1–3. Past the infrared region, in the direction of lower energies, is the microwave region. In this region, radiation is usually characterized by its frequency, ν, expressed in units of megahertz (MHz), where the unit, hertz (Hz), is a cycle per second. A typical microwave frequency is 2.0×10^4 MHz. Calculate the values of ν, λ, and E for this radiation and compare your results with those in Figure 1.11.

$$\nu = 2.0 \times 10^4\ \text{MHz}\left(\frac{1 \times 10^6\ \text{Hz}}{1\ \text{MHz}}\right) = 2.0 \times 10^{10}\ \text{s}^{-1}$$

$$\lambda = \frac{c}{\nu} = \frac{2.998 \times 10^8\ \text{m}\cdot\text{s}^{-1}}{2.0 \times 10^{10}\ \text{s}^{-1}} = 1.5 \times 10^{-2}\ \text{m}$$

$$E = h\nu = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.0 \times 10^{10}\ \text{s}^{-1}) = 1.3 \times 10^{-23}\ \text{J}$$

This is illustrated in Figure 1.11.

1–4. Planck's principal assumption was that the energies of the electronic oscillators can have only the values $E = nh\nu$ and that $\Delta E = h\nu$. As $\nu \to 0$, then $\Delta E \to 0$ and E is essentially continuous. Thus, we should expect the nonclassical Planck distribution to go over to the classical Rayleigh-Jeans distribution at low frequencies, where $\Delta E \to 0$. Show that Equation 1.2 reduces to Equation 1.1 as $\nu \to 0$. (Recall that $e^x = 1 + x + (x^2/2!) + \cdots$, or, in other words, that $e^x \approx 1 + x$ when x is small.)

$$d\rho(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/k_BT} - 1} \qquad (1.2)$$

For small x, $e^x \approx 1 + x$. As $\nu \to 0$, $h\nu/k_BT \to 0$, and

$$d\rho(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{1 + \frac{h\nu}{k_BT} - 1} = \frac{8\pi h\nu^3 k_B T d\nu}{c^3 h\nu}$$

$$= \frac{8\pi \nu^2 k_B T d\nu}{c^3}$$

which is the classical Rayleigh-Jeans distribution (Equation 1.1).

1–5. Before Planck's theoretical work on blackbody radiation, Wien showed empirically that (Equation 1.4)

$$\lambda_{max} T = 2.90 \times 10^{-3}\ \text{m}\cdot\text{K}$$

where λ_{max} is the wavelength at which the blackbody spectrum has its maximum value at a temperature T. This expression is called the Wien displacement law; derive it from Planck's theoretical expression for the blackbody distribution by differentiating Equation 1.3 with respect to λ. *Hint*: Set $hc/\lambda_{max}k_BT = x$ and derive the intermediate result $e^{-x} + (x/5) = 1$. This problem cannot be solved for x analytically but must be solved numerically. Solve it by iteration on a hand calculator, and show that $x = 4.965$ is the solution.

The Planck distribution law for blackbody radiation is

$$\rho_\lambda(T)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda k_BT} - 1} \qquad (1.3)$$

As suggested, we let $x = hc/(\lambda_{max}k_BT)$, and then

$$\frac{d}{d\lambda}\left[\rho_\lambda(T)\right] = \frac{d}{d\lambda}\left(\frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda k_BT} - 1}\right)$$

$$0 = 8\pi hc\left[\frac{-5}{\lambda_{max}^6(e^x - 1)} - \frac{e^x\left(\frac{-x}{\lambda_{max}}\right)}{\lambda_{max}^5(e^x - 1)^2}\right]$$

$$xe^x = 5(e^x - 1)$$

To solve, we iterate and find $x = hc/\lambda_{max}k_BT = 4.965$, or $\lambda_{max}T = 2.90 \times 10^{-3}$ m·K.

1–6. At what wavelength does the maximum in the radiant energy density distribution function for a blackbody occur if (a) $T = 300$ K? (b) $T = 3000$ K? (c) $T = 10\,000$ K?

From Equation 1.4,

a. $$\lambda_{max} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{300\ \text{K}} = 9.67 \times 10^{-6}\ \text{m}$$

b. $$\lambda_{max} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{3000\ \text{K}} = 9.67 \times 10^{-7}\ \text{m}$$

c. $$\lambda_{max} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{10\,000\ \text{K}} = 2.90 \times 10^{-7}\ \text{m}$$

1–7. Sirius, one of the hottest known stars, has approximately a blackbody spectrum with $\lambda_{max} =$ 260 nm. Estimate the surface temperature of Sirius.

