April 4, 2023 #1 Chem 3321 transport

- combine pg 81 of my notes with Table 14.1 of HJH:

process	potential	flow	state of equilibrium
electrical conduction	electrostatic potential	charge	uniform electrostatic potential
heat (thermal) conduction	temperature	heat (energy)	uniform temperature
diffusion	chemical potential	molecules (matter)	uniform chemical potential
electrophoresis	electrochemical potential	ions	uniform electrochem potential
sedimentation ~~viscosity~~	total potential = chemical potential + centrifugal potential energy	molecules	uniform total potential
viscosity	velocity	momentum	uniform velocity

- transport occurs in response to a gradient and the flux is generally proportional to the gradient

$$J_i = -L_i \frac{\partial u_i}{\partial x}$$

J_i: net amount of matter, energy, charge etc. passing through a unit area in unit time

L_i: proportionality constant

flow is "down" the gradient.

April 5, 2023 #4 3321 transport

- So far we used the kinetic theory of gases to derive transport properties but now examine liquids and biomolecules:

- It seems EC has a good presentation of the chemical potential being the driving force for diffusion so try to follow this:

- $F = \text{force} = -\frac{d\mu}{dx}$ = effective force arising from a concentration gradient.

- ideal solution has $\mu = \mu^\circ + RT \ln C \Rightarrow F = -RT \frac{d \ln C}{dx}$

- at this point we need a result about frictional resistance to movement.

- imagine a charged macromolecule (eg. DNA) in solution subjected to an electric field. A force acts parallel to the electric field.

- in the absence of solvent, this force would cause continued acceleration per $F = ma$, so that the macromolecule would move faster and faster the longer the field is left on.

- However the solvent provides frictional resistance to prevent the velocity from increasing indefinitely.

April 6 2023 #1 3321 Transport

- the friction is a force given by $-fv$ where v = velocity, f = frictional coefficient, and "$-$" because this force opposes the direction of v.

- the steady state velocity (terminal velocity) is given by

$$ma = m\frac{dv}{dt} = F - fv = 0 \Rightarrow v = F/f$$

- in solution this steady state is reached in $< 10^{-12}$ s $= 1$ ps.

- now, in $F = -RT\frac{d\ln C}{dx}$ since μ is the molar free energy this is the force per mole. With f the frictional coefficient per molecule, this means the transport velocity is

$$v = \frac{F}{N_A f} = -\frac{RT}{N_A f}\frac{d\ln C}{dx} = -\frac{RT}{C N_A f}\frac{dC}{dx}$$

$$\Rightarrow Cv = \text{flux} = -\frac{RT}{N_A f}\frac{dC}{dx} = -\frac{k_B T}{f}\frac{dC}{dx}$$

Therefore the diffusion coefficient is $D = \frac{k_B T}{f}$

D → macroscopic

f → molecular

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- diffusion can be thought of as a random walk for each molecule, for which Einstein showed that $\langle x^2 \rangle = 2D\Delta t$ for the mean squared displacement of a molecule over time Δt.

- how do we get transport from random movement?
  - because of the unequal spatial concentration.

- this can be illustrated by coloring some water molecules in a beaker of water:
  - in /Users/Son/Courses/Chem3321_2023s this is "vmd water.psf water.dcd
  - can turn on periodic images in $\pm x$ and $\pm y$ directions
  - show residue 1665 as an example of a random walk
  - show $x<0$ with $\pm x, \pm y$ images, and also $x<0$ and $x>-10$ and also $x<0$ and $x>-1$ (same residue as)
  - show $x^2+y^2+z^2<49$ where I do the selection both for the first and last frame, so you can see which molecules end up in the middle.

need to bring my laptop

bring laptop to class

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- using $\langle x^2 \rangle = 2D\Delta t$ we can see that diffusion is an effective transport mechanism over short distances only.

- average time for a protein molecule with $D = 1 \times 10^{-6}\ cm^2/s$ to move the length (1×10^{-4} cm) of a bacterial cell by diffusion.

$$\Delta t = \frac{x^2}{2D} = \frac{1\times10^{-8}\ cm^2}{2\cdot 1\times10^{-6}\ cm^2/s} = 5\times10^{-3}\ s = 5\ ms.$$

- time to cross a 100 Å lipid bilayer if $D = 1\times10^{-8}\ cm^2/s$?

$$\Delta t = \frac{1\times10^{-12}\ cm^2}{2\cdot 1\times10^{-8}\ cm^2/s} = 5\times10^{-5}\ s = 50\ \mu s$$

- But, time to move the full 1 m length of a nerve cell for a molecule with $D = 1\times10^{-6}\ cm^2/s$?

