Reference Groups and Product Line Decisions

An Experimental Investigation of Limited Editions and Product Proliferation

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Abstract

Some luxury goods manufacturers offer limited editions of their products while some others market multiple product lines. Researchers have found that reference groups shape consumer evaluations of these product categories. Yet little empirical research has examined how reference groups affect the product line decisions of firms. Indeed, in a field setting it is quite a challenge to isolate reference group effects from contextual effects and correlated effects. In this paper we propose a parsimonious model that allows us to study how reference groups influence firm behavior and that lends itself to experimental analysis. With the aid of the model, we investigate the behavior of consumers in a laboratory setting where we can focus on the reference group effects after controlling for the contextual and correlated effects. The experimental results show that in the presence of strong reference group effects, limited editions and multiple products can help improve firm's profits. Furthermore, the trends in the purchase decisions of our participants point to the possibility that they are capable of introspecting close to two steps of thinking at the outset of the game and then learning through reinforcement mechanisms.

Keywords: Reference Groups, Product Line, Experimental Economics, Game Theory.

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1 Introduction

Luxury goods manufacturers such as Hermès, Yves Saint Laurent, and Louis Vuitton focus their marketing efforts on the upper echelon of the market and offer limited editions of their products (Betts 2001, Branch 2004). On the other hand, some fashion designers offer multiple product lines. For example, fashion designer Armani offers Giorgio Armani Privé, Georgio Armani, Armani Collezioni, Emporio Armani and Armani Exchange. It well known that reference groups influence consumer's evaluations of these products (Bearden and Etzel 1982, Childers and Rao 1992). Furthermore, the elite consumers prefer products that help differentiate them from the masses while the masses like to emulate the choices of elite (Simmel 1957, Bourdieu 1984, Bryson 1996). The presence of such reference groups effects creates a dilemma for firms: a firm needs to sell its products to the elite to stimulate demand among the masses, but potential sales to the masses can deter the elite from even buying the product in the first place.

Recent theoretical research has attempted to examine how the presence of social influences could impact a firm's product line decisions (e.g., Balachander and Stock 2008, Amaldoss and Jain 2008). In theory, a firm may find it attractive to offer limited edition products and/or multiple product lines in the presence of social influences.[1] For example, Amaldoss and Jain (2008) suggest that a firm could actually sell more by offering a limited edition than without such a supply constraint. Thus, if the firm is selling x units without any supply constraint, it could increase its sales by committing to selling no more than $y > x$ units. From an empirical perspective, it is not immediately clear that consumers will behave according to this prediction. Indeed, if a firm's expected sales in the absence of any supply constraint is x, then its commitment not to sell more than x units could very well be ignored by the consumers, as this commitment seems quite superfluous. We know that it is not profitable for a firm to offer multiple products of the same quality as it is costly to launch new products. In theory, however, Amaldoss and Jain (2008) find that a firm could increase its profits by selling multiple variants of identical quality, if reference group effects are present. From an empirical perspective, it is again not clear that offering multiple variants of the same product will have any effect on consumers. Thus, in theory, reference groups could lead consumers as well as firms to behave in counterintuitive ways. As firms often offer limited edition products and multiple

[1] In the context of these models, these product line strategies are not valuable if social influences were absent.

product lines, it is important to empirically investigate the wisdom of using these strategies in markets where consumers are susceptible to reference group effects.

Though it is important to study reference group effects, it is a challenge to empirically investigate social interaction as pointed out by Manski (1993). This is because a consumer's product choice could be the consequence of the choices of her reference group (*endogenous social effect*), or the characteristics of her group (*contextual effect*). Furthermore, members of a reference group may behave in a similar manner because of common unobserved factors (*correlated effect*). Manski showed that standard econometric methods cannot draw a distinction between endogenous effects, contextual effects, and correlated effects, though these effects have very different managerial implications (see also Moffitt 2001). Researchers have attempted to address this concern by using experimental data, instrumental variables, and other identification strategies (e.g., Kremer and Levy 2001, Ioannides and Zabel 2002, Glaeser, Sacerdote and Scheinkman 2003, Krauth 2006, Sorensen 2006). The principal focus of the empirical literature on social interaction has been to establish the presence of social influences. Understanding how reference groups influence firm behavior is an even greater challenge as now one needs to carefully separate endogenous firm behavior from the social effects.

In this paper, we make a beginning in empirically investigating how the desire of *leaders* (elite) to contrast themselves from the *followers* (masses) and the countervailing goal of followers to assimilate with the leaders affect a firm's product line decisions. Toward this goal, we develop a discrete version of Amaldoss and Jain (2008) that is amenable to experimental analysis. Using the empirical model, we then investigate the behavior of consumers in a laboratory setting where we can focus on the reference group effects after controlling for the contextual and correlated effects.

In Study 1, we assess the effect of limited editions on consumer demand and firm's profits using a within-participant design. We find that offering a limited edition improves sales to leaders and increases firm's profits. In particular, sales increase even when the firm's quantity commitment was set at a higher level than the sales which were realized in the no limited edition condition. In Study 2 we examine the robustness of these results in a weaker information condition. The results of this study clarify that participants can adaptively learn to behave as predicted by the model. To further evaluate whether the natural preferences of participants will lead to outcomes consistent with the model predictions, we conducted an exploratory survey. The survey results indicate that consumer's intention to purchase a limited-edition car is higher than their intention to purchase a non-limited edition version of the same car. In Study 3 we examine how adding a product to a firm's product line can affect a firm's profits, when the products are of the same quality level. Keeping the product qualities at the same level helps to rule out heterogeneity in consumer's taste for quality as a potential explanation for the experimental results. We find that the total sales and profits increase when a firm offers two identical products in the presence of strong reference group effects. These experimental results are directionally consistent with the equilibrium predictions of our model. Finally, to gain insights into how our participants made their purchase decisions we fit the Cognitive Hierarchy model (Camerer et al. 2004) and the Sophisticated EWA model (Camerer et al. 2002) to the experimental data. We find that the choices of our participants in the very first trial can be accounted by very few steps of thinking, and the behavior in subsequent trials can be explained by reinforcement-based learning mechanism.

Related Literature. Our work is related to the empirical literature on social interaction (see Manski 2000 for an overview). The goal of this literature has been to establish the presence of social effects and to measure its magnitude. For example, Ioannides and Zabel (2003) estimated a model of housing demand with neighborhood effects. Using instrumental variable technique they assessed the magnitude of social interaction. Similarly, using data on the health insurance choices of employees of the University of California system, Sorensen (2006) estimated the impact of social interactions on choice of health plans. In contrast to this work, our focus is not on measuring social interaction. Instead, we induce social preferences in a laboratory setting and examine their impact on demand and firm's profits.

An emerging body of theoretical literature studies how social factors affect firm behavior. Balachander and Stock (2008) examine the issue of offering limited edition in a competitive setting. They find that the firm with a higher quality product will benefit from offering a limited edition while the firm with lower quality will be hurt by such an introduction. Using a different framework, Amaldoss and Jain (2008) show that limited editions can improve a firm's profits when reference group effects are strong. They also show that using multiple product variants of the same product can also improve firm's profits.[2] Both these theoretical models assume that consumer valuation

[2]For an alternate explanation based on signaling see for example Becker and Murphy (2000) and Pesendorfer (1995). In the signaling literature, consumers use their purchase decisions to signal a latent variable, such as wealth or status, which cannot be directly observed. In the behaviorial literature, however, social position is observable.

are distributed according to a continuous distribution, and this assumption makes it convenient to analyze the models. However, the simplifying assumption makes it difficult to implement these models in a laboratory setting. Our focus is different from these papers in that we are interested in empirically examining the strategic impact of reference group effects. Accordingly, we develop a discrete game with a finite number of participants and subject it to a laboratory test. Additionally, our analysis clarifies why product scarcity alone cannot account as to why firms offer limited editions. We also study how the equilibrium outcome may change if followers are budget constrained. These issues are not investigated in either Amaldoss and Jain (2008) or Balachander and Stock (2008).

Our work is grounded in the literature on social comparison (e.g., Hyman 1942, Sherif 1948, Brewer and Weber 1994, Di Maggio 1982; see Kemmelmeier and Oyserman 2001 for a review). Social psychologists have found that consumers have a natural tendency to engage in social comparisons and that these comparisons affect self image and behavior. In some instances, consumers engage in upward social comparisons with their aspirational group. When making these upward social comparisons, people seem to engage in assessing the similarities between the standard and the self and this produces an assimilation effect (Lockwood and Kunda 1997, Mussweiler et al. 2004). In other social contexts, individuals may make downward social comparisons with a lower social group. People making a downward social comparison focus on the dissimilarities between the standard and the self, and this often leads to 'aesthetic distancing' and 'symbolic exclusion' in the behavior of the higher social group (Bourdieu 1984 and Bryson 1996). In contrast to this body of literature, we are interested in product evaluation rather than evaluation of the self.

Another body of literature that is related to our paper is the work on product scarcity. According to this literature, consumers prefer products that are available in smaller quantities (e.g., Worchel et al 1975). One explanation for this finding is that consumers may believe that scarce products are of better quality (see Stock and Balachander 2005 for a signaling explanation for scarcity). In our model, as quality is known there is no need to signal quality. Note that leaders prefer a product more if it helps to distinguish them from followers, and followers prefer a product if it helps to

Furthermore, a follower emulates a leader in order to feel like the leader. For instance, consumers might buy what a famous movie star buys, not to signal that they are movie stars, but rather to feel like the movie star. In our model, we incorporate social influences directly into the utility formulation as consumption externalities. In the signaling literature, however, consumer utility is increased only if a person successfully signals her wealth (or other latent variable). Thus, our model formulation is very different from the signaling models.

assimilate with leaders. Thus our explanation is rooted in reference groups. The other explanation for scarcity is grounded in consumer need for uniqueness. Using a one-period model, Amaldoss and Jain (2005a) investigated the impact of consumer desire for uniqueness and conformism on demand, in the context of a monopoly. Amaldoss and Jain (2005b) extended this analysis in the context of a duopoly. In their framework, consumers wish to be different from *all* other consumers or wish to be similar to *all* other consumers. In contrast to this work, we focus on an inter-group phenomenon. Furthermore, the research question that we address is different. In particular, Amaldoss and Jain (2005a, 2005b) empirically investigates how the presence of desire for uniqueness and conformity can lead to an upward-sloping demand curve. Our paper, on the other hand, tests whether a firm can improve its profits by offering limited editions or introducing multiple product lines.

The rest of the paper is organized as follows: Section 2 introduces a model of limited editions and tests its predictive accuracy in a laboratory experiment. Section 3 analyzes multiple product lines and experimentally tests its predictions. In Section 4, we try to understand how participant's arrived at their decisions using learning models. Finally, Section 5 summarizes the findings and concludes by providing some directions for further research.

2 An Empirical Analysis of Limited Edition Products

In this section, we develop an empirical model of the game studied in Amaldoss and Jain (2008), as our primary purpose is to subject the model to an experimental test. While most of the insights obtained in Amaldoss and Jain (2008) continue to hold in this discrete version, there are also multiple equilibria including mixed-strategy equilibria which are not present in the continuous version studied in Amaldoss and Jain (2008). For completeness, we briefly present an analysis of the discrete version, which will form the basis of our empirical test.

Consider a two-period model where a firm produces a product at constant marginal cost c. In this two-period model, the firm sets its price in each period to maximize its total profits.[3] There are two groups of consumers, namely leaders and followers. The leaders come to the market in the first period and decide whether they want to buy the product. The followers enter the market only in the second period to make their purchases.

[3] To conserve on notation we assume that there is no discounting. However, using a discount factor does not change the basic nature of the results.