From Equation 1.4,

$$T = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{max}} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{260 \times 10^{-9}\ \text{m}} = 1.12 \times 10^4\ \text{K}$$

1–8. The fireball in a thermonuclear explosion can reach temperatures of approximately 10^7 K. What value of λ_{max} does this correspond to? In what region of the spectrum is this wavelength found (cf. Figure 1.11)?

From Equation 1.4,

$$\lambda_{max} = \frac{2.90 \times 10^{-3}\ \text{m}\cdot\text{K}}{1 \times 10^7\ \text{K}} = 3 \times 10^{-10}\ \text{m}$$

This corresponds to the X-ray region in the electromagnetic spectrum.

1–9. Calculate the energy of a photon for a wavelength of 100 pm (about one atomic diameter).

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{1 \times 10^{-10}\ \text{m}} = 2 \times 10^{-15}\ \text{J}$$

1–10. Express the Planck distribution law in terms of λ (and $d\lambda$) by using the relationship $\lambda\nu = c$.

$$\rho_\nu(T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/k_BT} - 1} \qquad (1.2)$$

We know that $\nu\lambda = c$, so $d\nu = -c d\lambda/\lambda^2$. Substituting, we obtain

$$\rho_\lambda(T)d\lambda = \frac{8\pi h}{c^3}\frac{-c^4 d\lambda}{\lambda^5(e^{hc/\lambda k_BT} - 1)}$$
$$= \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda k_BT} - 1}$$

where we have dropped the negative sign for convenience. It occurs because $d\nu$ and $d\lambda$ have opposite signs.

1–11. Calculate the number of photons in a 2.00 mJ light pulse at (a) 1.06 μm, (b) 537 nm, and (c) 266 nm.

a.

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}$$
$$= \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{1.06 \times 10^{-6}\ \text{m}}$$
$$= 1.87 \times 10^{-19}\ \text{J} \cdot \text{photon}^{-1}$$

Since 2.00 mJ of energy are contained in the light pulse,

$$\text{Number of photons} = \frac{2.00 \times 10^{-3}\ \text{J}}{1.87 \times 10^{-19}\ \text{J} \cdot \text{photon}^{-1}} = 1.07 \times 10^{16}\ \text{photons}$$

Parts (b) and (c) are done in the same manner to find

b. 5.41×10^{15} photons **c.** 2.68×10^{15} photons

1–12. The mean temperature of the Earth's surface is 288 K. Calculate the wavelength at the maximum of the Earth's blackbody radiation. What part of the spectrum does this wavelength correspond to?

From Equation 1.4,

$$\lambda_{\text{max}} = \frac{2.90 \times 10^{-3}\ \text{m·K}}{288\ \text{K}} = 1.01 \times 10^{-5}\ \text{m}$$

This is in the infrared region of the spectrum.

1–13. A helium-neon laser (used in supermarket scanners) emits light at 632.8 nm. Calculate the frequency of this light. What is the energy of a photon generated by this laser?

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{632.8 \times 10^{-9}\ \text{m}}$$
$$= 3.139 \times 10^{-19}\ \text{J}$$

1–14. The power output of a laser is measured in units of watts (W), where one watt is equal to one joule per second. ($1\ \text{W} = 1\ \text{J·s}^{-1}$) What is the number of photons emitted per second by a 1.00 mW nitrogen laser? The wavelength emitted by a nitrogen laser is 337 nm.

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}$$
$$= \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{337 \times 10^{-9}\ \text{m}}$$
$$= 5.89 \times 10^{-19}\ \text{J} \cdot \text{photon}^{-1}$$

$$1.00 \times 10^{-3}\ \text{J·s}^{-1}\frac{1}{5.89 \times 10^{-19}\ \text{J} \cdot \text{photon}^{-1}} = 1.70 \times 10^{15}\ \text{photon·s}^{-1}$$

1–15. A household lightbulb is a blackbody radiator. Many light bulbs use tungsten filaments that are heated by an electric current. What temperature is needed so that $\lambda_{\text{max}} = 550$ nm?