$$\Delta t = \frac{1\ m^2 \cdot \left(\frac{100\ cm}{1\ m}\right)^2}{2\cdot 1\times10^{-6}\ cm^2/s} = 5\times10^{9}\ s = 160\ \text{years}\ !$$

• April 12, 2023 #1 3321 transport

• Fick's 2nd law (diffusion) is $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$

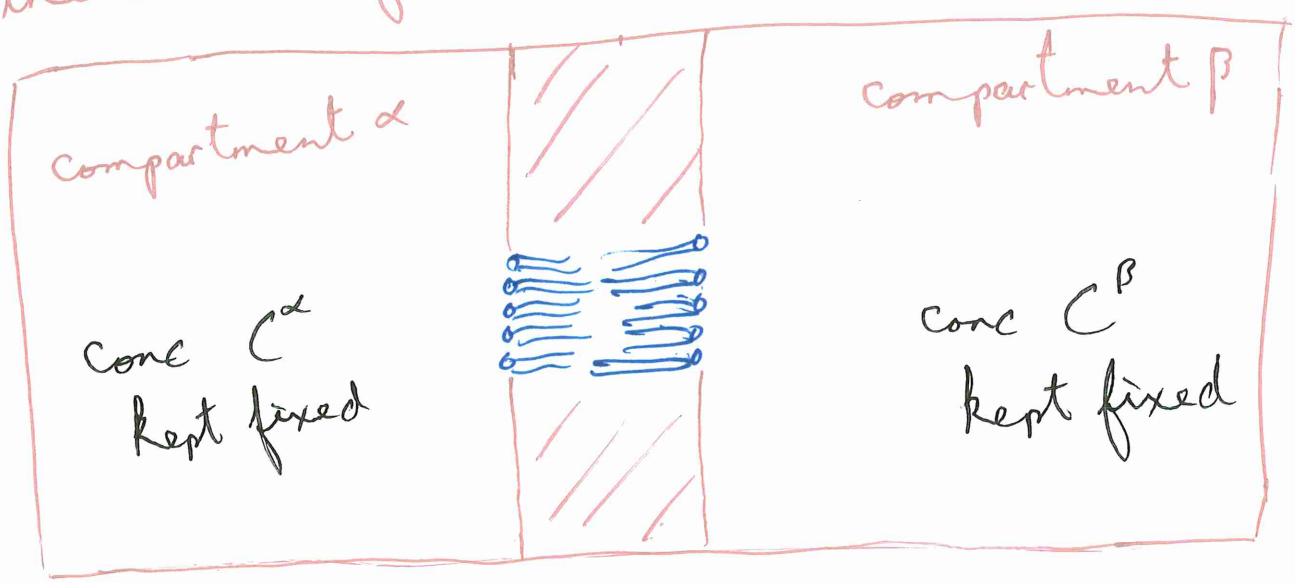
• at steady state $\frac{\partial c}{\partial t} = 0 \Rightarrow c(x)$ must be linear

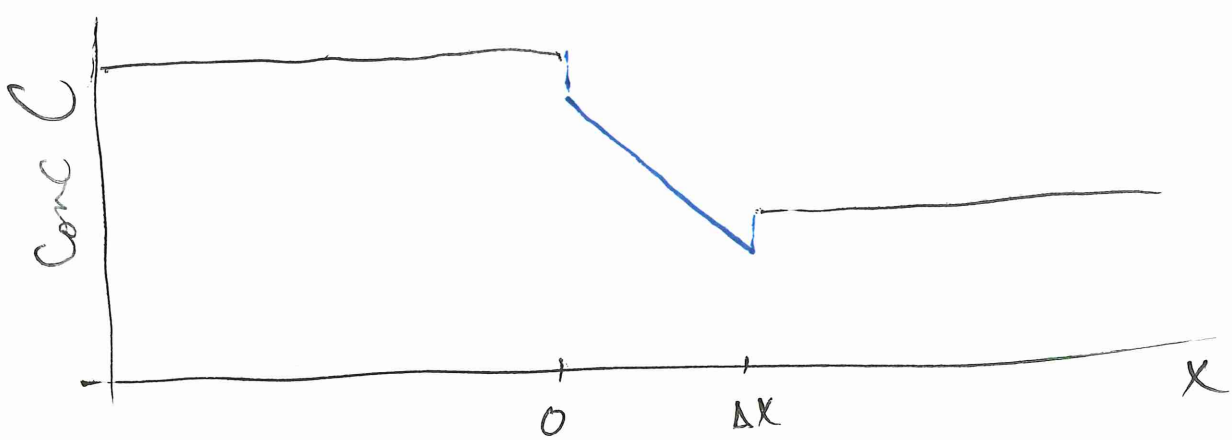
↓ show animations at www.dotpons.ac.uk

• Fick's 1st law is flux $= J_x = -D \frac{\partial c}{\partial x}$

• these results suggest that $\frac{\partial c}{\partial x}$ = non-zero constant is a steady state with transport. Does that make sense?