Leaders. There are N_l leaders. As the leaders desire to contrast themselves from the followers, the utility of a product declines if more followers buy the product. Specifically, we assume that the (indirect) utility derived by a leader from purchasing the product at a price p_1 is given by:

$$U_l(p) = \psi_l(y^e) - p_1 \tag{1}$$

where y^e is the number of followers expected to buy the product in the second period, and p_1 is the price of the product. As the utility derived from the product reduces with more followers adopting the product, we let $\psi_l'(\cdot) < 0$. In our formulation, the leaders find the product attractive enough to purchase it in the absence of social effects. More precisely, we assume that $\psi_l(0) > 0$. As we are interested in markets where social effects have a strong influence on purchase decisions, we focus on the case where if *all* the followers buy the product then its utility will decline so much so that none of the leaders will purchase the product.[4] That is, if the total number of followers in the market is N_f, then:

$$\psi_l(N_f) < 0. \tag{2}$$

Followers. The followers, who enter the market in the second period, have an opportunity to observe the purchases made by leaders. The (indirect) utility derived by a follower on purchasing the product is given by:

$$U_f(p) = \psi_f(x) - p_2 \tag{3}$$

where x is the number of leaders who bought the product in the first period and p_2 is the second-period price. We assume that $\psi_f' > 0$, suggesting that the attractiveness of a product to followers increases as more leaders purchase the product. We also assume that if no leader were to buy the product, then none of the followers will buy the product even if it is priced at marginal cost. This implies that:

$$\psi_f(0) - c < 0 \tag{4}$$

On the other hand, if all the leaders were to purchase the product, then the followers would find the product attractive enough to purchase, implying that:

$$\psi_f(N_l) - c \quad > \quad 0 \tag{5}$$

[4]While this assumption makes the analysis more clean, it is not a critical one. If this assumption is violated, leaders may be willing to adopt the product even if they anticipate all the followers to buy product. However, leaders' willingness to pay for the product will be lower since $\psi_l'(\cdot) < 0$. In such a case, it may still be profitable for the firm to develop strategies that will deter the followers from entering the market.

In our formulation, therefore, the utility derived by the followers on purchasing the product is essentially a consequence of the social effects.[5] We are also assuming that leaders and followers are homogeneous within each group so that we can analyze the effects of reference group without being distracted by heterogeneity in consumer preferences. Though this assumption is not critical for our analysis, it facilitates the exposition of the key results. We also assume that the leaders form rational expectations.

In the second period, the followers would purchase the product if the utility derived from the product is large enough to cover the price of the product, namely

$$\psi_f(x) - p_2 \geq 0 \tag{6}$$

The second-period price charged by the firm will be sufficient to at least cover its marginal cost, that is $p_2 \geq c$. Now for the followers to be willing to pay a price higher than the marginal cost, the first-period demand x should be high enough so that the utility derived from the product by followers rises above its marginal cost, namely $x \geq \psi_f^{-1}(c)$. Let y denote the sales to the followers. Hence, we have:

$$y = \begin{cases} N_f & \text{if } x \geq \psi_f^{-1}(c) \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Now define $\hat{x}$ as:

$$\hat{x} = \max_{x \in \mathbb{N}} \left\{ x | \psi_f^{-1}(x) - c < 0 \right\} \tag{8}$$

where $\mathbb{N}$ is the set of natural numbers. It follows that $y = 0$ if $x \leq \hat{x}$, suggesting that the second-period sales will be zero if the first-period sales is below a critical threshold. An important implication of (4) and (5) is that $\hat{x}$ exists and furthermore it must be strictly less than N_l. As no leader will purchase the product if $p_1 > \psi_l(0)$, it is reasonable to assume that $p_1 \leq \psi_l(0)$.

In our experimental investigation, we consider the case where $N_l = N_f = 10$, $\psi_l(y) = 46 - 5\sqrt{y}$, $\psi_f(x) = 15x, c = 32.5$ and $p_1 = 33.5$. Note that our choice of N_l and N_f is guided by the need to keep the population of consumers small enough that we can run these experiments in a laboratory setting, and yet accurately capture the coordination dilemmas faced by the participants. We chose the other parameter values so that constraints of our model, namely (2), (4) and (5), are satisfied,

[5] Thus (4) and (5) ensure in a simple way that leaders adopting the product is critical for followers to buy the product. These assumptions can also be further relaxed, as long the product remains attractive enough for followers to purchase it when leaders adopt it.

and furthermore the contrast between limited-edition treatment and no-limited edition treatments is high. A higher level of contrast increases the possibility that we may detect the treatment effect in the laboratory. We let the second-period price to depend on the first-period sales, namely $p_2 = \max(32.5, 13.5x)$.[6] With this setup, we have the following result which details the equilibrium behavior of leaders and followers.

Result 1 *a) In any pure-strategy Nash equilibrium of the game, $\hat{x}$ leaders (where $\hat{x} < N_l$) purchase the product but none of the followers purchase the product. For the parameter values we use in our experiment, $\hat{x} = 2$ and the expected profits for the firm are 2.*

b) In addition, there is a symmetric mixed-strategy equilibrium in which each leader purchases the product with probability ρ which is given by:

$$\psi_l(0) - [\psi_l(0) - \psi_l(N_f)] \sum_{x=\hat{x}}^{N_l-1} \binom{N_l - 1}{x} \rho^x (1-\rho)^{N_l-1-x} = p_1 \tag{9}$$

If more than 2 leaders adopt the product, then all followers buy the product in the second period. For the specific parameter values that we consider in the experiment, in the mixed-strategy equilibrium the leaders should purchase the product with a probability of $\rho = 0.292$ and the corresponding expected profits for the firm are 121.11.

In any pure-strategy solution, the leaders need to coordinate their purchases such that no follower finds it attractive to emulate the leaders by buying the product. In the symmetric mixed-strategy equilibrium, however, followers could buy the product in the second period. It is useful to contrast these results with those in Amaldoss and Jain (2008). In Amaldoss and Jain (2008), leaders' valuations are assumed to be continuously distributed and there are no mixed-strategy equilibria. Later in our experimental analysis, we examine the pure as well as the mixed-strategy equilibria of the game.

Limited Edition. Now consider the case when the firm produces a limited edition of its product. Assume that the firm promises not to sell more than Q units, and that Q is common knowledge. Further define $\hat{y}(p_1)$ as follows:

$$\hat{y}(p_1) = \max_{y \in \mathbb{N}} \{y | \psi_l(y) - p_1 > 0\} \tag{10}$$

[6] Theoretically, we could set $p_2 = 15x$ but setting a slightly lower price provides additional incentives to buy the product.

Thus, $\hat{y}(p_1)$ is the maximum number such that if $\hat{y}(p_1)$ followers adopt, the leaders would still prefer to buy the product. Clearly, $0 < \hat{y}(p_1) < N_l$. In our experiment, we set $Q = 14$ while retaining the other parameter values from the no-limited edition treatment. On examining the equilibrium behavior in the resulting game, we have the following finding:

Result 2 *a) If the firm offers a limited edition $Q > \hat{x}$ and $p_1 < \psi_l(0)$, then in the pure-strategy equilibrium the demand from leaders is given by:*

$$x(Q) = \begin{cases} \min(Q, N_l) & \text{if } Q \in [\hat{x}, \hat{x} + \hat{y}(p_1) + 1] \\ \{\hat{x}, \min(Q, N_l)\} & \text{if } Q \in (\hat{x} + \hat{y}(p_1) + 1, N_l + \psi_l^{-1}(p_1)) \\ \hat{x} & \text{otherwise.} \end{cases} \tag{11}$$

In our experiment, under pure strategy equilibrium all leaders should buy the product, and all the followers should desire the product although only four can purchase it. The expected profits for the firm is 420.

b) In addition there are mixed strategy equilibria in which each leader purchases the product with probability ρ_q. If $Q > \hat{x}$, then the mixed-strategy solution is implicitly defined by the equation:

$$\psi_l(0) - \sum_{x=\hat{x}}^{N_l-1} [\psi_l(0) - \psi_l(\min(N_f, Q - x - 1))] \begin{pmatrix} N_l - 1 \\ x \end{pmatrix} \rho_q^x (1 - \rho_q)^{N_l-1-x} = p_1 \tag{12}$$

If $x > 2$ leaders adopt the product, then all followers would want to buy the product but only $\max(Q - x, N_f)$ *followers can get the product. For the parameter values we use in our experiment, we find that leaders should buy the product with a probability* 0.74 *or* 0.30*; and the corresponding firm's profits are* 426.88 *and* 122.74*, respectively.*

The pure-strategy equilibrium shows that unless Q is too high, the first-period sales will be weakly higher when a firm offers a limited edition of its product. Note that if $Q \in (\hat{x} + \hat{y}(p_1) + 1, N_l + \psi_l^{-1}(p_1))$, the equilibrium in which only $\hat{x}$ leaders buy continues to hold. At the same time there is another equilibrium in which the first-period sales is higher, and the corresponding equilibrium sales is given by $\min(Q, N_l)$. In our experiment, we set $Q = 14$ and the corresponding pure strategy equilibrium sales to leaders should be 10. Recall that sales to leader was a mere 2, when no limited edition was offered. Unlike in Amaldoss and Jain (2008), in this discrete game there also exist additional symmetric mixed-strategy equilibria in which each leader purchases the product with probability ρ_q. As there can be multiple mixed-strategy equilibria that satisfy condition (12), let us define $\underline{\rho_q}$ as the minimum ρ_q which satisfies the condition. Then it is easy to show that $\underline{\rho_q} > \rho$. In other words, the probability of a leader purchasing a product rises when the product is offered in limited edition. Hence, the expected sales to leaders is higher when a firm offers a limited edition of its product (compared to the case with no quantity commitment).[7] Thus in both the pure-strategy as well as the mixed-strategy equilibria, offering a limited edition (weakly) improves the expected demand from leaders and the total profits.

The intuition for this result is simple. By offering a limited edition, a firm restricts the number of followers who can potentially purchase the product. This helps the firm to convince the leaders to buy the product. Consequently, more leaders buy the product in period 1.[8]

2.1 Study 1

A reduced-form regression analysis of survey data is not useful for testing our model as in such an analysis it is difficult to separate the endogenous social effects from the contextual effects and correlated effects. The multiplicity of equilibria further complicates the empirical analysis and there are no self-evident equilibrium selection criteria. Our interest is in studying how limited editions affect a firm's profits. This adds another level of complexity. Ideally we would like data obtained by randomly assigning firms to a strategy treatment. Such data is typically not available. Furthermore, the strategic choices of a firm would depend on the value of social effects.[9] Therefore, estimating the impact of the reference group parameters on sales and profits poses significant methodological and data availability problems.

[7] To see this note that from (9) and (12), we obtain:

$$[\psi_l(0) - \psi_l(N_f)] \sum_{x=\hat{x}}^{N_l-1} \begin{pmatrix} N_l - 1 \\ x \end{pmatrix} \rho^x (1-\rho)^{N_l-1-x} = \sum_{x=\hat{x}}^{N_l-1} [\psi_l(0) - \psi_l(\min(N_f, Q-x-1))] \begin{pmatrix} N_l - 1 \\ x \end{pmatrix} \rho_q^x (1-\rho_q)^{N_l-1-x}$$

Since $\psi_l(\cdot)$ is decreasing, this implies that:

$$\sum_{x=\hat{x}}^{N_l-1} \begin{pmatrix} N_l - 1 \\ x \end{pmatrix} \rho^x (1-\rho)^{N_l-1-x} \le \sum_{x=\hat{x}}^{N_l-1} \begin{pmatrix} N_l - 1 \\ x \end{pmatrix} \rho_q^x (1-\rho_q)^{N_l-1-x}$$

with the strict inequality being applicable if $Q < N_f$. This proves the assertion.

[8] In our framework, it is important to note that quantity commitment is of no value to the firm if social effects do not exist and that consumers cannot wait to purchase the product. Thus our result provides another explanation for why a firm may want to make a quantity commitment even if there are no Coase-like pricing issues that are seen in durable goods market. It is useful to note some important differences between our framework and the Coase problem where also quantity commitment can help. We show that a firm can use limited edition even when consumers cannot wait and prices increase over time. Contrary to durable goods models, in our framework offering a limited-edition product leads to increased total sales rather than reduced sales. Besides, limited edition can reduce the first-period price rather than increase the price as found in durable goods models.

[9] Consequently, we will need to estimate the parameter values of the social effects for a given pricing and limited-edition strategy while allowing for endogeneity of these strategic variables.