From Equation 1.4,

$$T = \frac{2.90 \times 10^{-3}\ \text{m·K}}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-3}\ \text{m·K}}{550 \times 10^{-9}\ \text{m}} = 5300\ \text{K}$$

1–16. The threshold wavelength for potassium metal is 564 nm. What is its work function? What is the kinetic energy of electrons ejected if radiation of wavelength 410 nm is used?

We will use Equation 1.7 to find ϕ and then use Equation 1.6 to find the kinetic energy.

$$\nu_0 = \frac{c}{\lambda_0} = \frac{2.998 \times 10^{8}\ \text{m·s}^{-1}}{564 \times 10^{-9}\ \text{m}} = 5.32 \times 10^{14}\ \text{Hz}$$
$$\phi = h\nu_0$$
$$= (6.626 \times 10^{-34}\ \text{J·s})(5.32 \times 10^{14}\ \text{Hz}) = 3.52 \times 10^{-19}\ \text{J}$$
$$\text{KE} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$
$$= \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{410 \times 10^{-9}\ \text{m}} - 3.52 \times 10^{-19}\ \text{J}$$
$$= 1.32 \times 10^{-19}\ \text{J}$$

1–17. Given that the work function of chromium is 4.40 eV, calculate the kinetic energy of electrons emitted from a chromium surface that is irradiated with ultraviolet radiation of wavelength 200 nm.

$$\phi = 4.40\ \text{eV}\left(\frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 7.05 \times 10^{-19}\ \text{J}$$

$$h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m}\cdot\text{s}^{-1})}{200 \times 10^{-9}\ \text{m}}$$

$$= 9.93 \times 10^{-19}\ \text{J}$$

$$\text{KE} = h\nu - \phi$$

$$= 9.93 \times 10^{-19}\ \text{J} - 7.05 \times 10^{-19}\ \text{J} = 2.88 \times 10^{-19}\ \text{J}$$

1–18. When a clean surface of silver is irradiated with light of wavelength 230 nm, the kinetic energy of the ejected electrons is found to be 0.805 eV. Calculate the work function and the threshold frequency of silver.

From Equation 1.6,

$$\phi = h\nu - \text{KE} = \frac{hc}{\lambda} - \text{KE}$$

$$= \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m}\cdot\text{s}^{-1})}{230 \times 10^{-9}\ \text{m}} - 0.805\ \text{eV}\left(\frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$= 8.64 \times 10^{-19}\ \text{J} - 1.29 \times 10^{-19}\ \text{J} = 7.35 \times 10^{-19}\ \text{J}$$

From Equation 1.7,

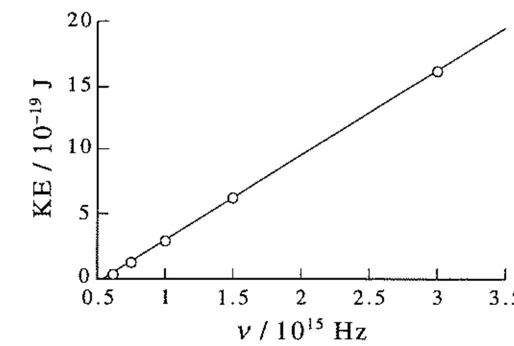
$$\nu_0 = \frac{\phi}{h} = \frac{7.35 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = 1.11 \times 10^{15}\ \text{Hz}$$

1–19. Some data for the kinetic energy of ejected electrons as a function of the wavelength of the incident radiation for the photoelectron effect for sodium metal are

λ/nm	100	200	300	400	500
KE/eV	10.1	3.94	1.88	0.842	0.222

Plot these data to obtain a straight line, and calculate h from the slope of the line and the work function ϕ from its intercept with the horizontal axis.

