only by new consumers (Mulpuru 2011; Bice 2012; Kumar and Rajan 2012). But observational learning cannot occur if experienced consumers do not buy the deal. Thus, based on our analysis, we can conclude that such restrictions can hurt in the case of a daily deal website that displays deal sales and leads to lower industry profitability.

Lastly, a daily deal website should explore ways to promote observational learning. As our model suggests, if more consumers visit the website frequently then it is beneficial. One way to attract consumers to the website frequently is by choosing the appropriate assortment of goods and services. Another way would be to use additional communication methods such as targeted emails and advertisements. Keeping the duration of the deal longer can also enhance the opportunities for observational learning.

12In our discussion with Groupon, we found that their sales team does not spend time qualifying merchants. Also, to keep the selling process simple, they do not offer a menu of contracts.
5.2 Relationship with Practice

How do our results tie in with stylized observations from practice? Our results suggest that managers of daily deal websites should reflect on whether or not to display deal sales since it is not a dominant strategy. Indeed, as the second opening quote of §1 indicates, the management at the daily deal website TroopSwap were worried that displaying dealing sales was hurting their business. They conducted a field experiment to evaluate the effect of the deal sales counter and found that it had a positive impact overall (Vasilaky 2012). We interpret that to mean that the website benefited from consumers being able to engage in observational learning.

As discussed earlier, displaying deal sales can be useful if the daily deal website finds it costly to qualify merchants. Indeed, in our discussions with Groupon, we found that it does not use its sales force to qualify or screen merchants. Moreover, the managers at Groupon also believed that consumers benefit from observing deal sales because it gives them an indication of the quality of a merchant or the desirability of the deal. It was for this reason that they decided not to eliminate the deal counter (Groupon 2011). In fact, Groupon conducted a field experiment to see if including deal sales information in their email communication would also be beneficial. They found that providing deal sales information significantly increased the effect of email communication on website traffic. This experiment further reinforces the relevance of our model and our results.

Lastly, one of the implications of our model is that under certain conditions it is optimal for the daily deal website to provide an upfront subsidy to the merchant. It is interesting to note that Groupon for example provides extensive support in developing the promotional material for the deal (Streitfeld 2011). Since there is no charge for this support and Groupon incurs costs to provide this support, it is clearly a subsidy for the merchant.

5.3 Limitations of our Approach

We should also note a few caveats with respect to our model. We do not explicitly model repeat business. It has been shown that, with repeat purchases, the high-type merchant may be able to separate through price alone (e.g., Milgrom and Roberts 1986). But such separation involves distortion compared to the outcomes under symmetric information. Similarly, it has been shown that making limited quantities of the product available can support separation (Stock and Balachander 2005). But this too would involve distortion. We conjecture that our result that separation can be accomplished without distortion by displaying deal sales will continue to hold even in these settings. Since it involves no distortion, this separation would also be preferable for the website.

In our model, consumer heterogeneity consists of experienced and new consumers, and new
consumers’ willingness to pay is less than experienced consumers. It is possible that there is heterogeneity amongst experienced consumers as well. While including this will make for a richer model, we conjecture that the impact of displaying deal sales on new consumers should continue to hold qualitatively. We leave it for future research to examine richer settings.

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Appendix A  Formal Description of the SUE Refinement

The SUE refinement differs from the intuitive criterion in that it constrains the beliefs in a candidate PBE using the beliefs in an alternative PBE. If the candidate PBE cannot be supported when the beliefs are so constrained, then it is said to be defeated by the alternative PBE. A SUE is a PBE that is not defeated by any other alternative PBE.

Consider a candidate PBE in which the deal price \( d \) is not used in equilibrium. Suppose there exists an alternative PBE in which the deal price \( d \) is used in equilibrium and at least one of the merchant types that sets this deal price realizes higher profits than in the candidate PBE. Then, the refinement requires that the beliefs following the deal price \( d \) in the candidate PBE do not assign lower probability than the alternative PBE to the merchant types that are strictly better off in the alternative PBE. Formally, let \( T \) denote the set of merchant types that set the deal price \( d \) in the alternative PBE. Let \( T_1 \subset T \) denote the set of types that realize strictly higher profits in the alternative PBE than in the candidate PBE. Let \( \mu (t \mid d) \) denote the belief in the candidate equilibrium that the merchant’s type is \( t \) at some information set following a deal price \( d \). Let \( \mu' (t \mid d) \) denote the corresponding belief in the alternative equilibrium. The candidate PBE is not defeated by the alternative PBE iff \( \mu (t \mid d) \geq \mu' (t \mid d) \) for any \( t \in T_1 \).