In an attempt to directly test the theory, we use the tools of experimental economics to induce reference group effects by exogenously manipulating the externalities experienced by leaders and followers. Further to avoid contextual effects, we randomly assign participants to play the role of leaders and followers. With this control on the demand side of the empirical model, we exogenously vary firm's strategy of offering limited edition and study its impact on consumer's product choices and firm's profits. As our laboratory setting closely parallels the empirical model, it is easy to compare the observed behavior against the equilibrium predictions. This also gives us an opportunity to use observed behavior as an equilibrium selection criterion and better understand its practical implications (See Wang and Krishna 2006, Lim et al 2007, Haruvy and Stahl 2007 for more empirical tests of theoretical models)

In Study 1, as a first step in assessing the predictive accuracy of the model, we test the predictions regarding offering a limited edition. Specifically, we seek answers for the following three questions:

1. *Does offering a limited edition of a product increase total sales and sales to leaders?*
2. *Is it profitable for a firm to offer a limited edition of a product?*
3. *Are the expectations of leaders consistent with the market outcomes?*

Participants. Business school students were recruited to participate in the study. They were paid a show-up fee of $5 in addition to a monetary reward contingent on their performance. All transactions were in an experimental currency called 'francs' which were converted into US dollars at the end of the experiment. Participants earned $20.20 on average in the study.

Experimental Design. We ran two groups of participants with limited edition as a within-participant variable. In one group there was no quantity restriction in the first thirty trials (1-30) and a limited edition of fourteen units was offered in the next thirty trials (31-60). In the other group, the order of presentation of the limited edition was reversed. By letting participants play the two-period game repeatedly, we allow for the possibility that participants could learn to conform to the equilibrium prediction. Allowing participants to play multiple trials is a common practice in experimental economics (e.g., see Holt 1995 for a brief survey). But playing the game repeatedly does not change the equilibrium prediction. In the empirical analysis, we discuss the average behavior of our participants over the several iterations of the game, as well as their behavior in the very first trial of each experimental session.

Procedure. In our experiment, participants played the role of buyers while the computer played the role of a seller.[10]

Seller. In the laboratory, the seller posted a price and promised to sell a fixed number of units of the product to buyers who were willing to pay the posted prices (see Holt 1995 for a discussion on posted prices market). The posted-price market is akin to retail markets where buyers cannot negotiate the price with the seller. The first-period price was fixed at 33.5 francs. The second-period price, however, varied depending on the number of leaders who purchased the product in the first period, and it was given by the function $p_2 = Max\{35, 13.5x\}$. Thus the seller always took advantage of the incremental value that accrued to followers because of the sales to leaders.

Buyers. Each participant was randomly assigned to play the role of either a Type A or Type B buyer, and her type remained fixed throughout the game.

Type A buyers. There were ten Type A buyers, and they came to the market in the first period. The Type A buyers value the product *less* if more Type B buyers purchase the product in the second period. For example, if five Type B buyers purchase the product, the actual value of the product for a Type A buyer will be 34.82 francs (that is, $46 - 5\sqrt{5} = 34.82$), which is more than the price of the product. But if eight Type B buyers purchase the product, then its value will decline further to 31.86 francs (that is, $46 - 5\sqrt{8} = 31.86$). It is important to note that the value of the product to Type A buyers depends on the number of Type B buyers who actually purchase the product in the second period.

Type B buyers. Type B buyers value the product *more* when more Type A buyers own the product. For example, if five Type A buyers had purchased the product in period 1, then the value of the product to a Type B buyer will be 75 francs (that is, $15 \times 5 = 75$), which is more than the purchase price. On the other hand, if eight Type A buyers had purchased the product in period 1, then the value of the product grows further to 120 francs ($15 \times 8 = 120$).

At the commencement of each trial, participants were endowed with 35 francs so that they had sufficient funds to pay for the product if they decided to buy it. We let the endowments be the same for both Type A and Type B buyers as our theoretical results are driven by the reference

[10] The detailed experimental instructions are available from the authors.

group effects, not by any difference in income or endowment.[11] In the experiment, participants were informed about their valuation, the number of units of the product available for sale, product cost, and the price of the product. The type of participant (A or B) and the total number of participants were fixed for all the sixty trials. The quantity of the product available for sale, however, changed after thirty trials.

In the first period of every trial, each Type A buyer had to decide whether or not to purchase the product. We also wanted to track in a less intrusive way the sales expectations that guided the product choices of our players. Toward this goal, Type A buyers were asked to provide estimates of the number of Type B buyers who might buy the product in the second period. The computer displayed to each individual Type A buyer the likely value of the product based on her individual sales projection. Each Type A buyer could revise her sales projection and assess how the likely sales changed the value of the product. Note that the likely product value displayed to an individual Type A buyer relied only on her sales projection, not on the the sales projections of other Type A buyers. Thus participants were not informed about the sales expectations of other players. After all the Type A buyers made their decisions, the game advanced to the second period. Notice that the actual value of the product to Type A buyers depends on the number of Type B buyers who purchase the product in the second period. Furthermore, the second period sales could be different from the expectation of any individual Type A buyers.

In the second period of every trial, the price of the product for Type B buyers was announced based on the number of Type A players who bought the product in period 1. Then each Type B buyer decided whether or not to buy the product. After all the Type B players made their decisions, the results for that trial were announced.

The payoff for participants who bought the product was: endowment + actual value of the product - price paid. The participants who did not buy the product kept the endowment. At the end of every trial, each participant was informed of the number of Type A and Type B buyers who purchased the product, and the payoff for the trial.

In order to make participants familiar with the structure of the game, they were allowed to play three practice trials for which they received no monetary reward. Then they played sixty trials,

[11] While it is possible that the leaders are better endowed, we are interested in understanding how reference group effects influence behavior when a product can be afforded by both types of consumers.

with the limited-edition condition changing after thirty trials. The number of units available for sale was either 20 (no limited edition) or 14 (limited edition). At the end of sixty trials, participants were paid according to their cumulative earnings. Finally, they were debriefed and dismissed.

Results. We first analyze the sales to Type A buyers (leaders) and Type B buyers (followers). Then we investigate firm's profits and Type A buyers' sales expectations. The experimental results suggest that participants behaved in a manner consistent with the qualitative predictions of the model. On average, firm's profits and sales to leaders increased when a limited edition was offered. Further, the expectations of participants were directionally consistent with the model predictions.

Analysis of Sales. Table 1 presents the mean sales to the two types of buyers. We report separately the sales to Group 1 and Group 2 of Type A and Type B buyers. The observed sales pattern is in keeping with the theoretical predictions. However, we see variation among participants over the several iterations of the game.

Mean Sales. In equilibrium, the sales to leaders should increase if the firm offers a limited edition of the product. On average, the Type A buyers purchased 3.75 units in the absence of a limited edition. But upon offering a limited edition, the corresponding sales grew to 9.23 units. We can reject the null hypothesis that these two sales levels are the same ($F_{(1,118)} = 657.09$, $p < 0.001$). A similar sales pattern is also observed in each of the two groups. In Group 1, the mean sales to Type A buyers saw a significant increase from 3.4 to 9.20 units on offering a limited edition ($F_{(1,58)} = 385.09$, $p < 0.001$). Likewise, in Group 2 the corresponding mean sales increased from 4.10 to 9.27 units ($F_{(1,58)} = 295.60$, $p < 0.001$).

According to the equilibrium prediction, the overall sales should increase on offering a limited edition. The overall sales increased from 11.30 to 14 units on offering a limited edition, and the shift in mean sales is significant ($F_{(1,118)} = 19.66$, $p < 0.001$). At the level of individual groups, we note that the overall sales to Group 1 rose from 10.10 to 14 units ($F_{(1,58)} = 21.74$, $p < 0.001$). In Group 2 the overall sales increased from 12.50 to 14 units, but this difference is only marginally significant ($F_{(1,58)} = 3.17$, $p < 0.081$).

Distribution of Sales. The aggregate distribution of sales to Type A buyers over the several iterations of the game is presented in Figure 1. In both groups, the distribution of sales in the absence of a limited edition did not overlap with the distribution of sales when a limited edition was offered. Hence, we saw a perceptible upward shift in sales to Type A buyers on offering limited edition.

Figure 2 compares the mixed-strategy equilibrium prediction of $\rho = 0.292$ with the observed distribution of sales across the two groups. In the absence of a limited edition, the distribution of sales to Type A buyers was only marginally different from the equilibrium prediction ($D_{60} = 0.22$, $p > 0.05$). Upon offering a limited edition, the sales to Type A buyers was higher than the mixed-strategy equilibrium prediction of $\rho = 0.742$ and the departure is highly significant ($D_{60} = 0.57$, $p < 0.01$). These departures are even stronger from the mixed-strategy equilibrium prediction of $\rho = 0.305$. These findings imply that the observed behavior is more closely aligned with the pure strategy prediction of ten units.

Figure 3 presents the sales to Type B buyers. Recall that all Type B buyers should buy the product if three or more Type A buyers purchased the product in the first period (that is, $x \geq 3$). In the absence of a limited edition, the sales to Type B buyers were bimodal, ranging all the way from zero to ten. Upon offering a limited edition, the sales range was reduced and extended only from 4 to 7. Given the departures in the behavior of Type A buyers from the mixed-strategy equilibrium predictions, it is expected that sales in the second period will not accord well with the equilibrium prediction. Accordingly, we see in Figure 4 that the observed sales pattern is different from the mixed-strategy equilibrium prediction of $\rho = 0.742$ ($p < 0.01$). The observed behavior, however, is directionally consistent with a best reply to the actual sales to Type A buyers.

Trends in Sales. In an attempt to detect trends in the sales pattern, we divided the thirty trials into six blocks of five trials each and conducted an ANOVA to test for block effects. When a limited edition was offered, the mean sales to Type A buyers started at 9 units in the first block, grew to 10 units by the fourth block, and then declined to 9.4 units in the final block (See Figure 5). We can reject the null hypothesis that these block means are the same ($F_{(54,5)} = 3.54$, $p < 0.01$). This analysis suggests that participants potentially learned over trials when a limited edition was offered, but they were not able to lock into the symmetric pure-strategy equilibrium when they reached it in the fourth block of trials. When a limited edition was not offered, the shifts in sales over the six blocks of trials were not significant (Type A Buyer: $F_{(54,5)} = 0.19$, $p > 0.2$; Type B Buyer: $F_{(54,5)} = 0.42$, $p > 0.20$).

Note that this trend analysis focuses on the average behavior over blocks of five trials. The behavior in the first trial of each experimental session is of special interest as there is no scope for learning in the very first trial. Consequently, the observed behavior is more likely to be the result of some introspection about the structure of the underlying game. In both Group 1 and Group 2, Type A buyers bought four units of the product, when supply was not restricted. But with limited edition, Type A participants in Group 1 and Group 2 purchased nine and eight units of the product, respectively. This informal analysis suggests that even in the first trial Type A buyers purchased more when the product was in limited edition, implying that the basic intuition of our participants was probably consistent with the model prediction.

Individual Differences. Thus far we have examined the aggregate sales pattern. Figure 6 presents the distribution of purchase probabilities of individual participants. The distribution of probabilities was qualitatively consistent with the equilibrium solution. As predicted, the purchase probabilities of Type A buyers increased when a limited edition was offered. On the other hand, Type B buyers got to buy the product less often when a limited edition was offered.

In equilibrium, Type A buyers should buy the product with a probability of 0.292 if a limited edition is not offered. In actuality, the purchase probability of individual Type A buyers ranged from from 0.03 to 1. Interestingly, the modal frequency fell in the probability class 0.2-0.3. Upon offering a limited edition, the mixed-strategy purchase probability predictions are 0.305 and 0.74. But the corresponding symmetric pure-strategy prediction is 1; and according to the asymmetric pure-strategy prediction the purchase probability should be 0.2 on average. The actual purchase probability of Type A buyers in the presence of limited edition ranged from 0.6 to 1 with the mode in the probability class 0.9-1. This implies that in the presence of a limited edition, the purchase probabilities of most participants were closer to the symmetric pure-strategy solution rather than the asymmetric pure-strategy or the mixed-strategy predictions.