• in the context of a membrane it does:





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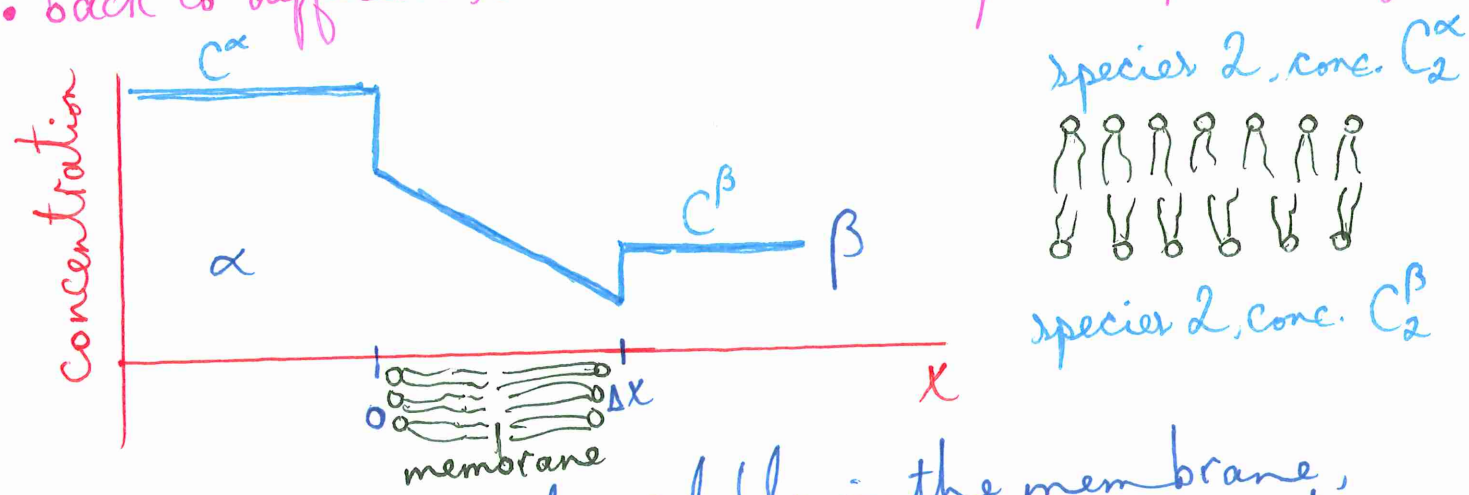
- back to diffusion, but this time through a lipid bilayer.



- Since species 2 is less soluble in the membrane, we have the steady state concentrations as shown

$$C(x) = \begin{cases} C^\alpha & \text{for } x < 0 \\ C^\beta & \text{for } x > \Delta x \\ C^\alpha f + \dfrac{(C^\beta - C^\alpha) f}{\Delta x} \cdot x & \text{for } 0 < x < \Delta x \end{cases}$$

- here f is the partition coefficient which captures the lower solubility in the bilayer.

- the flux is then $J_2 = -D_2 f_2 \dfrac{(C_2^\beta - C_2^\alpha)}{\Delta x}$

- however for biological membranes D_2, f_2, and even Δx are unknown or known inexactly.

- therefore usually we use a directly measurable quantity called the permeability P

- $J_2 = -P_2 (C_2^\beta - C_2^\alpha)$ and $P_2 = \dfrac{D_2 f_2}{\Delta x}$

See Table 14.2 HJH

April 9, 2023 #1 3321 transport

- Some of the values in Table 14.2 seem quite high. These are examples of facilitated transport, due to carrier molecules or pores and channels, which is still a thermodynamically spontaneous process.
- namely the transport corresponds to a decrease in the free energy of the system.
- specifically, let us transfer one mole of solute "2" across a membrane from compartment α at concentration C_2^α to compartment β at concentration C_2^β.

$$\Delta G = \mu_2^\beta - \mu_2^\alpha = RT \ln C_2^\beta - RT \ln C_2^\alpha = RT \ln \frac{C_2^\beta}{C_2^\alpha} < 0.$$

- for $\boxed{\text{active transport}}$, the ΔG above is > 0 and the transport process needs to be coupled to ATP hydrolysis or some other event.
- for sodium ions Na^+, there is also the electrical potential gradient across the membrane which means we need to include this in the ΔG.

