at $x(\frac{1}{2})$ using procedure *RK45_Adaptive* to obtain the desired solution to nine decimal places. Compare with the true solution:

$$x = t^3 - \frac{9}{2}t^2 + \frac{13}{2}t$$

6. (Continuation) Repeat the previous problem for $x(-\frac{1}{2})$.

7. It is known that the fourth-order Runge-Kutta method described in Equation (12), Section 8.2, has a local truncation error that is $\mathcal{O}(h^5)$. Devise and carry out a numerical experiment to test this. *Suggestions*: Take just one step in the numerical solution of a nontrivial differential equation whose solution is known beforehand. However, use a variety of values for h, such as 2^{-n}, where $1 \leqq n \leqq 24$. Test whether the ratio of errors to h^5 remains bounded as $h \to 0$. A multiple-precision calculation may be needed. Print the indicated ratios.

8. Compute the numerical solution of

$$\begin{cases} x' = -x \\ x(0) = 1 \end{cases}$$

using the **Midpoint Method**

$$x_{n+1} = x_{n-1} + 2hx_n'$$

with $x_0 = 1$ and $x_1 = -h + \sqrt{1 + h^2}$. Are there any difficulties in using this method for this problem? Carry out an analysis of the stability of this method. *Hint*: Consider fixed h and assume $x_n = \lambda^n$.

9. Tabulate and graph the function $[1 - \ln v(x)]v(x)$ on $[0, e]$, where $v(x)$ is the solution of the initial-value problem $(dv/dx)[\ln v(x)] = 2x$, $v(0) = 1$. *Check value*: $v(1) = e$.

10. Determine the numerical value of

$$2\pi \int_4^5 \frac{e^s}{s}\, ds$$

using a procedure for solving an ordinary differential equation.

11. Compute and print a table of the function

$$f(\phi) = \int_0^\phi \sqrt{1 - \frac{1}{4}\sin^2\theta}\, d\theta$$

by solving an appropriate initial-value problem. Cover the interval $[0, 90°]$ with steps of 1° and use the Runge-Kutta method of order 4. *Check values*: Use $f(30°) = 0.51788\,193$, and $f(90°) = 1.46746\,221$. *Note*: This is an example of an elliptic integral of the second kind. It arises in finding an arc length on an ellipse and in many engineering problems.

12. By solving an appropriate initial-value problem, make a table of the function

$$f(x) = \int_{1/x}^\infty \frac{dt}{te^t}$$