Two implications follow from such a belief restriction. First, a candidate PBE will be defeated if the type \( H \) merchant is better off in the alternative PBE. This is because consumer beliefs at
the deal price $d$ must be at least as optimistic as in the alternative PBE, i.e., we require that $\mu(H \mid d) \geq \mu'(H \mid d)$. Given any beliefs that satisfy this restriction, since consumers derive higher utility from the type $H$ merchant and their decisions must be sequentially rational in a PBE, consumer demand for the type $H$ merchant at the deal price $d$ cannot be lower than that in the alternative PBE. It follows that it will be profitable for the type $H$ merchant to deviate to $d_H = d$ in the candidate PBE, because it can earn at least as much profits as in the alternative PBE. Hence, the candidate PBE cannot be supported under these belief restrictions and is defeated by the alternative PBE.

A second implication is that a candidate PBE will not be defeated if only the type $L$ merchant is better off in the alternative PBE. In this case, we require that consumer beliefs are not more optimistic than in the alternative PBE, i.e., $\mu(L \mid d) \geq \mu'(L \mid d)$. But this allows for the beliefs to be pessimistic, i.e., $\mu(L \mid d) = 1$. Since to be a PBE, the candidate PBE must have survived under such pessimistic beliefs, it follows that it is not defeated by the alternative PBE.

It follows from these two implications that a PBE that yields the highest profits for the type $H$ merchant: (a) will defeat any other PBE that yields lower profits for the type $H$ merchant, and (b) is itself not defeated by any other PBE. Thus, only the PBE that yields the highest profits for the type $H$ merchant can be a SUE.

**Appendix B  Proofs for Lemmas**

**Proof for Lemma 1:** We show that website profits cannot exceed $\Pi_{SI}^W$ in any potential PBE. First, suppose that the equilibrium is a separating PBE in which both merchant types offer a deal and $d_H \neq d_L$. New consumers must hold correct beliefs about the merchant’s type on the equilibrium path. Therefore, if $d_t > \alpha_t$, new consumers will not buy the deal in equilibrium. But then $R_t^D \leq R_t^O$ and offering a deal is not profitable, which is a contradiction. Hence, it must be that $d_t \leq \alpha_t$ in any separating PBE. Further, it must be that $d_L = \alpha_L$ since all new consumers will buy the deal at this price even if they hold pessimistic beliefs $\theta = 0$. Now, if both merchant types set $d_t = \alpha_t$ in equilibrium and all experienced consumers and new consumers buy the deal, then $R_t^D = \alpha_t (\alpha_t + N)$ as in the symmetric information benchmark. Hence, website profits will be equal to $\Pi_{SI}^W$. It follows that website profits will be strictly lower than $\Pi_{SI}^W$ in any other potential separating PBE since $d_H < \alpha_H$. It also follows from the same arguments that if only one of the merchant types offers a deal in equilibrium, then $R_t^D \leq \alpha_t (\alpha_t + N)$ for that merchant type and $R_t^D = 0$ for the other merchant type. Therefore, website profits must be strictly lower than $\Pi_{SI}^W$.

Next, suppose that the equilibrium is a pooling PBE in which both merchant types offer a deal
and $d_H = d_L = d$. In a pooling PBE, at the equilibrium deal price, new consumers must maintain their prior belief that $\theta = \tilde{\theta}$. Suppose, that $d > \tilde{\alpha}$. In period 1, frequent- and early-new consumers will not buy the deal because their expected surplus $\tilde{\alpha} - d$ is negative. Only frequent- and early-experienced consumers will buy, resulting in period 1 sales of $(1 - \frac{1}{2} \beta) \alpha_t$. We note that the period 1 sales depends on the merchant’s type. In period 2, if deal sales are not displayed, frequent- and late-new consumers will maintain their prior belief $\tilde{\theta}$ and will not buy the deal. If deal sales are displayed, then frequent- and late-new consumers will correctly identify the merchant’s type from period 1 sales. They will not buy the deal from the type $L$ merchant since their expected surplus $\alpha_L - d$ is negative. Thus, whether or not deal sales are displayed, new consumers will not buy the deal from the type $L$ merchant in equilibrium. But then $R^D_H \leq R^O_L$, which is a contradiction. Therefore, $d \leq \tilde{\alpha}$ in any pooling PBE. If in the pooling PBE, $d = \tilde{\alpha}$ and all experienced consumers and new consumers buy, then website profits are given by

$$\Pi^W = (1 - \lambda) \tilde{\alpha} \left( \theta \alpha_H + (1 - \tilde{\theta}) \alpha_L + N \right).$$  \hspace{1cm} (15)

The above profits are strictly lower than $\Pi^W_{SI}$ since

$$\Pi^W_{SI} - \Pi^W = (1 - \lambda) \tilde{\theta} (1 - \tilde{\theta}) (\alpha_H - \alpha_L)^2.$$ \hspace{1cm} (16)

It follows that in any potential pooling PBE in which $d \leq \tilde{\alpha}$, $\Pi^W < \Pi^W_{SI}$. Thus, $\Pi^W = \Pi^W_{SI}$ only in the case of a separating PBE in which $d_t = \alpha_t$ and all consumers buy the deal.