Analysis of Profits. Table 2 presents the mean profits. In equilibrium, profits improve if the firm offers a limited edition. On average, the firm earned 189.05 francs in the absence of a limited edition. On offering a limited edition of 14 units, firm's profits improved to 437.97 francs. We can reject the null hypothesis that these two profit levels are the same ($F_{(1,118)} = 135.6$, $p < 0.001$). The results are similar at the level of individual groups (Group 1: $F_{(1,118)} = 139.3$, $p < 0.001$; Group 2: $F_{(1,118)} = 38.03$, $p < 0.001$).

In theory, offering a limited edition could improve the profits from each consumer segment. Accordingly, the average profits from leaders increased from 3.75 to 9.23 francs ($F_{(1,118)} = 657.1$,

$p < 0.001$). Similarly, the profits from followers saw a significant increase ($F_{(1,118)} = 131.72$, $p < 0.001$). The theoretical prediction holds at the level of individual groups as well ($p < 0.001$).

Analysis of Expectations. Each individual Type A buyer should make her purchase decision based on the number of Type B buyers expected to buy the product in the following period. In our experiment we collected from Type A buyers their individual estimates of the second-period sales to Type B buyers. It is useful to note that we did not provide financial incentives to truthfully reveal their expectations. On the other hand, there was no incentive to intentionally provide false expectations. Providing additional monetary incentives might improve our results, and thus we provide a conservative analysis of rational expectations. Consistent with our equilibrium prediction, the average expected second-period sales declined from 7.38 to 4.55 on offering a limited edition ($F_{(118,1)} = 171.85$, $p < 0.001$). Corresponding actual sales dropped from 7.55 to 4.76 units. Upon making a trialwise paired comparison of the mean expectation of Type A buyers against the actual sales to Type B buyers, we cannot reject the null hypothesis that the expectations and actual sales were the same when no limited edition was offered ($t_{59} = 0.30$, $p > 0.2$). When a limited edition was offered, the difference between average expectations and actual sales was low but marginally significant ($t_{59} = 2.28$, $p < 0.05$).

Discussion. In summary, this empirical analysis suggests that the sales pattern is directionally consistent with the equilibrium prediction. Offering a limited edition improved sales to Type A buyers and also profits. The expectations of Type A buyers were also qualitatively consistent with the model predictions. However, we observe differences across participants.

We also note with interest that on offering a limited edition, the participants seemed to make their purchase decisions more rapidly. On average, the decision time of Type A buyers declined from 26.48 seconds to 24.33 seconds in the presence of a limited edition. When a limited edition was offered, on average only 4.76 Type B buyers could buy the product though 9.33 Type B buyers wanted to purchase the product. Therefore, in such circumstances slow responses on the part of Type B buyers could potentially deny them an opportunity to buy the product. Accordingly, we note that Type B buyers reduced their decision time from 10.36 to 6.27 seconds when a limited edition was offered. This observation is consistent with the view that a limited edition might cause a buying frenzy among Type B buyers. Next we venture to test the predictions of Proposition 1 and Proposition 2 under a weaker information condition.

2.2 Study 2

It is possible that buyers do not know the precise costs of the products they buy and may also have no idea of future prices. To allow for such a poor information condition, in Study 2 we do not inform leaders about the cost of the product or the second-period prices. Further, in an attempt to sharply focus the empirical investigation on the behavior of leaders, in Study 2 we automate the purchase decisions of Type B buyers such that all Type B buyers would want to purchase the product if more than two leaders purchase the product in period 1. In addition, we do not seek the expectations of individual Type A buyers and this simplifies the experimental protocol. Thus Study 2 is another effort to assess the predictive accuracy of the model. Next we describe the experimental procedure of the study.

Procedure. We ran two groups of of ten participants with limited edition as a within-participant variable. Students were recruited to play the role of Type A buyers. The role of Type A buyers and their compensation was as in the previous study, except that now they do not know the cost of the product and second-period prices. The computer played the role of Type B buyers. Note that Type A buyers were not explicitly told the rule used by Type B buyers to make their purchase decisions, though they could well infer it from the payoff structure of Type B buyers.

The experimental procedure closely followed the previous study. The number of units available for sale was either 14 units (limited edition) or 20 units (no limited edition). As in the previous study, participants played three practice trials to familiarize them with the structure of the game. Then, they played thirty trials in limited-edition condition and another thirty trials in no-limited-edition condition. The order of presentation of the treatments were reversed in the two groups. At the end of the experiment, the cumulative earnings of the participants were converted to US dollars and paid accordingly.

Results. The sales to Type A buyers and profits increased on offering a limited-edition product. These findings suggest that the model can account for the behavior of leaders even in a weaker information condition.

Analysis of Sales. Table 3 summarizes the sales to the two segments of consumers. On examining the mean sales across the two groups, we find that offering a limited-edition product increased the sales to Type A buyers from 2.85 to 9.18 units ($F_{(1,118)} = 752.75$, $p < 0.001$). Furthermore, in the presence of limited edition, total sales increased from 9.68 to 13.78 units ($F_{(1,118)} = 29.07$, $p < 0.001$). Next we compare the behavior of Type A buyers against that of Type B buyers in each trial of the study. In the absence of limited edition, on average, 2.85 units were sold to Type A buyers whereas 6.83 units were sold to Type B buyers ($t = 8.01$, $p < 0.001$). The results were reversed in the presence of limited edition — now the average sales to Type A buyers were 9.18, greater than the 4.6 units sold to Type B buyers ($t = 20.52$, $p < 0.001$). These findings are consistent with the sales pattern observed in Study 1 and the qualitative predictions of the model.

In the presence of limited edition, the sales to Type A buyers typically ranged from 6 to 10 units, but in the absence of limited edition it varied between 1 to 6 units (See Figure A1).[12] On comparing the observed distribution of sales against the mixed-strategy equilibrium prediction of $\rho = 0.742$ (See Figure A2), we note that the sales to Type A buyers closely followed the symmetric mixed strategy solution in the absence of limited edition ($D_{60} = 0.11$, $p < 0.05$) but not in the presence of limited edition ($D_{60} = 0.64$, $p < 0.01$). It seems that the pure strategy solution of ten units can better account for the sales when limited edition was offered.

On examining the trends in the sales to Type A buyers, we find that in the second block of five trials, sales reached the equilibrium level of 10 units in some trials, but failed to sustain at that level (see Figure A3). We observed a similar difficulty to lock into the equilibrium in Study 1. Like in the previous study, we observe individual level differences (see Figure A3, panel 2).

Analysis of Profits. Consistent with the equilibrium predictions, profits increased when a limited edition was offered. On average, the firm earned 100.27 francs in the absence of a limited edition. But upon offering a limited edition, profits improved to 437.97 francs. We can reject the null hypothesis that these two profit levels are the same ($F_{(1,118)} = 414.54$, $p < 0.001$). We obtain similar results at the level of individual groups (Group 1: $F_{(1,58)} = 116.41$, $p < 0.001$; Group 2: $F_{(1,58)} = 516.19$, $p < 0.001$).

We also find that, as predicted by theory, offering a limited edition improved the profits derived from each consumer segment. The average profits from leaders increased from 2.85 to 9.18 francs ($F_{(1,118)} = 752.75.1$, $p < 0.001$). Similarly, the average profits from followers rose from 97.42 to 420.20 francs ($F_{(1,118)} = 407.47$, $p < 0.001$).

Discussion. To conserve space, the preceding analysis of experimental results focused on

[12] To save space we have placed the figures related to this study in the Technical Appendix.

the average behavior of Group 1 and Group 2. We obtain similar results at the level of individual groups. Taken together with the previous study, these experimental findings lead us to place greater confidence in the predictive power of the model. Interestingly, we find that observed behavior is closer to the pure-strategy equilibrium when limited edition is offered but closer to the mixed-strategy equilibrium if limited edition is not offered (see also Haruvy and Stahl 2004).

Note that both Study 1 and Study 2 investigated the predictive accuracy of our model in a context where preferences were induced to be consistent with the model assumptions. To assess whether natural preferences of consumers will lead to outcomes consistent with the model predictions, we conducted a preliminary survey of consumers. The brief survey attempted to answer to the question: Will offering a limited-edition product increase consumers' intentions to purchase the product? Recall that in our model we assumed that neither the leaders nor the followers are budget constrained. In some situations, however, followers are unable to purchase the product due to budget constraints. Furthermore, our model is based on the notion of reference groups, which is an inter-group phenomenon. This may make one wonder whether consumer desire for scarce product can motivate firms to offer limited editions. In an attempt to address these issues, we next a) discuss a brief survey consumers' natural preference, b) examine the strategic implications of followers having limited budgets, and c) highlight the difference between the effect of consumers' desire for scarce goods and the effect reference groups.

A survey of consumer preferences. We invited fifty students to participate in a computerized survey for a fee of $7. The survey first described the styling, performance, Consumer Report ratings and price of a car, and then asked respondents to indicate their intention to purchase the car on a ten-point scale.[13] After respondents stated their purchase intention, they performed several distraction tasks.[14] Upon completing these tasks, respondents were requested to read the description of a second car and indicate their intention to purchase the car. The descriptions of the two cars were identical except that one was a limited-edition product and the other was not. The computer did not allow respondents to revisit their previous choices so that respondents remained focused on the current task described on the screen.

[13]In this scale, choosing 1 indicated that the respondent "will definitely not buy" whereas selecting 10 indicated that the respondent "will definitely buy" the car.

[14]In these distraction tasks, they read descriptions of several products within a product category and chose one of them. They made several such choices in very divergent product categories.

We rotated the order of presentation of the stimuli between participants so that one half of the participants first viewed the limited edition car and the other half viewed it later. By keeping limited edition as a within-participant variable, we were able to better control for typical heterogeneity in preferences across respondents and afford to test for the treatment effect with a smaller sample of respondents. The average intention to purchase the limited edition car was 7.08 whereas the corresponding intention to purchase the non-limited edition car was 6.28. Using a paired-comparison test, we can reject the null hypothesis that our respondents are indifferent to limited edition ($t = 3.04$, $p < 0.01$).[15] While the survey results are consistent with our model findings, it is useful to note that the survey in itself is not a test of the model and there is a need for more corroborating field research.

Budget constraint. To understand the implications of followers being budget constrained, assume that the income of followers is I. Followers can use their income to purchase one unit of the product at price p and some quantity of an outside good priced at 1 unit. The utility that a follower derives from consuming z units of the outside good is given by $u(z)$ where $u(\cdot)$ is monotonically increasing in z. If the consumer chooses not to buy one unit of the product, then she invests all her income in purchasing the outside good and the utility derived by the consumer in this case is $u(I)$. But, if the consumer were to buy one unit of the product then her utility will be $u(I - p) + \psi_f(x)$. Hence the consumer will purchase the product if:

$$u(I - p) + \psi_f(x) \geq u(I) \tag{13}$$

This implies that we must have:

$$p \leq I - u^{-1}\left[u(I) - \psi_f(x)\right] = \Psi_f(x) \tag{14}$$

Note that:

$$\frac{\partial \Psi_f(x)}{\partial x} = (u^{-1})' \cdot \psi_f'(x) > 0 \tag{15}$$

This result has two implications for our earlier analysis. First, the main results of our earlier analysis will continue to hold if we just replace $\psi_f(x)$ with $\Psi_f(x)$. Second, the budget constraint limits a firm's ability to extract surplus from the followers. This could even make it more attractive

[15] Our respondents agreed that the car described in the survey could be an exclusive product ($mean = 6.5$) that helped to distinguish them ($mean = 7.34$), and indicate a person's social position ($mean = 9.02$).

for the firm to sell only to leaders. However, if $\Psi_f(N_l) > c$, it is profitable to sell to the followers. In such cases offering limited editions will strictly be beneficial to the firm, although followers are budget constrained.