April 9, 2023 #2 3321 transport

- the free energy of transporting one mole of Na^+ out of a cell with membrane potential $\Delta\psi$ is

$$\Delta G_{in\to out} = \underbrace{RT \ln \frac{[Na^+]^{out}}{[Na^+]^{in}}}_{5.93} + \underbrace{\mathcal{F}\,\Delta\psi}_{6.75}$$

$\mathcal{F}$ → Faraday constant

- for $\frac{[Na^+]^{out}}{[Na^+]^{in}} = 10$, $\Delta\psi = 70\,mV$, $T = 37°C$, $\Delta G = 12.7\,kJ/mol$

- but $\Delta G_{ATP} \approx -30\,kJ/mol$ or more
- so in fact several Na^+ can be transported for each ATP.
- this is how the sodium-potassium pump is able to work
- K^+ is transported in but at a low free energy cost because the electrical potential is favorable.

April 7, 2023 #2 3321 transport

- look at equilibrium sedimentation, which is extensively used to determine macromolecular weight.

- ultracentrifuge used at speeds up to 100,000 rpm, corresponding to forces 100000 × that of gravity.

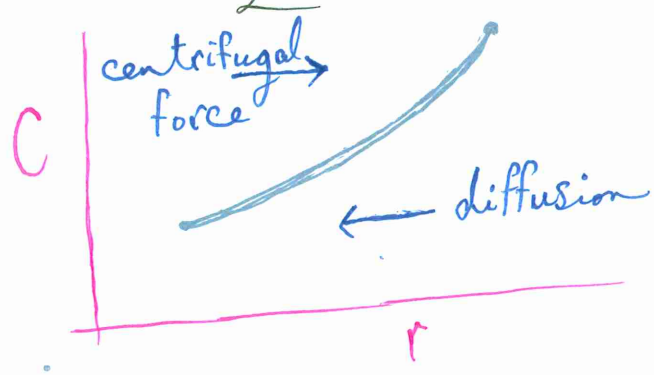
- depending on the speed, a balance is established between the centrifugal force and the force due to a concentration gradient.

- need to account for the concentration and centrifugal effects in the chemical potential.

$$\mu_r = \mu_{chemical} + U(r)$$, $F = m\omega^2 r$ where ω = rotation speed and r = distance from axis of rotation

- the work required to move the molecule from $r=0$ to r is

$$U(r) = \int_0^r -F(r')\,dr' = -\frac{m\omega^2 r^2}{2}$$



- solvent: A
- solute: B

- we also need the buoyancy correction:

with $\bar{v}_B$ the specific volume ($cm^3 g^{-1}$) of the macromolecule and ρ_A the density of the solvent ($g\,cm^{-3}$), the excess of mass of the molecule over the liquid it displaces is

$$m(1 - \bar{v}_B \rho_A)$$

April 8, 2023 #1 3321 transport

- put this into the $U(r)$ equation and change m to M_B (per mole)

$$U(r) = -\frac{M_B(1-\bar{v}_B\rho_A)r^2\omega^2}{2}$$

- using $\mu_{chemical} = \mu_B^\circ + RT\ln C_B$ we have

$$\mu_r = \mu_B^\circ + RT\ln C_B - M_B(1-\bar{v}_B\rho_A)r^2\omega^2/2$$

- at equilibrium $\frac{d\mu_r}{dr} = 0 \Rightarrow \boxed{RT\frac{d\ln C_B}{dr} = M_B(1-\bar{v}_B\rho_A)r\omega^2}$

- using $r\,dr = \frac{1}{2}d(r^2)$, get

$$\frac{d\ln C_B}{d(r^2)} = \frac{M_B\omega^2}{2RT}(1-\bar{v}_B\rho_A)$$

- a plot of $\ln C_B$ vs r^2 has slope $\frac{M_B\omega^2}{2RT}(1-\bar{v}_B\rho_A)$.
- if $\bar{v}_B$ and ρ_A are measured, can get the molecular weight M_B.

See Table 13.2 HJH

from $\frac{1}{C_B}$ from $\frac{d\ln C_B}{dr}$

before equil., this expression yields

$$J_B = s_B\omega^2 r C_B - D_B\frac{\partial C_B}{\partial r}$$

sedimentation coefficient

diffusion coefficient

which describes the transport due to sedimentation and diffusion, respectively.

steady state: $F = \text{force} = -\frac{d\mu_r}{dr} = vf$ and $\text{flux} = Cv \Rightarrow$

$$\text{flux} = \underbrace{\frac{M_B(1-\bar{v}_B\rho_A)}{f}}_{s_B} r\omega^2 C_B - \underbrace{\frac{RT}{f}}_{D_B}\frac{\partial C_B}{\partial r}$$