**Proof for Lemma 4:** As noted in the proof of 1, the deal price cannot exceed $\tilde{\alpha}$ in a pooling equilibrium. We will show that a pooling PBE in which $d_t = d < \tilde{\alpha}$ cannot be a SUE. Consider a pooling PBE in which $d_t = d \in (\alpha_L, \tilde{\alpha})$. In this equilibrium, all experienced consumers buy the deal as they obtain a positive surplus. Early-new consumers must maintain their prior belief $\tilde{\theta}$ and will buy as they obtain a non-negative expected surplus $\tilde{\alpha} - d$. While frequent-new consumers will also derive a non-negative expected surplus if they buy in period 1, they can do better if they wait till period 2 to learn the merchant’s type by observing deal sales. This is because they obtain a negative expected surplus $\alpha_L - d$ if the merchant’s type is $L$. In period 2, frequent- and late-new consumers will buy if period 1 sales equals $\tau_H$, which would indicate that the merchant’s type is $H$, and will not buy if it equals $\tau_L$, which would indicate that the merchant’s type is $L$. Therefore, the equilibrium revenues for a type $H$ and type $L$ merchant are given by

$$R^D_H = d (\alpha_H + N), \quad R^D_L = d \left( \alpha_L + \frac{1}{2} \beta N \right).$$ \hspace{1cm} (17)

This pooling PBE exists iff the following conditions hold: (i) $R^D_t > R^O_t = p \alpha_t$ so that offering a deal...
is profitable, and (ii) \( R^D_L \geq \alpha_L (\alpha_L + N) \) so that the type \( L \) merchant does not have an incentive to deviate to \( d_L = \alpha_L \) to sell to all new consumers. It follows therefore that if a pooling PBE in which \( d \in (\alpha_L, \tilde{\alpha}) \) exists, then a pooling PBE in which \( d = \tilde{\alpha} \) must also exist because both merchant types derive higher revenues in the latter case. Moreover, the PBE in which \( d \in (\alpha_L, \tilde{\alpha}) \) cannot be a SUE since the type \( H \) merchant’s profits are strictly higher in the PBE in which \( d = \tilde{\alpha} \).

Next, suppose in a pooling PBE \( d_t = \alpha_L < \tilde{\alpha} \). In this case new consumers will derive non-negative expected surplus even if the merchant’s type is \( L \). Therefore, all experienced consumers and new consumers buy. Equilibrium revenues for the type \( t \) merchant are given by \( R^D_t = \alpha_t (\alpha_t + N) \). This pooling PBE exists iff \( R^D_t > R^O_t \). We rule out this candidate PBE by showing that whenever it exists, one of the following alternative PBE in which the type \( H \) merchant’s profits are strictly higher also exists: (i) a pooling PBE in which \( d = \tilde{\alpha} \), or (ii) a separating PBE in which \( d_H = \tilde{\alpha} \) and \( d_L = \alpha_L \). Consider first the alternative pooling PBE. Merchant revenues are given by equation (17) for \( d = \tilde{\alpha} \). The PBE exists iff the type \( L \) merchant’s revenues are (weakly) higher than in the candidate pooling PBE, i.e., iff \( \tilde{\alpha} (\alpha_L + \frac{1}{2} \beta N) \geq \alpha_L (\alpha_L + N) \). Otherwise, the type \( L \) merchant will find it profitable to deviate from \( d_L = \tilde{\alpha} \) to \( d_L = \alpha_L \).