Scarcity. To explore the issue of product scarcity, consider a consumer whose value of a good decreases as more consumers buy it. The utility derived by this consumer on purchasing a product is given by:

$$U(p) = v - \zeta(x^e) - p \tag{16}$$

where x^e is the expected number of customers who will buy the product and $\zeta(\cdot)$ is a function such that $\zeta(0) = 0$ and $\zeta'(\cdot) < 0$. Since consumers are homogeneous, the pure-strategy equilibrium would imply that $\hat{x}_s$ consumers will buy the product if:

$$\hat{x}_s = \max_{x \in \mathbb{N}} \{x | v - \zeta(x) - p > 0\} \tag{17}$$

The corresponding profits earned by the firm are given by:

$$\Pi = \hat{x}_s(p - c) \tag{18}$$

In this setting, if the firm announces a limited edition $Q > \hat{x}_s$ then consumers will ignore the limited-edition since the consumer who buys the $(\hat{x}_s + 1)$st product receives negative utility. If the firm announces $Q < \hat{x}_s$ then it makes strictly lower profits than in the case with no limited edition. Hence offering limited-editions is not profitable if consumer desire scarce products. In sum, imposing a consumer budget constraint does not affect the key qualitative results of our original model, and the notion of product scarcity alone cannot induce firms to offer limited editions.

In Study 1 and Study 2, we have looked at how offering a limited edition can help a firm manage the reference group effects and improve its profits. Alternatively, the firm could introduce new product lines to handle the social effects. We explore this issue in the next section.

3 An Empirical Analysis of Product Line Design

Consider the case where the firm could introduce k products at an additional fixed cost of ϕ per new product line. We assume that when consumers are presented multiple products that provide the same level of utility, they are equally likely to choose any of the product variants. Further, a consumer buys only one product variant. As we want to focus on situations in which the firm introduces at least one product line, we assume that the resulting profits exceed the fixed cost:

$$\hat{x}(\psi_l(0) - c) > \phi \tag{19}$$

In the setting described above, if the social effects are absent, the firm would never find it profitable to offer multiple products. This is because consumers are homogeneous and there is no heterogeneity in consumer's taste for quality. But, in the presence of social effects, the firm might prefer to introduce new products which are identical in quality, even when it has to incur some incremental fixed costs for doing so. In our experiment, we use the same parameter values that we used in Study 1 and Study 2, but set $\phi = 1$ and consider a case where subjects could buy Product 1, Product 2 or nothing.[16] We have the following result:

Result 3 *If the firm introduces $k = 2$ variants and $\psi_l(N_f/2) - p_1 < 0$, in the unique pure-strategy equilibrium $\hat{x}$ leaders buy Product 1 and $\min(N_l - \hat{x}, \hat{x})$ leaders buy Product 2. However, if $\psi_l(N_f/2) - p_1 \geq 0$, there are multiple pure-strategy equilibrium in which x leaders buy Product 1 and another x leaders buy Product 2 where $x = \{\hat{x}, \hat{x}+1, \ldots, \frac{N_l}{2}\}$. Our experimental game has multiple pure-strategy equilibria: $(2,2), (3,3), (4,4), (5,5)$. When the equilibrium is $(2,2)$ none of the followers purchase the product. In all other cases, 5 followers purchase each of the two products. The profits corresponding to these pure-strategy equilibria are $4, 87.0, 224.0$ and 360.0 respectively.[17] In addition, there is a mixed strategy equilibrium in which leaders buy either of the products with probability 0.5 and make expected profits of 311.21. The followers do not buy the product if less than 2 leaders buy either product. If the sales of one of the product is higher than the other and exceeds 2, all followers buy that product. Otherwise, 5 followers purchase each of the product.*

Notice that when it is profitable for the leaders to buy the product even when $N_f/2$ followers choose to purchase the product, the game has multiple pure-strategy equilibria. In our experimental investigation we compare the case where the firm offers just one type of product (Product 1) against the case where the firm offers two types of products (Product 1 and Product 2). The following study will help us understand which equilibria may survive an experimental test.

[16]Specifically, we assume that $N_l = N_f = 10$, $\psi_l(y) = 46 - 5\sqrt{y}$ and $\psi_f(x) = 15x$ with $c = 32.5$, $\phi = 1$.
[17]These numbers ignore ϕ in order to make comparison between the two cases easy.

3.1 Study 3

Result 3 suggests that a firm can better manage reference group effects by adding a product to its portfolio, even if the new product is of the same quality and it is costly to introduce the new product. In the current study, we test this prediction in a controlled laboratory setting. Specifically, we contrast the behavior when the number of product variants $k = 2$ against the case when $k = 1$.

We ran two groups of twenty participants. In Group 1, one product variant was available for sale in the first thirty trials and two product variants were available in the next thirty trials. In Group 2, participants could choose from a set of two product variants in the first thirty trials and in the later thirty trials the set size was reduced to one. Thus we treated k as a within-participant variable in this study. The experimental protocol for the case when $k = 1$ was identical to that in Study 1. Below we discuss the procedure for the case when $k = 2$.

Procedure. Students played the role of buyers while the computer played the role of a seller. The seller posted its prices for Product 1 and Product 2 and offered to sell them to any consumer willing to pay the list price. The first-period price was fixed at 33.5 francs for both products, while the second-period price of a product changed according to the number of leaders who purchased that specific product in the first period. Specifically, the second period price of product i was $p_2^i = Max\{35, 13.5x_i\}$. The seller listed twenty units of each of the products for sale, though the number of participants in the game was only twenty. Hence there was no binding supply restriction in this study.

As in the previous study, ten participants played the role of Type A buyers and another randomly chosen ten participants played the role of Type B buyers. At the beginning of each trial, each participant was endowed with 35 francs so that they could afford to purchase the products. Furthermore, participants were informed about their valuations, the number of products in the seller's product line, quantity of each product available for sale, product costs, and the first-period prices of the product.

In the first period of every trial, each Type A buyer had to decide whether to purchase Product 1, Product 2 or nothing. Although the base values and the list prices of the two products were the same, the realized value of these products might not be the same if the number of Type B buyers buying Product 1 and Product 2 in the second period were different. For example, if five Type B buyers were to purchase Product 1, then the actual value of the product for a Type A buyer would be 34.82 francs (that is, $46 - 5\sqrt{5} = 34.82$). But, if only four Type B buyers were to purchase Product 2, then its value would be 36 francs (that is, $46 - 5\sqrt{4} = 36$).[18] In the case of this example, it should be preferable to purchase Product 2. To better understand the rationale for the choices of Type A buyers, we also recorded the beliefs of each individual Type A buyer about the number of Type B buyers who might buy Product 1 and Product 2 in the second period. It is important to note that the actual sales of Product 1 and Product 2 to Type B buyers in the second period could be different from the expectations of any individual Type A buyer. Furthermore, the payoff to a Type A buyer for purchasing a product only depended on the actual sales of that product to Type B buyers, not the reported expectations of the Type A buyer.

In the second period of every trial, the prices of Product 1 and Product 2 for Type B buyers were announced based on the number of Type A players who bought those products in the first period. Then each Type B buyer decided whether to purchase Product 1, Product 2 or nothing. Type B buyers value a product more when more Type A buyers own the product. For instance, if five Type A buyers had purchased Product 1 in the first period, then the value of the product to a Type B buyer would be 75 francs (that is, $15 \times 5 = 75$). If four Type A buyers had purchased the Product 2 in the first period, then the value of the product would become 60 francs ($15 \times 4 = 60$). In this case, Type B buyers should prefer to purchase Product 1. After all the Type B players made their decisions, the results for that trial were announced. The payoff for participants who bought a product was computed by adding the endowment to the actual value of the specific product purchased and then deducting the price paid. The participants who did not buy any of the products kept their endowment.

Participants played three practice trials so that they became conversant with the decision structure of the game, and they received no monetary reward for these trials. After the practice trials, participants played sixty trials for monetary reward with the number of products variants offered for sale changing after thirty trials. At the end of the experiment, participants were paid their cumulative earnings and dismissed.

Results. We find that increasing the number of products in a firm's product line improved firm's sales and profits. We first analyze the sales and then discuss the impact on firm's profits.

[18] In these examples, 46 is the base value and $5\sqrt{y_i}$ captures leaders desire to contrast from the followers.

Analysis of Sales. The left panel of Table 5 presents the average sales to Type A and Type B buyers, when the number of products in a firm's product line was one ($k = 1$). The right panel shows the average sales of Product 1 and Product 2 to Type A and Type B buyers when the firm sold two product variants ($k = 2$).

Mean Sales. In equilibrium, the sales to leaders should increase if the firm increases the number of product variants from $k = 1$ to $k = 2$. On average, the total sales to Type A buyers increased from 3.56 to 8.06 units on offering two product variants ($F_{(1,118)} = 308.37$, $p < 0.001$). We obtained similar results in each of the two groups. In Group 1, the mean sales to Type A buyers increased from 3.67 to 9.0 units as the number of product variants rose from one to two ($F_{(1,58)} = 272.94$, $p < 0.001$). Similarly, in Group 2 the mean sales increased from 3.46 to 7.13 units ($F_{(1,58)} = 131.52$, $p < 0.001$).

Looking at the pure-strategy equilibrium, equilibrium behavior also implies that the overall sales should increase on offering multiple product variants. Similar observation holds for the mixed-strategy equilibria. Accordingly, the overall sales increased from 11.03 units to 18.06 units on offering two products ($F_{(1,119)} = 89.36$, $p < 0.001$). On examining the behavior at the level of individual groups, we find that in Group 1 the overall sales grew from 11.33 to 19 units ($F_{(1,58)} = 53.23$, $p < 0.001$). In Group 2 the overall sales increased from 10.73 to 17.13 units ($F_{(1,58)} = 37.86$, $p < 0.001$).

Distribution of Sales. Over the thirty iterations of the game, we observed variations in sales and Figure 7 presents the distribution of total sales to Type A buyers. In Group 1, the sales to Type A buyers ranged all the way from zero to six when $k = 1$, whereas the corresponding sales when $k = 2$ ranged from seven to ten. In Group 2, we observed a similar upward shift in the distribution of sales to Type A buyers as k increased from one to two.

In Figure 8, we compare the observed aggregate distribution of sales to Type A buyers against the symmetric mixed-strategy equilibrium prediction. We find that, when $k = 1$, the observed distribution of sales to Type A buyers was close to the equilibrium prediction but marginally skewed to the right ($D_{60} = 0.20$, $p < 0.1$). When $k = 2$, the sales to Type A buyers was lower, and the distribution was marginally different ($D_{60} = 0.24$, $p < 0.05$).

When a firm offers two product variants, as noted earlier, there are three candidate pure strategy solutions: any two, three, four or five leaders could purchase each of the two product variants. An important implication of these pure strategy equilibria is that the sales of Product 1 and Product 2 to Type A buyers should be positively correlated. On the other hand, the symmetric mixed-strategy equilibrium suggests that all leaders should purchase either Product 1 or Product with the mixing probability being 0.5. According to this mixed-strategy solution, the sales of Product 1 and Product 2 should be negatively correlated. Among the Type A buyers, the correlation between the sales of the two products was -0.895 ($p < 0.001$) and -0.752 ($p < 0.001$) in Group 1 and Group, respectively. Perhaps, the behavior of our participants could be better accounted by the mixed-strategy solution rather than the pure strategy equilibria. On probing further, we found that the pure strategy solutions can account for the observed sales to leaders on only six of the sixty trials across the two groups.

<u>Trends in Sales.</u> When $k = 1$, the average sales to Type A buyers was 3.7 units in the first block of five trials and it reached 4 units in the final block of trials. When $k = 2$, the average sales increased from 8.3 to 7.9 units over the course of six blocks of trials. These temporal variations in demand were not statistically significant ($F_{(1,54)} = 0.52$, $p > 0.75$). In the case of Type B buyers, the average sales when $k = 1$ typically varied between 7 and 9 units with a sharp drop to 4.9 in the third block of trials. But when $k = 2$, the sales to Type B buyers was consistently 10 units. Overall, we did not detect significant trends in the sales to Type B buyer ($F_{(1,54)} = 01.05$, $p > 0.39$).