First, we use the equation $\nu\lambda = c$ to convert wavelength to frequency and the conversion factor $1\ \text{eV} = 1.6022 \times 10^{-19}$ J. Then we can plot kinetic energy (in joules) as a function of frequency (in Hz):



The best fit of a line to the data is $y = -3.5891 \times 10^{-19}\ \text{J} + (6.5918 \times 10^{-34})\ \text{J}\cdot\text{s}$, which gives a slope of $h = 6.60 \times 10^{-34}\ \text{J}\cdot\text{s}$. The threshhold freqency is the frequency when the kinetic energy is equal to zero, and can be obtained from the x-intercept:

$$y = 0 = -3.5891 \times 10^{-19}\ \text{J} + \left(6.5918 \times 10^{-34}\ \text{J}\cdot\text{s}\right) x$$

Solving for x,

$$x = \nu_0 = \frac{3.5891 \times 10^{-19}\ \text{J}}{6.5918 \times 10^{-34}\ \text{J}\cdot\text{s}} = 5.44 \times 10^{14}\ \text{Hz}$$

Equation 1.7 then gives

$$\phi = h\nu_0 = \left(6.60 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(5.44 \times 10^{14}\ \text{Hz}\right) = 3.59 \times 10^{-19}\ \text{J}$$

1–20. Use the Rydberg formula (Equation 1.10) to calculate the wavelengths of the first three lines of the Lyman series.

$$\tilde{\nu} = \frac{1}{\lambda} = 109\,680\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\ \text{cm}^{-1} \qquad (n_2 > n_1)$$

To find the first line in the Lyman series, substitute 1 for n_1 and 2 for n_2:

$$\tilde{\nu}_2 = 109\,680\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\ \text{cm}^{-1} = 82\,260\ \text{cm}^{-1}$$

$$\lambda_2 = \frac{1}{\tilde{\nu}_2} = 121.566\ \text{nm}$$

The second line occurs when $n_2 = 3$, at 102.571 nm, and the third line occurs when $n_2 = 4$, at 97.2526 nm.

1–21. A line in the Lyman series of hydrogen has a wavelength of 1.03×10^{-7} m. Find the original energy level of the electron.

In the Lyman series, $n_1 = 1$ in Equation 1.10. Thus

$$\frac{1}{\lambda} = 109\,680\left(1 - \frac{1}{n_2^2}\right)\ \text{cm}^{-1}$$

$$\frac{109\,680\ \text{cm}^{-1}}{n_2^2} = -\frac{1}{1.03 \times 10^{-5}\ \text{cm}} + 109\,680\ \text{cm}^{-1}$$

$$n_2 = 2.98 \approx 3$$

because n must be an integer.

1–22. A ground-state hydrogen atom absorbs a photon of light that has a wavelength of 97.2 nm. It then gives off a photon that has a wavelength of 486 nm. What is the final state of the hydrogen atom?

First, we find the value of n_2, the state of the hydrogen atom that is obtained upon absorption, by using Equation 1.10 with $n_1 = 1$.

$$\frac{1}{97.2 \times 10^{-7}\ \text{cm}} = 109\,680\left(1 - \frac{1}{n_2^2}\right)\ \text{cm}^{-1}$$

$$n_2 = 4$$

We can now use Equation 1.10 with $n_1 = 4$ to find the final state of the hydrogen atom:

$$\frac{1}{97.2 \times 10^{-7}\ \text{cm}} = 109\,680\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)\ \text{cm}^{-1}$$

$$n_2 = 2$$

The final state of the hydrogen atom is $n = 2$.

1–23. Show that the Lyman series occurs between 91.2 nm and 121.6 nm, that the Balmer series occurs between 364.7 nm and 656.5 nm, and that the Paschen series occurs between 820.6 nm and 1876 nm. Identify the spectral regions to which these wavelengths correspond.

We can use the Rydberg formula (Equation 1.10) to determine the ranges of the specified series. The maximum wavelength can be found by taking the smallest n_2 allowed ($n_2 = n_1 + 1$) and the minimum wavelength can be found by using the largest n_2 allowed ($n_2 = \infty$).

Lyman series

maximum wavelength

$$\frac{1}{\lambda} = 109\,680\left(1 - \frac{1}{2^2}\right)\ \text{cm}^{-1}$$

$$\lambda = 121.6\ \text{nm}$$

minimum wavelength

$$\frac{1}{\lambda} = 109\,680\left(1 - \frac{1}{\infty}\right)\ \text{cm}^{-1}$$

$$\lambda = 91.2\ \text{nm}$$

This corresponds to the ultraviolet region of the spectrum.

Balmer series

maximum wavelength

$$\frac{1}{\lambda} = 109\,680\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\ \text{cm}^{-1}$$

$$\lambda = 656.5\ \text{nm}$$

minimum wavelength

$$\frac{1}{\lambda} = 109\,680\left(\frac{1}{2^2} - \frac{1}{\infty}\right)\ \text{cm}^{-1}$$

$$\lambda = 364.7\ \text{nm}$$

This corresponds to the near ultraviolet region of the spectrum.