If instead \( \tilde{\alpha} (\alpha_L + \frac{1}{2} \beta N) < \alpha_L (\alpha_L + N) \), then we can construct the alternative separating PBE as follows. If the deal price is \( \alpha_L \), then all experienced consumers and new consumers buy. If the deal price is \( \alpha_H \), then all experienced consumers and early-new consumers buy. Frequent- and late-new consumers buy in period 2 only if deal sales are \( \tau_H \). The type \( L \) merchant’s revenues in this PBE are the same as in the candidate pooling PBE while the type \( H \) merchant’s revenues are higher. Since \( \tilde{\alpha} (\alpha_L + \frac{1}{2} \beta N) < \alpha_L (\alpha_L + N) \), the type \( L \) merchant will not have an incentive to mimic the type \( H \) merchant. Therefore, one of the alternative PBE always exists and yields higher profits than the candidate PBE for the type \( H \) merchant. It follows that the candidate pooling PBE cannot be a SUE.

**Proof for Lemma 5:** Lemma 4 describes the deal price and merchant revenues in a pooling equilibrium if deal sales are displayed. The pooling PBE will exist iff:

(i) Offering a deal is profitable for both merchant types. Therefore, we have,

\[
R^D_{H|pooling} \geq R^O_H \implies \tilde{\alpha} (\alpha_H + N) \geq p\alpha_H; \tag{18}
\]

\[
R^D_{L|pooling} \geq R^O_L \implies \tilde{\alpha} (\alpha_L + \frac{1}{2} \beta N) \geq p\alpha_L; \tag{19}
\]

(ii) The type \( L \) merchant does not deviate to \( d_L = \alpha_L \) to sell to all new consumers, which yields

\[
\tilde{\alpha} (\alpha_L + \frac{1}{2} \beta N) > \alpha_L (\alpha_L + N); \tag{20}
\]
Condition (20) essentially ensures that the type L merchant will mimic the type H merchant in equilibrium. We wish to show that if condition (20) holds, then the no-mimicking constraint cannot hold in any separating PBE in which the type H merchant sets \(d_H \in [\bar{\alpha}, \alpha_H]\). First, as noted in the proof of Proposition 2, there cannot be a separating PBE in which only the type H merchant offers a deal as the type L merchant will find it profitable to mimic the type H merchant. Next, consider a separating PBE in which both merchant types offer a deal. As shown in the proof of Lemma 1, in this PBE we require that \(d_L = \alpha_L\) and \(d_H \leq \alpha_H\). On the equilibrium path, all experienced consumers and new consumers will buy the deal as they obtain non-negative surplus. Hence, \(R^D_L = \alpha_L(\alpha_L + N)\). If the type L merchant mimics the type H merchant, early-new consumers will still buy the deal as they cannot observe deal sales, while frequent- and late-new consumers can avoid buying the deal if they condition their buying decisions on the realized period 1 sales. Therefore, a necessary condition for mimicking to be unprofitable in a separating PBE is

\[
\alpha_L(\alpha_L + N) \geq d_H \left(\alpha_L + \frac{1}{2}\beta N\right).
\]

But conditions (20) and (21) cannot both hold for \(d_H \in [\bar{\alpha}, \alpha_H]\). Therefore, if the pooling PBE exists, then there is no other separating PBE in which the type H merchant earns higher profits. Hence, the pooling PBE is the unique SUE whenever it exists.

We note that condition (20) is sufficient for condition (19) because of condition (6). Now suppose that \(\bar{\theta} \to 1\). We have that \(\bar{\alpha} \to \alpha_H\). Then, condition (18) holds because of condition (6), and condition (20) holds because \(\alpha_L < \alpha_1\). We further note that conditions (18) and (20) are linear in \(\bar{\theta}\). Therefore, by continuity, there exists \(\theta_1 < 1\) such that the pooling PBE exists iff \(\bar{\theta} < \theta_1\). It is straightforward to verify that condition (18) requires that \(\bar{\theta} < 1 - \frac{\alpha_L(\alpha_L + N)}{(\alpha_L + \frac{1}{2}\beta N)(\alpha_H - \alpha_L)}\), condition (20) requires that \(\bar{\theta} < \frac{\alpha_L(\alpha_L + N)}{(\alpha_L + \frac{1}{2}\beta N)(\alpha_H - \alpha_L)}\). Therefore, \(\theta_1\) is the minimum of these two bounds.

**Proof for Lemma 6:** We construct the separating equilibrium that can occur if \(\alpha_L < \alpha_1\). We first show that \(d_H^* \in (\alpha_L, \alpha_H)\). As shown in the proof of Proposition 1, \(\alpha_L(\alpha_L + N) < \alpha_H(\alpha_L + \frac{1}{2}\beta N)\) if \(\alpha_L < \alpha_1\) because the no-mimicking constraint (10) will not hold. Therefore, there exists a deal price \(d \in (\alpha_L, \alpha_H)\) such that \(\alpha_L(\alpha_L + N) = d(\alpha_L + \frac{1}{2}\beta N)\). This deal price \(d\) equals \(d_H^*\) defined in the statement of the Lemma. Suppose that in a separating PBE, \(d_H = d_H^*\) and \(d_L = \alpha_L\) and consumers adopted the following strategies:

- Experienced consumers buy the deal at either deal price regardless of the merchant’s type, with frequent visitors buying in period 1.