<u>Individual Differences.</u> According to the symmetric mixed-strategy solution, all players should purchase the product on 29.2% of the occasions if $k = 1$, and on 50% of the trials if $k = 2$. In reality, all participants did not behave in the same manner and we observed substantial heterogeneity in their purchase probabilities (see Figure 9).[19]

Though Type A buyers should buy the product with a probability of 0.292 when $k = 1$, the actual purchase probabilities of individual Type A buyers ranged from 0.03 to 0.66 with the probability class 0.4-0.5 being the mode. On offering two product variants, participants should purchase Product 1 or Product 2 with the mixing probability being 0.5. The actual purchase probabilities of Type A buyers in this case ranged all the way from 0.1 to 0.9 with half of all the participants falling in the probability interval 0.4-0.6. It is easy to see in Figure 13 that the probability of Type A buyer purchasing a product increased on offering multiple variants. We observed such an upward shift in the distribution of purchase probabilities among Type B buyers.

[19]When $k = 2$, we separately computed the average probability of purchasing Product 1 and Product 2.

Analysis of Profits. Table 6 presents the mean profits from each type of buyers and the total profits when the number of product variants is $k = 1$ and $k = 2$.[20] As predicted, firm's total profits increased from increased from 185.51 to 369.80 on offering two product variants ($F_{(1,118)} = 45.46$, $p < 0.001$). We similar results in Group 1 and Group 2 (Group 1: $F_{(1,58)} = 34.97$, $p < 0.001$; Group 2: $F_{(1,58)} = 16.18$, $p < 0.001$). In theory, this general result should also hold within each segment of buyers. We find that the average profits from Type A increased from 3.57 to 8.07 ($F_{(1,118)} = 308.37$, $p < 0.001$). The profits from Type B buyers rose substantially from 189.07 to 377.87 ($F_{(1,118)} = 43.96$, $p < 0.001$). Such shifts in profits can be discerned at the level of individual groups ($p < 0.001$).

Analysis of Expectations. Recall that each individual Type A buyer should decide on buying a product based on the number of Type B buyers expected to buy that product in the following period. Along the lines discussed in Study 1, we tracked the beliefs of each Type A buyer about the number of Type B buyers who might purchase Product 1 and Product 2. The actual sales to Type B buyers were 7.47 and the corresponding average expected sales were 6.75 when $k = 1$ ($t_{(59)} = 1.07$, $p > 0.28$). But when $k = 2$, the actual sales of Product 1 and Product 2 were 5.32 and 4.68 units, respectively. The corresponding average expected sales were 4.29 and 4.38 (Product 1: $t_{(59)} = 1.71$, $p > 0.09$; Product 2: $t_{(59)} = 0.49$, $p > 0.62$)). Thus the average beliefs of leaders about the likely sales to followers were closely aligned with the actual sales to followers.

Discussion. In summary, this empirical analysis suggests that the sales pattern is directionally consistent with the equilibrium prediction. Offering multiple product variants increased sales to Type A buyers and also profits. The expectations of Type A buyers were also qualitatively consistent with the model predictions. However, we observe differences across participants.

4 *k*-Step Thinking and Adaptive Learning

In this section, we attempt to understand how our participants arrived at their decisions. Toward this end, we first explore the level of thinking that can account for the purchase decisions of our subjects in the very first trial. We also examine the learning process that can explain the dynamics in the subsequent trials.

[20] To help compare these results against that of Study 1, we focus on the gross profits before deducting any fixed costs.

In the first trial of Study 1, 40% Type A buyers (leaders) purchased the product, when limited edition was not offered; but 85% of the Type A buyers bought the product when a limited edition was offered. Similarly, in Study 2 only 30% of the Type A buyers purchased the product when limited edition was not offered, but 55% of them purchased it, when a limited edition was offered. Thus, in the very first trial we observe a shift in sales in the direction predicted by the equilibrium solution. What level of strategic thinking can account for this behavior? To seek an answer to this question, we attempted to fit a *step-k* model of thinking. While several *step-k* models have been advanced in the literature, we fitted a non-parametric version of Cognitive Hierarchy model (Camerer, Ho and Chong 2004, see also Stahl and Haruvy 2008). In our model, a zero-step thinker randomly makes her purchase decisions. A one-step thinker assumes that all other players are zero-step thinkers and chooses her strategy to maximizes her payoff. A two-step thinker assumes that all other players are either zero-step or one-step thinkers and chooses her strategy to maximizes her payoff. Similarly, a k-step thinker maximizes her payoffs by choosing a strategy which maximizes her utility under the assumption that all other players are zero-step to $k-1$ step thinkers. We pooled the data from Study 1 and Study 2 to estimate the average steps of thinking across these studies. Our analysis suggests that on average, subjects were using 1.79 steps. We use these estimates to predict the expected decisions in Study 3. In Study 3, where we offered multiple products, random choice would predict a market share of 0.33 for each product. If we use the estimates of steps of thinking from Study 1 and Study 2, our model predicts that the average market share for each product would be 0.383 while the observed market share is 0.425. Thus, the model does reasonably well in predicting data from first trial observation for Study 3.

Learning. The trends in the average sales over the several iterations of the game raise the possibility that subjects could have learned over the course of each experimental session. In order to examine which type of learning mechanisms can account for the observed behavior of the participants, we estimate the SEWA model proposed by Camerer, Ho and Chong (2002). SEWA model allows for both adaptive learning mechanisms, such as reinforcement learning and belief learning, and sophisticated learning mechanisms, such as QRE and the Nash equilibrium concept. Thus, using SEWA model, it is possible to understand which class of the commonly known learning mechanisms can track the purchase decisions of our participants. We first provide a brief description of the SEWA model (for more details we refer the reader to Camerer, Ho and Chong 2002).

Next, we discuss the overall model fit, interpretation of parameter estimates, and performance in validation samples.

The SEWA Learning Model. The model allows for both adaptive and sophisticated players. The adaptive players best respond to past actions. The sophisticated players, on the other hand, form expectations of the behavior of others and then best respond to their expectations.

Adaptive players. Recall that in every trial of Study 1 and Study 2 each player has to decide whether or not to purchase the product. In Study 3, each player needs to decide whether to buy Product 1, or buy Product 2 or purchase nothing. The probability of adaptive player i choosing strategy j from the m available strategies on trial $t+1$ is given by the logit function:

$$p_i^j(a,t+1) = \frac{e^{\lambda A_i^j(a,t)}}{\sum_{k=1}^m e^{\lambda A_i^k(a,t)}} \tag{20}$$

where $A_i^j(a,t)$ is the attraction for an adaptive player i to choose strategy j at time t, and λ is a measure of payoff sensitivity. At the end of every trial, a player updates the attractiveness of a strategy based on the actual payoff and also the expected payoffs for strategies that were not chosen. The attraction of choosing strategy j, namely $A_i^j(a,t)$, is a weighted average of the payoff for period t and the previous attraction $A_i^j(a,t-1)$, which is given by:

$$A_i^j(a,t) = \frac{\phi N(t-1)A_i^j(a,t-1) + [\delta + (1-\delta)I(s_i^j, s_i(t))]\pi_i(s_i^j(t), \mathbf{s_{-i}}(t))}{N(t)} \tag{21}$$

where $\pi_i(s_i^j(t), \mathbf{s_{-i}}(t))$ is the payoff received by player i by choosing strategy j in period t given that the other players chose $\mathbf{s_{-i}}(t)$ in time period t, and δ is the weight given to foregone payoffs. The $I(s_i^j, s_i(t))$ function is an indicator variable, which is 1 if $s_i(t) = s_i^j$ and 0 otherwise. The experience at time t is given by:

$$N(t) = (1-\kappa)\cdot\phi\cdot N(t-1) + 1, \quad t \geq 1 \tag{22}$$

where ϕ and κ are depreciation parameters.

Sophisticated Players. Unlike the adaptive learners, these players form expectations about the likely behavior of other players, and then best respond to their forecast. Let α fraction of the players be sophisticated. While forming expectation about the likely behavior of other players, the sophisticated players assume that $(1-\alpha')$ proportion of players are adaptive, where α and α' can be different. These players update attractions of strategies as follows:

$$A_i^j(s,t) = \sum_{k=1}^{m_{-i}} [\alpha' P_{-i}^k(s,t+1) + (1-\alpha') \cdot P_i^k(a,t+1)] \cdot \pi_i(s_i^j, s_{-i}^k) \tag{23}$$

where s_{-i}^k is the strategy vector $k \in \mathbf{S_{-i}}$ chosen by all players except i, where $\mathbf{S_{-i}}$ is the available strategy space for all players except i, m_{-i} is the cardinality of $\mathbf{S_{-i}}$, $P_{-i}^k(s,t+1)$ is the probability that the sophisticated players (except i) will choose strategy vector k at time $(t+1)$ and $P_{-i}^k(a,t+1)$ is the corresponding probability for adaptive players. Note that $P_{-i}^k(a,t+1)$ can be easily derived from (20). Now the probability of a sophisticated player i using strategy j on period $t+1$ is given by:

$$p_i^j(s,t+1) = \frac{e^{\lambda A_i^j(s,t)}}{\sum_{k=1}^{m} e^{\lambda A_i^k(s,t)}} \tag{24}$$

Notice that (23) and (24) define a set of recursive equations which need to be solved at each time period in order to determine the probabilities that a player would choose strategy j at time $(t+1)$ and to derive the updated attractiveness.

Results

We estimated the SEWA model using the maximum likelihood method. Following prior research, we used the purchase decisions made in the first trial of each treatment to initialize the model.[21] The model was calibrated on the next twenty trials, and validated on the last nine trials of each condition. We conducted separate analyses for each treatment in each study. Tables 7-9 presents the parameter estimates and the fit statistics corresponding to each study..

–Insert Table 7-9 –

Overall Model Fit. Tables 7-9 reports the fit statistics for SEWA learning model, EWA learning, reinforment-based learning, and belief-based learning. We are not reporting the fit statistics for QRE model as it performs even worse than a random model. For model comparison, we use log-likelihood (LL), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC). While AIC adjusts the log-likelihood to account for the number of model parameters, BIC corrects the likelihood for both number of parameters and sample size.[22] In each of the three studies, we

[21] For a detailed discussion of the computation of initial attractions see Camerer et al. (2002, p 18) and also Amaldoss and Jain (2005a).

[22] $AIC = LL - k$, and $BIC = LL - (k/2)log(M)$, where k is the number of degrees of freedom and M is the sample size.

note that reinforcement learning can provide the most parsimonious account of the purchase decisions of our participants. This finding is consistent with prior research, which has shown that often reinforcement learning can account for the behavior of participants in mixed strategy as well as pure strategy games (e.g., Roth and Erev 1995 and 1998, Rapoport and Amaldoss 2000, Amaldoss and Jain 2002 and 2005a).

Interpretation of the parameter values. The parameters of the SEWA model are κ, ϕ, δ, λ, α and α'. Below we discuss the estimates of these parameters.

Depreciation parameters ϕ and κ. The parameter ϕ measures the extent to which attractions of strategies wear out over trials. The parameter κ suggests the extent to which past experience accumulates over trials. We note that ϕ is over 0.772 except in one case where it is 0.494, suggesting that attractions do not wear out rapidly. The estimated value of κ is close to 1 in no-limited edition treatments but zero in the limited edition treatments of Study 1 and Study 2, implying that experience accumulates more in limited edition treatments. In Study 3, the estimated value of $\kappa = 0.949$ and $\phi = 0.839$. Thus, the decisions of our participants are less stationary and possibly more adaptive.

Imagination Parameter δ. This parameter indicates the relative importance given to foregone payoffs, compared to actual payoffs, while updating the attraction for a particular strategy. It can also be interpreted as a kind of 'imagination' of foregone payoffs. Reinforcement learning is the special case where $\delta = 0$ and $\kappa = 1$. Thus in reinforcement learning participants do not consider foregone payoffs and past reinforcement accumulate over trials. However, in belief learning the same weight is placed on both actual and foregone payoffs ($\delta = 1$ and $\kappa = 0$). Though the estimated value of δ is positive in the SEWA model in all cases except in the no-limited edition treatment of Study 2, the reinforcement learning model fits the data best according to Bayesian Inference Criterion, implying that reinforcement-based learning provides the most parsimonious account for our data.