Paschen series

maximum wavelength

$$\frac{1}{\lambda} = 109\,680\left(\frac{1}{3^2} - \frac{1}{4^2}\right)\ \text{cm}^{-1}$$

$$\lambda = 1875.6\ \text{nm}$$

minimum wavelength

$$\frac{1}{\lambda} = 109\,680\left(\frac{1}{3^2} - \frac{1}{\infty}\right)\ \text{cm}^{-1}$$

$$\lambda = 820.6\ \text{nm}$$

This corresponds to the near infrared region of the spectrum.

1–24. Calculate the wavelength and the energy of a photon associated with the series limit of the Lyman series.

We have found that the minimum wavelength of the Lyman series is 91.2 nm (Problem 1–23). So

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J·s})(2.998 \times 10^{8}\ \text{m·s}^{-1})}{91.2 \times 10^{-9}\ \text{m}} = 2.18 \times 10^{-18}\ \text{J}$$

1–25. Calculate the de Broglie wavelength for (a) an electron with a kinetic energy of 100 eV, (b) a proton with a kinetic energy of 100 eV, and (c) an electron in the first Bohr orbit of a hydrogen atom.

We use Equation 1.12 ($\lambda = h/p$) in all cases to find λ.

a.

$$\text{KE} = \frac{mv^2}{2}$$

$$100\ \text{eV}\left(\frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = \frac{(v^2)(9.109 \times 10^{-31}\ \text{kg})}{2}$$

$$v = 5.93 \times 10^{6}\ \text{m·s}^{-1}$$

So

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9.109 \times 10^{-31}\ \text{kg})(5.93 \times 10^{6}\ \text{m·s}^{-1})}$$

$$= 1.23 \times 10^{-10}\ \text{m} = 0.123\ \text{nm}$$

b. Replace m_e with m_p in (a) to find $\lambda = 2.86 \times 10^{-3}$ nm.

c. We must first determine the velocity of an electron in the first Bohr orbit of a hydrogen atom. From Equation 1.16, we see that

$$v = \frac{nh}{2\pi m_e r} \tag{1.16}$$

and we know

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m_e e^2} \tag{1.17}$$

Substituting Equation 1.17 for r into Equation 1.16 gives

$$v = \frac{e^2}{2nh\varepsilon_0}$$

For $n = 1$,

$$\begin{aligned} v &= \frac{(1.602 \times 10^{-19}\ \text{C})^2}{2(1)(6.626 \times 10^{-34}\ \text{J·s})(8.854 \times 10^{-12}\ \text{C}^2\ \text{J}^{-1}\ \text{m}^{-1})} \\ &= 2.188 \times 10^6\ \text{m·s}^{-1} \end{aligned}$$

So

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9.109 \times 10^{-31}\ \text{kg})(2.188 \times 10^6\ \text{m·s}^{-1})} \\ &= 3.32 \times 10^{-10}\ \text{m} = 0.332\ \text{nm} \end{aligned}$$

1–26. Calculate (a) the wavelength and kinetic energy of an electron in a beam of electrons accelerated by a voltage increment of 100 V and (b) the kinetic energy of an electron that has a de Broglie wavelength of 200 pm (1 picometer = 10^{-12} m).

---

a.

$$\text{Electron charge} \times \text{Potential} = \text{KE/electron}$$
$$(1.602 \times 10^{-19}\ \text{C})(100\ \text{V}) = 1.602 \times 10^{-17}\ \text{J/electron}$$

$$v = \sqrt{\frac{2\,(\text{KE})}{m}} = \sqrt{\frac{2\,(1.602 \times 10^{-17}\ \text{J})}{9.109 \times 10^{-31}\ \text{kg}}} = 5.93 \times 10^6\ \text{m·s}^{-1}$$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9.109 \times 10^{-31}\ \text{kg})(5.93 \times 10^6 \text{m·s}^{-1})} = 1.23 \times 10^{-10}\ \text{m}$$

b. Use Equation 1.12 to find the velocity of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}\ \text{J·s}}{(9.109 \times 10^{-31}\ \text{kg})(200 \times 10^{-12}\text{m})} = 3.64 \times 10^6\ \text{m·s}^{-1}$$

$$\text{KE} = \frac{mv^2}{2} = \frac{(9.109 \times 10^{-31}\ \text{kg})(3.46 \times 10^6\ \text{m·s}^{-1})^2}{2} = 6.02 \times 10^{-18}\ \text{J}$$

1–27. Through what potential must a proton initially at rest fall so that its de Broglie wavelength is 1.0×10^{-10} m?