- If the deal price is \(\alpha_L\), all new consumers buy the deal with frequent visitors buying in period 1.
If the deal price is \( d^*_H \), then early-new consumers buy in period 1 and frequent-new consumers wait till period 2. In period 2, frequent- and late-new consumers buy if period 1 sales equals \( \tau_H \) and do not buy if period 1 sales equals \( \tau_L \).

In equilibrium, all experienced consumers and new consumers buy the deal. It is straightforward to verify that given the merchant strategies, consumers do not have an incentive to deviate. By construction, the type \( L \) merchant will not have an incentive to mimic the type \( H \) merchant. The merchant revenues and website profits are as given in the statement of the Lemma. This separating PBE exists iff \( R^D_1 \mid \text{separation} > R^C_1 \) so that it is profitable to offer a deal. Therefore, we have

\[
d^*_H (\alpha_H + N) > p\alpha_H, \tag{22}
\]

\[
\alpha_L (\alpha_L + N) > p\alpha_L. \tag{23}
\]

We note that condition (23) holds because of condition (6). If \( \alpha_L \to \alpha_1 \), then \( d^*_H \to \alpha_H \) and condition (22) holds because of condition (6). If \( \alpha_L \to 0 \), then \( d^*_H \to 0 \) and condition (22) cannot hold. Therefore, by continuity there exists \( \alpha_2 \in (0, \alpha_H) \) such that condition (22) holds iff \( \alpha_L \in (\alpha_2, \alpha_1) \). Further condition (22) must hold as an equality if \( \alpha_L = \alpha_2 \), from which we obtain \( \alpha_2 \) as defined in the statement of the Lemma.

We show that the separating PBE is an SUE iff \( d^*_H \geq \bar{\alpha} \). As shown in the proof of Lemma 1, in any separating PBE we require that \( d_L = \alpha_L \) and \( d_H \leq \alpha_H \). By construction, a separating PBE in which the type \( H \) merchant charges a higher price than \( d^*_H \) cannot exist since the corresponding no-mimicking constraint will not hold. If a separating PBE in which \( d_H < d^*_H \) exists, then the separating PBE in which \( d_H = d^*_H \) will also exist and lead to higher profits for the type \( H \) merchant. Therefore, a separating PBE in which \( d_H < d^*_H \) cannot be the SUE. As shown in the proof of Lemma 5, if \( d^*_H \geq \bar{\alpha} \), then the pooling PBE in which \( d_t = \bar{\alpha} \) cannot exist. This is because the mimicking condition (20) will not hold and the the type \( L \) merchant will prefer to separate than to pool with the type \( H \) merchant. Conversely, if \( d^*_H < \bar{\alpha} \), then the pooling PBE will exist whenever the separating PBE exists. This is because the mimicking condition (20) will be satisfied, and condition (18) holds if condition (22) holds. Moreover, the type \( H \) merchant’s profits are higher in the pooling PBE since \( d^*_H < \bar{\alpha} \). Therefore, the separating PBE we constructed is an SUE if \( d^*_H \geq \bar{\alpha} \). Further, no other PBE that leads to a different equilibrium outcome can be a SUE if \( d^*_H \geq \bar{\alpha} \). Thus the SUE outcome is unique. We note that \( d^*_H < \bar{\alpha} \) if \( \bar{\theta} \to 1 \) and \( d^*_H > \bar{\alpha} \) if \( \bar{\theta} \to 0 \). Therefore, by continuity, there exists \( \theta_2 \in (0, 1) \) such that \( d^*_H \geq \bar{\alpha} \) iff \( \bar{\theta} \in (0, \theta_2] \), where \( \theta_2 \) is as defined in the statement of

\[13\] While there can be other separating PBE in which \( d_H = d^*_H \) that lead to the same outcome, they exist for a narrower range of parameters since their no-mimicking constraint will be stricter than that in the PBE we have constructed.
the Lemma.