Payoff sensitivity parameter λ. This parameter indicates the extent to which participants are sensitive to payoff. An alternative interpretation is that it measures the level of noise in the choice process. The estimated value of λ ranges from 0.247 to 0.014, with the level of payoff sensitivity being lower in the no-limited edition treatments. This finding is consistent with the tendency of our participants to mix strategies in the no-limited edition treatments. In Camerer et al. (2002)

we also see that the estimates of λ are lower in mixed strategy games.

Sophistication parameters α and α'. If $\alpha = \alpha' = 1$, we obtain the Quantal Response equilibrium model as a special case. The Nash equilibrium is a special case of SEWA where $\alpha = \alpha' = 1$ and λ is infinitely large. We find that $\alpha = \alpha' = 0$ in Study 3 as well as in the no-limited edition treatments of Study 1 and Study 2. Even in the limited edition treatments, reinforcement learning can provide a parsimonious explanation for the data. This suggests that we can summarize the trends in the purchase decisions of our participants with the aid of an adaptive learning mechanism without invoking any sophistication.

Model Validation. We assessed the predictive accuracy of the model in the last 9 trials of each treatment. Tables 7-9 report the log-likelihood, AIC, and BIC. Even in the validation sample, the reinforcement model performs better. In Study 1, the reinforcement model accurately predicts the purchase decisions on 62.22% and 92.78% of the occasions in the no limited-edition and limited-edition treatments, respectively. The hit ratios for the corresponding calibration samples are 68.5% and 91.25%, and it closely parallels those in the validation sample. Similarly, in the two treatments of Study 2 the hit ratios are 67% and 91.25% in the validation sample and 68% and 93.39% in the calibration sample. Perhaps, it is easier to predict the pure strategy play in the limited-edition treatments rather than the mixed strategy play in the no limited-edition treatments. This finding is consistent with the notion that mixed strategy adds noise to the purchase decisions thus making it less predictable. In Study 3, the hit ratios corresponding to the calibration and validation samples are 48.25% and 55%, respectively. The lower predictive accuracy of this study could be ascribed to the fact that now players have three choices: buy product 1, buy product 2 or buy neither.

In sum, the behavior of our participants in the first trial of the game implies that subjects are capable of sophisticated thinking. However, purchases in the subsequent trials can be accounted for by adaptive decision making. Reinforcement learning tracks the purchase decisions of our participants in the limited edition treatments more accurately than those in no limited edition treatments.

5 Conclusion

In this paper we experimentally investigated the implications of reference groups for product line decisions. Our investigation shows that, in the presence of strong reference group effects, a firm can increase its sales and profits by offering limited editions or introducing multiple product lines.

In Study 1, we tested whether offering limited editions increase sales to leaders and improve firm's profits. The laboratory setting conformed to the model structure, but did not control for assumptions about consumer behavior, such as the ability to form rational expectations. On the aggregate, the experimental results are consistent with the qualitative predictions of the model. Further, the average expectations of participants were in keeping with the model predictions. Experimental results lean towards the pure strategy equilibrium in the case of limited edition treatment and the mixed strategy equilibrium in the case of no limited edition treatment. This finding makes intuitive sense since limited edition enables coordination to the symmetric pure strategy equilibrium. Such coordination is difficult in the case of no-limited edition treatment because the pure strategy equilibrium is an asymmetric equilibrium. Thus, the symmetric players in our game favor the symmetric equilibrium.

Study 2 points out that the model is able to account for observed behavior even in a weaker information condition. Additionally, we conducted an exploratory survey to understand how limited editions shape consumer's intention to purchase a product. The results of this preliminary research are consistent with the predictions of our model. Across the two studies, we find that reinforcement learning can better track the purchase decisions of our participants. Furthermore, it is easier to predict the pure-strategy play in the limited-edition treatments rather than the mixed strategy play in the no limited-edition treatments. A potential explanation for this finding is that in a mixed strategy equilibrium players randomize their choices thus making it more difficult to predict their decisions in the no-limited edition treatments. In Study 3, we examined how offering multiple product lines can impact sales and firm profits. Consistent with the theoretical model, we find that sales to leaders and the total sales increase when the firm offers multiple product lines. The profits of the firm are also higher when it offers multiple product lines, even though the two products are of identical quality. Furthermore, on probing the purchase decisions of our participants, we find that their decisions in the first trial can be accounted by approximately 1.79 steps of iterative reasoning, and all subsequent decisions can be explained by reinforcement learning.

In the current investigation, we did not consider how other marketing mix variables can be impacted by reference group effects. For example, consider a firm's advertising decisions. A firm needs to decide whether and how much to advertise to either of the two groups. In the presence of reference group effects, advertising to one group could either positively or negatively affect the demand from the other group. Relatedly, firms need to decide on the intensity of product distribution. Future research can examine these issues both from a theoretical and empirical perspective. Social influences also manifest in a variety of other contexts such as consumer network formation and new product adoption. The profit implications of these social effects need further scrutiny. While we tested our model in a laboratory setting, future research can attempt to test our model in a field setting to the extent possible (e.g., Krishna and Unver 2007, Lim et. al. 2007 and Simester et. al. forthcoming). Such field studies can augment our understanding of the impact of reference groups on firm's profits.

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Table 1: Study 1 – Mean Sales

Type of Buyer	No Limited Edition			Limited Edition		
	Group 1	Group 2	Both	Group 1	Group 2	Both
Type A	3.40 (1.30)	4.10 (1.42)	3.75 (1.40)	9.20 (0.96)	9.27 (0.83)	9.23 (0.89)
Type B	6.70 (3.58)	8.40 (3.56)	7.55 (3.64)	4.80 (0.96)	4.73 (0.83)	4.77 (0.89)
Total	10.10 (4.58)	12.50 (4.61)	11.30 (4.72)	14.00 (0.00)	14.00 (0.00)	14.00 (0.00)

Table 2: Study 1 – Mean Profits

Type of Buyer	No Limited Edition			Limited Edition		
	Group 1	Group 2	Both	Group 1	Group 2	Both
Type A	3.40 (1.30)	4.10 (1.42)	3.75 (1.40)	9.20 (0.96)	9.27 (0.83)	9.23 0.90
Type B	134.75 (137.38)	235.85 (174.13)	185.30 (163.23)	428.10 (18.91)	429.37 (18.75)	428.73 (18.68)
Total	138.15 (137.64)	239.95 (175.54)	189.05 (164.60)	437.30 (19.87)	438.63 (18.07)	437.97 (17.97)

Table 3: Study 2 – Mean Sales

Type of Buyer	No Limited Edition			Limited Edition		
	Group 1	Group 2	Both	Group 1	Group 2	Both
Type A	2.90 (1.32)	2.80 (1.00)	2.85 (1.62)	9.13 (1.78)	9.23 (0.77)	9.18 (1.36)
Type B	6.33 (4.90)	7.33 (4.50)	6.83 (4.69)	4.43 (1.22)	4.77 (0.77)	4.60 (1.03)
Total	9.23 (6.00)	10.13 (5.33)	9.68 (5.65)	13.56 (2.37)	14.00 (0.00)	13.78 (1.68)

Table 4: Study 2 – Mean Profits

Type of Buyer	No Limited Edition			Limited Edition		
	Group 1	Group 2	Both	Group 1	Group 2	Both
Type A	2.90 (1.32)	2.80 (1.00)	2.85 (1.62)	9.13 (1.78)	9.23 (0.77)	9.18 (1.36)
Type B	109.16 (131.37)	85.66 (82.22)	97.42 (109.30)	408.97 (79.50)	431.43 (18.09)	420.20 (58.27)
Total	112.06 (132.62)	88.46 (83.09)	100.27 (110.36)	418.10 (80.92)	440.67 (17.47)	429.38 (59.14)

Notes:

- The terms in parentheses are the standard deviations.
- When no limited edition is offered, leaders should buy 2 units and the firm's profits should be 2 in the pure strategy equilibrium; but in the mixed-strategy equilibrium on average 2.92 leaders should buy and the firm's profits should 121.1. When the firm offers limited edition, all leaders should buy the product and the firm should earn 420 under the pure strategy equilibrium; but under the mixed-strategy equilibrium, leaders should buy with probability 0.74 or 0.30 with the associated profits are 122.74 and 426.88, respectively.

Table 5: Study 3 – Mean Sales

Type of Buyer	$k=1$			$k=2$								
	Group 1	Group 2	Both	Group 1			Group 2			Both		
				Pt1	Pt2	Total	Pt1	Pt2	Total	Pt1	Pt2	Total
Type A	3.67 (1.60)	3.47 (1.46)	3.57 (1.52)	4.73 (1.66)	4.27 (1.55)	9.00 (0.74)	3.40 (1.48)	3.73 (1.14)	7.13 (0.97)	4.07 (1.70)	4.00 (1.38)	8.07 (1.27)
Type B	7.67 (4.30)	7.27 (4.46)	7.47 (4.35)	5.73 (4.65)	4.27 (4.65)	10.00 (0.00)	4.90 (5.00)	5.10 (5.00)	10.00 (0.00)	5.32 (4.80)	4.68 (4.80)	10.00 (0.00)
Total	11.33 (5.71)	10.73 (5.61)	11.03 (5.62)	10.47 (6.00)	8.53 (5.97)	19.00 (0.74)	8.30 (6.33)	8.83 (5.94)	17.13 (0.97)	9.38 (6.21)	8.68 (5.91)	18.07 (1.27)

Note: Pt1 and Pt2 refer to Product 1 and Product 2 respectively. When $k = 1$, only 2 leaders should buy the product in the pure strategy equilibrium, but under the mixed strategy 2.92 leaders should buy the product. When $k = 2$ there are multiple pure strategy equilibria: $(2,2),(3,3),(4,4),(5,5)$. In the mixed-strategy equilibrium leaders should buy either product with probability 0.5.

Table 6: Study 3 – Mean Profits

Type of Buyer	$k=1$			$k=2$								
	Group 1	Group 2	Both	Group 1			Group 2			Both		
				Pt1	Pt2	Total	Pt1	Pt2	Total	Pt1	Pt2	Total
Type A	3.67 (1.60)	3.47 (1.46)	3.57 (1.52)	4.73 (1.66)	4.27 (1.55)	9.00 (0.74)	3.40 (1.48)	3.73 (1.14)	7.13 (0.97)	4.07 (1.70)	4.00 (1.38)	8.07 (1.27)
Type B	205.33 (159.74)	165.68 (163.30)	185.51 (161.40)	256.47 (249.72)	183.08 (225.09)	439.55 (152.10)	149.90 (163.85)	150.15 (163.87)	300.05 (90.05)	203.18 (216.18)	166.62 (195.90)	369.80 (142.49)
Total	209.00 (161.27)	169.15 (164.70)	189.07 (162.85)	261.20 (251.23)	187.35 (226.45)	448.55 (152.37)	153.30 (165.21)	153.88 (164.90)	307.18 (90.53)	207.25 (217.71)	170.62 (197.12)	377.87 (143.25)

Note: Pt1 and Pt2 refer to Product 1 and Product 2 respectively. When $k = 1$ the expected profits are 2 under the pure strategy equilibrium but 121.11 under the mixed strategy equilibrium. When $k = 2$, the pure strategy equilibria are $(2,2),(3,3),(4,4),(5,5)$ and the corresponding profits are 4, 87.0, 224.0 and 360.00, respectively. When $k = 2$, the expected profits under the mixed strategy equilibrium are 311.21.