We will use Equation 1.12 to find the velocity of the proton. Then we will calculate the kinetic energy and the potential needed to supply the kinetic energy.

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}\ \text{J·s}}{(1.672 \times 10^{-27}\ \text{kg})(1 \times 10^{-10}\ \text{m})} = 4.0 \times 10^3\ \text{m·s}^{-1}$$

Thus

$$\begin{aligned} \text{KE} &= \frac{mv^2}{2} = \frac{(1.672 \times 10^{-27}\ \text{kg})(4.0 \times 10^3\ \text{m·s}^{-1})^2}{2} \\ &= 1.3 \times 10^{-20}\ \text{J} \end{aligned}$$

and

$$\text{Potential} = \frac{\text{KE/proton}}{\text{proton charge}} = \frac{1.3 \times 10^{-20}\ \text{J}}{1.602 \times 10^{-19}\ \text{C}} = 0.082\ \text{V}$$

1–28. Calculate the energy and wavelength associated with an α particle that has fallen through a potential difference of 4.0 V. Take the mass of an α particle to be 6.64×10^{-27} kg.

An alpha particle is a helium nucleus, so it has a +2 charge.

$$\alpha \text{ particle charge} \times \text{Potential} = \text{KE}/\alpha \text{ particle}$$
$$[2\,(1.602 \times 10^{-19}\ \text{C})]\,(4.0\ \text{V}) = 1.28 \times 10^{-18}\ \text{J}/\alpha \text{ particle}$$

$$\begin{aligned} \text{KE} &= \frac{mv^2}{2} = \frac{p^2}{2m} \\ p &= \sqrt{2(\text{KE})m} = \sqrt{2\,(6.64 \times 10^{-27}\ \text{kg})(1.28 \times 10^{-18}\ \text{J})} \\ &= 1.30 \times 10^{-22}\ \text{kg·m·s}^{-1} \\ \lambda &= \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J·s}}{1.30 \times 10^{-22}\ \text{kg·m·s}^{-1}} = 5.08 \times 10^{-12}\ \text{m} \\ &= 5.08\ \text{pm} \end{aligned}$$

1–29. One of the most powerful modern techniques for studying structure is neutron diffraction. This technique involves generating a collimated beam of neutrons at a particular temperature from a high-energy neutron source and is accomplished at several accelerator facilities around the world. If the speed of a neutron is given by $v_n = (3k_B T/m)^{1/2}$, where m is the mass of a neutron, then what temperature is needed so that the neutrons have a de Broglie wavelength of 50 pm? Take the mass of a neutron to be 1.67×10^{-27} kg.

We are given

$$v_n = \left(\frac{3k_B T}{m}\right)^{1/2}$$

and so (Equation 1.12)

$$\lambda = \frac{h}{mv_n} = \frac{h}{(3mk_BT)^{1/2}}$$

Solving for T gives

$$\begin{aligned} T &= \frac{h^2}{3mk_B\lambda^2} \\ &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{3\,(1.67\times10^{-27}\ \text{kg})\,(1.381\times10^{-23}\ \text{J}\cdot\text{K}^{-1})\,(50\times10^{-12}\ \text{m})^2} \\ &= 2500\ \text{K} \end{aligned}$$

1–30. Show that a small change in the speed of a particle, Δv, causes a change in its de Broglie wavelength, $\Delta\lambda$, of

$$|\Delta\lambda| = \frac{|\Delta v|\lambda_0}{v_0}$$

where v_0 and λ_0 are its initial speed and de Broglie wavelength, respectively.

For a small change, $\Delta v = dv$ and $\Delta\lambda = d\lambda$. Then

$$\begin{aligned} \lambda &= \frac{h}{mv} \\ d\lambda &= \frac{-h}{mv_0^2}dv = \frac{-h}{mv_0}\frac{dv}{v_0} \\ |d\lambda| &= \frac{h}{mv_0}\frac{|dv|}{v_0} \\ |\Delta\lambda| &= \lambda_0\frac{|\Delta v|}{v_0} \end{aligned}$$

1–31. Derive the Bohr formula for $\tilde{\nu}$ for a nucleus of atomic number Z.