**Proof for Lemma 7:** We make two observations that will be useful in our analysis. First, \( R^O_H > R^O_L \), since a larger number of experienced consumers buy from the type \( H \) merchant. Second, \( R^D_H > R^D_L \), since the type \( H \) merchant sets a (weakly) higher deal price and realizes (strictly) higher deal sales in any equilibrium. Now, the equilibrium contract is one that maximizes the website’s profits subject to the IR constraints (14) and the feasibility constraint \( \lambda \in [0, 1] \). Obviously, the IR constraint must bind for at least one of the types, i.e., the website must fully extract the surplus of at least one of the merchant types. First, consider the case in which the website fully extracts the surplus of both merchant types. We require that,

\[
\lambda R^D_i - F = R^O_i \quad \implies \quad \lambda = \frac{R^O_H - R^O_L}{R^D_H - R^D_L}, \quad F = \frac{R^D_H R^O_L - R^D_L R^O_H}{R^D_H - R^D_L}. \quad (24)
\]

For the contract in equation (24) to be feasible we require that \( \lambda \in [0, 1] \). Since \( R^O_H > R^O_L \) and \( R^D_H > R^D_L \), we have \( \lambda > 0 \). We have \( \lambda \leq 1 \) iff \( R^D_H - R^O_H \geq R^D_L - R^O_L \). Lastly, the contract will involve a subsidy \( (F < 0) \) iff \( \frac{R^H_H}{R^H_O} > \frac{R^D_H}{R^D_O} \). In fact, we also have that \( \lambda < 1 \) if \( \frac{R^H_H}{R^H_O} > \frac{R^D_H}{R^D_O} \) since,

\[
\frac{R^D_H}{R^O_H} > \frac{R^D_L}{R^O_L} \implies \frac{R^D_H - R^O_H}{R^O_H} > \frac{R^D_L - R^O_L}{R^O_L} \implies R^D_H - R^O_H > R^D_L - R^O_L \quad (25)
\]

where the last step follows because \( R^O_H > R^O_L \).

Next, consider the case in which the website fully extracts the surplus of only one of the merchant types. From our analysis above, we require that \( R^D_H - R^O_H < R^D_L - R^O_L \). We also have \( R^D_H > R^D_L \). Consequently, we have,

\[
\lambda R^D_H - F \geq R^O_H \quad \implies \quad (R^D_H - R^O_H) - (1 - \lambda) R^D_H - F \geq 0,
\]

\[
\implies \quad (R^D_L - R^O_L) - (1 - \lambda) R^D_L - F > 0,
\]

\[
\implies \quad \lambda R^D_L - F > R^O_L. \quad (26)
\]

Thus, any contract that will be accepted by the type \( H \) merchant cannot fully extract the surplus of the type \( L \) merchant. Therefore, in equilibrium, the website can fully extract the surplus of only the type \( H \) merchant. Moreover, the type \( L \) merchant will always accept such a contract since it will obtain positive surplus. Now consider a revenue-sharing contract that fully extracts the surplus of the type \( H \) merchant in which \( \lambda \in (0, 1) \). We can construct an alternative contract with a higher revenues-sharing rate \( \lambda' > \lambda \) and a higher fixed-fee \( F' = F + (\lambda' - \lambda) R^D_H \). This contract also fully extracts the surplus of the type \( H \) merchant. It will extract a higher portion of the type \( L \) merchant’s surplus since \( R^D_H > R^D_L \). Therefore, website profits are higher under the alternative contract. It follows that the optimal contract is one in which \( \lambda = 1 \) and \( F = R^D_H - R^O > 0 \), which
does not involve a subsidy. Hence, a subsidy occurs iff \( \frac{R^D_H}{R^D_H} > \frac{R^D_L}{R^D_L} \).

Appendix C  Proofs for Propositions

**Proof for Proposition 3:** If \( \alpha_L < \alpha_1 \) then separation occurs in the parameter region described in Lemma 6 and website profits are equal to \( \Pi^W|_{separation} \). In this parameter region, if deal sales are not displayed, then there are two possibilities: (i) both merchant types offer a deal at a price \( d_t = \bar{\alpha} \) as described in Lemma 3, or (ii) only the type \( L \) merchant offers a deal at \( d_L = \alpha_L \).\(^{14}\) We note that website profits in the latter case must strictly be lower than \( \Pi^W|_{separation} \) since the type \( H \) merchant’s revenues are zero while the type \( L \) merchant’s revenues are the same as \( R^D_L|_{separation} \).

Therefore, displaying deal sales can hurt profits only if not displaying deal sales leads to a pooling equilibrium in which \( \Pi^W|_{separation} < \Pi^W|_{pooling} \).

