contribute to such understanding, assist in the identification of necessary and sufficient boundary conditions, and help in formulation and selection of computational algorithms.

PROBLEMS

1.1. Classify the following differential equations (1) as ordinary or partial; (2) as to order; (3) as to degree; (4) as linear, nonlinear, or quasilinear; and (5) as homogeneous or nonhomogeneous.

(a) $\dfrac{\partial u}{\partial t} + v\dfrac{\partial u}{\partial x} - \dfrac{\partial}{\partial x}\left(\mathcal{D}\dfrac{\partial u}{\partial x}\right) + ku^2 = 0$

where v, $\mathcal{D}$, and k are constants

(b) $\dfrac{\partial T}{\partial t} - \dfrac{\partial}{\partial x}\left(\kappa\dfrac{\partial T}{\partial x}\right) = 0$

where $\kappa = \kappa(T)$

(c) $\dfrac{\partial \zeta}{\partial t} + h\dfrac{\partial u}{\partial x} = 0$

$\dfrac{\partial u}{\partial t} + \tau u + g\dfrac{\partial \zeta}{\partial x} = 0$

where h, τ, and g are constants

(d) $\dfrac{d^2 f}{dx^2} + M\left[1 + \left(\dfrac{df}{dx}\right)^2\right]^{1.5} = 0$

where $M = M(x)$

(e) $S\dfrac{\partial h}{\partial t} - \dfrac{\partial}{\partial x}\left(T\dfrac{\partial h}{\partial x}\right) - \dfrac{\partial}{\partial y}\left(T\dfrac{\partial h}{\partial y}\right) = Q$

where

$Q = Q(x, y, t)$
$T = T(x, y, h)$
$S = S(x, y)$

1.2. Given the differential equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0$$

where $u = u(x, t)$, x has units of feet, and t has units of seconds, determine the following solutions, if possible.

(a) If $u(0, t) = 1$, $u(x, 0) = 0$, and $v = 2$ ft/sec, find $u(1, 1)$, $u(3, 2)$, and $u(7, 2)$.

(b) If $u(0, t) = 1$, $u\left(x, (x-2)/v\right) = 2$, and $v = 1$ ft/sec, find $u(1, 3)$, $u(1, 2)$, $u(4, 1)$, and $u(4, 4)$.

(c) If $u(0, t) = t/\text{sec}$, $u(x, 0) = x/\text{ft}$, and $v = 3$ ft/sec, find $u(1, 1)$, $u(3, 2)$, $u(4, 4)$, and $u(1, 3)$.

(d) If $u(0, t) = t/\text{sec}$, $u(x, 0) = x/\text{ft}$, and $v = x/\text{sec}$, find $u(1, 1)$, $u(3, 2)$, $u(4, 4)$, and $u(1, 3)$.

(e) If $u(0, t) = 1$ for $3 \geq t > 0$ and $u(x, x/2 - 1) = 2$, and $v = 1$ ft/sec, find the domain where a solution may be obtained and the solution within that domain.