Table 7: Learning Model

Study 1					
Treatment	Parameter	SEWA	EWA	Reinforcement	Belief
No Limited Edition	κ	1.000	1.000	1.000	0.000
	ϕ	0.901	0.901	0.952	0.955
	δ	0.812	0.812	0.000	1.000
	λ	0.039	0.039	0.006	0.330
	α'	0.000	0.000	0.000	0.000
	α	0.000	0.000	0.000	0.000
Calibration(20 trials)	Log-Likelihood	-245.046	-245.046	-246.246	-275.650
	AIC	-251.046	-249.046	-248.246	-277.650
	BIC	-260.624	-255.431	-251.439	-280.843
	χ^2		0.000	1.201	30.605
	(p-value, dof)		(1,2)	(0.878,4)	(0.00, 4)
Validation(9 Trials)	Log-Likelihood	-102.131	-102.131	-101.684	-21.555
	AIC	-108.131	-106.131	-103.684	-23.555
	BIC	-117.710	-112.517	-106.877	-26.748
	χ^2		0.000	0.447	80.576
	(p-value, dof)		(1,2)	(0.978,4)	(0.00,4)
Limited Edition	κ	0.000	0.000	1.000	0.000
	ϕ	0.494	0.610	0.630	0.353
	δ	0.701	0.900	0.000	1.000
	λ	0.247	0.647	0.035	1.000
	α'	1.000	0.000	0.000	0.000
	α	0.264	0.000	0.000	0.000
Calibration(20 trials)	Log-Likelihood	-102.507	-105.875	-108.737	-137.254
	AIC	-108.507	-109.875	-110.737	-139.254
	BIC	-118.086	-116.261	-113.930	-142.446
	χ^2		3.368	6.230	34.747
	(p-value, dof)		(0.1856,2)	(0.1856,4)	(0.00, 4)
Validation (9 Trials)	Log-Likelihood	-44.467	-44.877	-45.037	-54.917
	AIC	-50.467	-48.877	-47.037	-56.917
	BIC	-60.046	-55.263	-50.230	-60.110
	χ^2		0.411	0.570	10.450
	(p-value, dof)		(0.8144,2)	(0.9663,4)	(0.03,4)

Note: The estimated parameters are significant at $p < 0.001$.

Table 8: Learning Model

Study 2					
Treatment	Parameter	SEWA	EWA	Reinforcement	Belief
No Limited Edition	κ	0.980	0.980	1.000	0.000
	ϕ	0.772	0.772	0.772	1.000
	δ	0.000	0.000	0.000	1.000
	λ	0.014	0.014	0.014	0.543
	α'	0.000	0.000	0.000	0.000
	α	0.000	0.000	0.000	0.000
Calibration(20 trials)	Log-Likelihood	-226.260	-226.260	-226.259	-274.079
	AIC	-232.260	-230.260	-228.259	-276.079
	BIC	-241.839	-236.646	-231.452	-279.271
	χ^2		0.000	0.001	47.819
	(p-value, dof)		(1,2)	(0.541,4)	(0.00, 4)
Validation(9 Trials)	Log-Likelihood	-92.875	-92.875	-92.875	-132.487
	AIC	-98.875	-96.875	-94.875	-134.487
	BIC	-108.454	-103.261	-98.068	-137.680
	χ^2		0.000	0.000	39.612
	(p-value, dof)		(1,2)	(1,4)	(0.00,4)
Limited Edition	κ	0.000	0.000	1.000	0.000
	ϕ	1.000	1.000	0.702	0.531
	δ	0.334	0.461	0.000	1.000
	λ	0.148	0.197	0.028	1.000
	α'	1.000	0.000	0.000	0.000
	α	0.249	0.000	0.000	0.000
Calibration(20 trials)	Log-Likelihood	-107.933	-110.370	-115.494	-135.856
	AIC	-113.933	-114.370	-117.494	-137.856
	BIC	-123.512	-120.756	-120.687	-141.049
	χ^2		2.437	7.561	27.922
	(p-value, dof)		(0.2957,2)	(0.1090,4)	(0.00, 4)
Validation (9 Trials)	Log-Likelihood	-44.894	-45.490	-42.947	-49.498
	AIC	-50.894	-49.490	-44.947	-51.498
	BIC	-60.473	-55.876	-48.140	-54.691
	χ^2		0.596	1.948	4.604
	(p-value, dof)		(0.7424,2)	(0.7454,4)	(0.3304,4)

Table 9: Learning Model

Study 3					
Treatment	Parameter	SEWA	EWA	Reinforcement	Belief
	κ	0.949	0.949	1.000	0.000
	ϕ	0.839	0.839	0.864	0.864
	δ	0.717	0.717	0.000	1.000
	λ	0.030	0.030	0.008	1.000
	α'	0.000	0.000	0.000	0.000
	α	0.000	0.000	0.000	0.000
Calibration(20 trials)	Log-Likelihood	-290.739	-290.739	-293.838	-313.765
	AIC	-296.739	-294.739	-295.838	-315.765
	BIC	-306.317	-301.124	-299.031	-318.958
	χ^2		0.000	3.099	23.027
	(p-value, dof)		(1,2)	(0.541,4)	(0.00, 4)
Validation(9 Trials)	Log-Likelihood	-130.654	-130.654	-129.564	-149.968
	AIC	-136.654	-134.654	-131.564	-151.968
	BIC	-146.233	-141.040	-134.757	-155.161
	χ^2		0.000	1.090	19.314
	(p-value, dof)		((1,2)	(0.896,4)	(0.00,4)

Note: The estimated parameters are significant at $p < 0.001$.

Technical Appendix

Proof of Proposition 1

To show that this is an equilibrium, note that if one leader deviates and buys then all the followers enter the market. Consequently, such deviation is unprofitable. If a leader deviates and does not buy, then he makes zero. However, by assumption:

$$\psi_l(0) - p_1 \geq 0 \tag{A1}$$

This ensures that deviation is not strictly profitable. If the inequality in (A1) is strict, then the equilibrium is unique.

For the symmetric mixed-strategy equilibrium, note that the leaders should be indifferent between buying and not buying. If each leader purchases the product with probability ρ, then the expected utility of purchasing the product for a leader when the other $N_l - 1$ leaders are using the mixed strategy is given by:

$$\psi_l(0)\sum_{x=0}^{\hat{x}-1}\begin{pmatrix} N_l - 1 \\ x \end{pmatrix}\rho^x(1-\rho)^{N_l-1-x} + \psi_l(N_f)\sum_{x=\hat{x}}^{N_l-1}\begin{pmatrix} N_l - 1 \\ x \end{pmatrix}\rho^x(1-\rho)^{N_l-1-x} - p_1 = 0 \tag{A2}$$

where the first part of (A2) refers to the case when only $\hat{x} - 1$ leaders have bought the product and therefore no follower buys the product even if the focal leader buys the product. The second term refers to the case when $\hat{x}$ leaders have bought the product and therefore adoption by the focal leader will lead to all followers buying the product. Equation (A2) reduces to:

$$\psi_l(0) - [\psi_l(0) - \psi_l(N_f)]\sum_{x=\hat{x}}^{N_l-1}\begin{pmatrix} N_l - 1 \\ x \end{pmatrix}\rho^x(1-\rho)^{N_l-1-x} - p_1 = 0 \tag{A3}$$

Note that for $\rho = 0$ the left hand side of (A3) is positive but becomes negative for $\rho = 1$. Thus, existence is assured by intermediate value theorem. Uniqueness is also guaranteed as the left hand side of (A3) is monotonically decreasing in ρ. □

Proof of Proposition 2

We will prove the proposition via a series of claims.

Claim 1 *If $\hat{x} < Q \leq \hat{x} + \hat{y}(p_1) + 1$ then in a pure strategy equilibrium* $\min(Q, N_l)$ *leaders buy the product.*

Proof: First consider the case when $\min(Q, N_l) = Q$. In this case, if the leaders adopt then they all get indirect utility $\psi_l(0) - p_1 > 0$; and, therefore, this is an equilibrium. We also want to argue that there is no other pure strategy equilibrium for this case. Consider the case when $\hat{x}$ leaders adopt. In this case a leader can deviate and get utility $\psi_l(\hat{y}(p_1)) - p_1 > 0$. So this is not an equilibrium. Similar logic rules out any pure strategy equilibrium in which $Q > x > \hat{x}$ since the non adopting leader can be better off by adopting.

Second, consider the case when $\min(Q, N_l) = N_l$. If a leader deviates, he gets zero while the utility under the equilibrium is given by:

$$\psi_l(Q - N_l) - p_1 \geq \psi_l(\hat{x} + \hat{y}(p_1) + 1 - N_l) - p_1 \geq \psi_l(\hat{y}(p_1)) - p_1 > 0 \tag{A4}$$

where the first inequality follows since $Q \leq \hat{y}(p_1) + \hat{x} + 1$, the second inequality follows since $N_l \geq \hat{x} + 1$ and the third inequality follows by definition of $\hat{y}(p_1)$. Thus, the leader will not deviate and this is an equilibrium. Note that $x = \hat{x}$ cannot be an equilibrium in this case because a leader can deviate and buy the product as:

$$\psi_l(\hat{y}(p_1)) - p_1 > 0 \tag{A5}$$

Similar logic applies for all $x \in (\hat{x} + 1, \min(Q, N_l))$. Thus, we have established the pure strategy equilibrium for the case $Q \leq \hat{x} + \hat{y}(p_1) + 1$. □

Claim 2 *If $\hat{x} + \hat{y}(p_1) + 1 < Q < N_l + \psi_l^{-1}(p_1)$ then in one pure strategy equilibrium $\hat{x}$ leaders buy the product. In another equilibrium $\min(Q, N_l)$ leaders buy the product.*

Proof: First note that $\hat{x}$ in this case is an equilibrium since if a leader deviates and buys his utility will be:

$$\psi_l(Q - \hat{x} - 1) - p_1 \leq \psi_l(\hat{y}(p_1) + 1) - p_1 \leq 0 \tag{A6}$$

where the first inequality follows since $Q \geq \hat{x} + \hat{y}(p_1) + 2$. Thus, only $\hat{x}$ leaders buying is an equilibrium.

Now consider the case when $\min(Q, N_l)$ leaders buy. First assume that $N_l > Q$. In this case, Q leaders buying is clearly an equilibrium since they make $\psi_l(0) - p_1 > 0$. Also, note that any $x \in (\hat{x}, N_l)$ cannot be an equilibrium for $Q < N_l$ since some leader can always benefit by deviating either by buying or by not buying.

Next, we consider the case when $Q > N_l$. In this case, if N_l leaders adopt, then at most $\psi_l^{-1}(p_1)$ followers can adopt. The utility that the leaders get in this case is at least:

$$\psi_l(\psi_l^{-1}(p_1)) - p_1 = 0 \tag{A7}$$

Thus no leader can strictly benefit by deviating and this is an equilibrium. □

Claim 3 *If $Q > N_l + \psi_l^{-1}(p_1)$ then in the pure strategy equilibrium $\hat{x}$ leaders buy the product.*

Proof: In this case, $\hat{x}$ is clearly an equilibrium. However, any $x > \hat{x}$ cannot be an equilibrium since in this case the leaders who adopt make:

$$U_l \leq \psi_l(\psi_l^{-1}(p_1) + 1) - p_1 < 0 \tag{A8}$$

where the first inequality follows since the second period sale is at least $\psi_l^{-1}(p_1) + 1$ and the second inequality follows since $\psi_l(\cdot)$ is monotonically decreasing in its argument and $\psi_l(\psi_l^{-1}(p_1)) - p_1 = 0$. □

The result in the proposition immediately follows from Claims 1, 2 and 3. □

Proof of Proposition 3

First consider the case when $\psi_l(N_f/2) - p_1 < 0$. In this case, if more than $\hat{x}$ leaders adopt a product, at least $N_f/2$ followers will buy and this would lead to negative utility for the leaders. Thus, the only pure strategy equilibrium in this case is $(\hat{x}, \hat{x})$. Now consider the case when $\psi_l(N_f/2) - p_1 > 0$. In this case, if the sale of both the products in the first period is the same then $N_f/2$ followers will buy the product and since $\psi_l(N_f/2) - p_1 > 0$ no leader can benefit by deviating. However, if the sale of the two products in the first period is not the same, then the followers will all purchase the product with the higher sale and the leaders will receive negative utility. Thus, all symmetric equilibria in which x leaders buy product 1 and x leaders buy product 2 where $x = \hat{x}, \hat{x} + 1, \ldots, \frac{N_l}{2}$ are valid. □