Now, we have from the proof of Lemma 6 that \( d^*_H = \bar{\alpha} \) iff \( \bar{\theta} = \theta_2 \), where \( \theta_2 \) is defined in Lemma 6. If \( d_H = \bar{\alpha} \), then from Lemma 6 and Lemma 4 we have \( R^D_L|_{separation} < R^D_L|_{pooling} \) and \( R^D_H|_{separation} = R^D_H|_{pooling} \). Given that the separating equilibrium exists (if deal sales are displayed), we have \( R^D_t|_{separation} > R^D_t|_{pooling} \). It follows that the pooling PBE must exist (if deal sales are not displayed) since \( R^D_t|_{pooling} \geq R^D_t|_{separation} > R^D_t \). Moreover, the pooling PBE must be the SUE as there is no other equilibrium in which the type \( H \) merchant can earn higher revenues. Furthermore, \( \Pi^W|_{separation} < \Pi^W|_{pooling} \) since the type \( L \) merchant revenues are strictly higher in the pooling equilibrium. Therefore, displaying deal sales lowers website profits if \( \bar{\theta} = \theta_2 \). If \( \bar{\theta} < \theta_2 \), then \( d^*_H > \bar{\alpha} \) and we have

\[
\Pi^W|_{separation} - \Pi^W|_{pooling} = (1 - \lambda) \bar{\theta} \left( d^*_H (N + \alpha_H) - N \alpha_H - (2 \alpha_H - \alpha_L) \alpha_L - (\alpha_H - \alpha_L)^2 \bar{\theta} \right)
\]

which is decreasing in \( \bar{\theta} \). We also have that \( d^*_H - \bar{\alpha} \) is decreasing in \( \bar{\theta} \). By continuity, it follows that there exist \( \theta_3 \in (0, \theta_2) \) and \( \delta > 0 \) such that \( d^*_H - \bar{\alpha} < \delta \), \( \Pi^W|_{separation} < \Pi^W|_{pooling} \) and \( R^D_t|_{pooling} > R^D_t \) iff \( \bar{\theta} > \theta_3 \).

**Proof for Proposition 4:** Suppose the website does not display deal sales. In this case, the only equilibrium in which both merchant types offer a deal is the pooling equilibrium that is described in Lemma 3. In this equilibrium, we have

\[
\frac{R^D_H}{R^D_H} - \frac{R^D_L}{R^D_L} = -\frac{N \bar{\alpha} (\alpha_H - \alpha_L)}{p \alpha_H \alpha_L} < 0.
\]

Therefore, from Lemma 7, the contract will not involve a subsidy. Suppose the website displays deal

\(^{14}\)As noted in the proof of Proposition 2, it is not possible that only the type \( H \) merchant offers a deal as the type \( L \) merchant will find it profitable to mimic the type \( H \) merchant. It is also not possible that neither merchant types offer a deal since from condition (6) offering a deal will be profitable for the type \( L \) merchant.
sales. From Proposition 1, if $\alpha_L \geq \alpha_1$, then the equilibrium is a separating equilibrium in which $d_t = \alpha_1$. We have,

$$\frac{R_H^D}{R_H^O} - \frac{R_L^D}{R_L^O} = \frac{\alpha_H - \alpha_L}{p} > 0. \quad (29)$$

Therefore, the contract always involves a subsidy. If $\alpha_L < \alpha_1$ and the equilibrium is a separating equilibrium as described in Lemma 6, we have

$$\frac{R_H^D}{R_H^O} - \frac{R_L^D}{R_L^O} = \frac{N (N + \alpha_L) (\alpha_L - \frac{1}{2} \beta \alpha_H)}{p \alpha_H (\alpha_L + \frac{1}{2} \beta N)}, \quad (30)$$

which is positive iff $\alpha_L \geq \alpha_3 = \frac{1}{2} \beta \alpha_H$. If instead the equilibrium is a pooling equilibrium as described in Lemma 4, we have

$$\frac{R_H^D}{R_H^O} - \frac{R_L^D}{R_L^O} = \frac{N \alpha (\alpha_L - \frac{1}{2} \beta \alpha_H)}{p \alpha_H \alpha_L}, \quad (31)$$

which is positive iff $\alpha_L \geq \alpha_3$. It is straightforward to verify that $\alpha_1 > \alpha_3 > \alpha_2$, where $\alpha_2$ is a bound that appears in Lemmas 5 and 6. It follows that the contract involves a subsidy if $\alpha_L \geq \alpha_3$.

**Proof for Proposition 5:** If deal sales are not displayed, then as before the only equilibrium in which both merchant types offer a deal is a pooling equilibrium in which $d_t = \bar{\alpha}$. Merchant revenues and website profits are as given in Lemma 2. We wish to show that there exists $t^* \in (0, 1)$ such that if $\frac{\tau_L}{\tau_H} > t^*$ then $R_t^D < R_t^D|_{pooling}$ in any PBE that can occur if deal sales are displayed. Since, $R_t^D > R_t^O$ in any PBE, it follows that $R_t^D|_{pooling} > R_t^O$. Therefore, the equilibrium if deal sales are not displayed is a pooling equilibrium and displaying deal sales leads to lower website profits.

First, consider a pooling PBE in which $d_t = \bar{\alpha}$. In this equilibrium, early- and frequent-experienced consumers and early-new consumers buy the deal in period 1. Frequent-new consumers will wait till period 2 since this allows them to avoid buying the deal if it is not sufficiently popular. In particular, if they observe realized deal sales less than $2\tau_L$ then as per Bayes-rule they must update their belief to $\theta < \bar{\theta}$ since such a sales level is more likely if the merchant’s type is $L$. Then, their expected surplus from buying the deal is negative if the realized sales level is less than $2\tau_L$. Therefore, it is worthwhile for them to wait till period 2 to make a more informed buying decision. We have

$$R_{H}^D|_{pooling} = \bar{\alpha} \left( \alpha_H + \frac{1}{2} \beta N + E \left[ M \left\{ \left(1 - \frac{1}{2} \beta \right) \Pr \left[ M > 2 (N + 1) \frac{\tau_L}{\tau_H} \right] \right\} \right] \right),$$

$$R_{L}^D|_{pooling} = \bar{\alpha} \left( \alpha_L + \frac{1}{2} \beta N \right). \quad (32)$$

Therefore, $R_{t}^D|_{pooling} < R_{t}^D|_{pooling}$. 

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Next, consider a separating PBE in which \( d_t = \alpha_t \). In equilibrium, all experienced and new consumers buy at the deal price \( \alpha_L \). At the deal price \( \alpha_H \), all experienced and early-new consumers buy the deal. Frequent- and late-new consumers will buy the deal at the deal price \( \alpha_H \) iff the realized period 1 deal sales exceed \( 2\tau_L \). Therefore, expected merchant revenues are given by

\[
R^D_{H(1)\text{separating}} = \alpha_H \left( \alpha_H + \frac{1}{2} \beta N + \frac{N}{N+1} \left( 1 - \frac{1}{2} \beta \right) \left( N + 1 \right) \left( 1 - \frac{\tau_L^2}{\tau_H} \right) \right); \quad (34)
\]

\[
R^D_{L(1)\text{separating}} = \alpha_L \left( \alpha_L + N \right); \quad (35)
\]

We note that \( R^D_{L(1)\text{separating}} < R^D_{L(0)\text{pooling}} \) and, if \( \frac{\tau_L}{\tau_H} \to 1 \), then \( R^D_{H(1)\text{separating}} < R^D_{H(0)\text{pooling}} \). It follows by continuity that there exists \( t^* \in (0, 1) \) such that if \( \frac{\tau_L}{\tau_H} > t^* \), then \( R^D_{L(1)\text{separating}} < R^D_{L(0)\text{pooling}} \).

Lastly, note that a separating equilibrium in which \( d_L = \alpha_L \) and \( d_H < \alpha_H \) cannot occur. In such an equilibrium, all frequent- and early-new consumers must buy the deal in period 2 on the equilibrium path as they obtain strictly positive surplus. But this implies that they do not condition their buying decisions on realized deal sales and the no-mimicking constraint cannot hold.

**Proof for Proposition 6:** Consider a separating PBE in which \( d_t = \alpha_t r_t \). In equilibrium, all new consumers and experienced consumers will buy the deal and we have \( R^D_t = \alpha_t r_t \left( N + \alpha_t \right) \). The type \( H \) merchant does not have an incentive to deviate as it cannot sell to new consumers at a higher price. Suppose the type \( L \) merchant deviates to \( d_L = \alpha_H r_H \). If \( r_L \geq \alpha_H r_H \), then all experienced consumers and new consumers will still buy its deal. Therefore, mimicking is profitable and such a separating PBE cannot exist. If \( r_L < \alpha_H r_H \) then only new consumers will buy its deal. For mimicking to be unprofitable we require that

\[
\alpha_L r_L \left( N + \alpha_L \right) \geq \alpha_H r_H N; \quad (36)
\]

Therefore the separating PBE exists iff \( \alpha_H r_H > r_L \geq \frac{\alpha_L}{(N+\alpha_L)\alpha_L} r_H \) and is the unique SUE since the type \( H \) merchant cannot earn higher profits in any other PBE. This yields the conditions in the statement of the proposition.