For a nucleus of charge Z, the attractive force of the nucleus is Ze and Equation 1.14 becomes

$$\frac{(Ze)(e)}{4\pi\varepsilon_0r^2} = \frac{m_ev^2}{r}$$

In the subsequent formulas, e^2 is replaced by Ze^2, and Equation 1.22 becomes

$$E_n = \frac{-m_eZ^2e^4}{8\varepsilon_0^2h^2n^2}$$

Likewise, Equation 1.24 becomes

$$\tilde{\nu} = \frac{m_eZ^2e^4}{8\varepsilon_0^2ch^3}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = Z^2R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

which is the Bohr formula for $\tilde{\nu}$ for a nucleus of atomic number Z.

1–32. The series in the He^+ spectrum that corresponds to the set of transitions where the electron falls from a higher level into the $n = 4$ state is called the Pickering series, an important series in solar astronomy. Derive the formula for the wavelengths of the observed lines in this series. In what region of the spectrum does it occur? (See Problem 1–31.)

From Problem 1–31, we have

$$\tilde{\nu} = Z^2R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

For the Pickering series in the helium spectrum, $Z = 2$ and $n_2 = 4$. Thus

$$\tilde{\nu} = 4(109\,680\ \text{cm}^{-1})\left(\frac{1}{4^2}-\frac{1}{\text{n}_1^2}\right) \qquad \text{n}_1 = 5, 6, 7, \ldots$$

For $n_1 = 5$, $\tilde{\nu} = 9871\ \text{cm}^{-1}$, or $\lambda = 1.0131\times10^{-6}$ m. This is in the visible region of the electomagnetic spectrum.

1–33. Using the Bohr theory, calculate the ionization energy (in electron volts and in $\text{kJ}\cdot\text{mol}^{-1}$) of singly ionized helium.

To find the ionization energy of singly ionized helium, we can consider the case where we move an electron from the ground state ($n = 1$) to an infinite distance from the nucleus. Then, from Problem 1–31,

$$\begin{aligned} \tilde{\nu} &= Z^2R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \\ &= 2^2(109\,680\ \text{cm}^{-1})\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right) \\ &= 4.3872\times10^5\ \text{cm}^{-1} \end{aligned}$$

or

$$E = hc\tilde{\nu} = 8.72\times10^{-18}\ \text{J} = 5250\ \text{kJ}\cdot\text{mol}^{-1} = 54.4\ \text{eV}$$

1–34. Show that the speed of an electron in the nth Bohr orbit is $v = e^2/2\varepsilon_0nh$. Calculate the values of v for the first few Bohr orbits.

We derived this relationship in Problem 1.25(c), where we found that $v = 2.188\times10^6\ \text{m}\cdot\text{s}^{-1}$ for $n = 1$. Likewise, for $n = 2$, $v_2 = 1.094\times10^6\ \text{m}\cdot\text{s}^{-1}$, and for $n = 3$, $v_3 = 7.292\times10^5\ \text{m}\cdot\text{s}^{-1}$.

1–35. If we locate an electron to within 20 pm, then what is the uncertainty in its speed?

By definition, $\Delta p = m\Delta v$, and Heisenberg's Uncertainty Principle (Equation 1.26) states that $\Delta x \Delta p \geq h$. Then

$$\begin{aligned}\Delta x(m\Delta v) &\geq h \\ \Delta v &\geq \frac{h}{m\Delta x} \\ &\geq \frac{6.626 \times 10^{-34}\ \mathrm{J\cdot s}}{(9.109 \times 10^{-31}\ \mathrm{kg})(20 \times 10^{-12}\ \mathrm{m})} \\ &\geq 3.6 \times 10^{7}\ \mathrm{m\cdot s^{-1}}\end{aligned}$$

The minimum uncertainty in the velocity of the electron is $3.6 \times 10^7\ \mathrm{m\cdot s^{-1}}$.

1–36. What is the uncertainty of the momentum of an electron if we know its position is somewhere in a 10 pm interval? How does the value compare to momentum of an electron in the first Bohr orbit?

Using Equation 1.26:

$$\begin{aligned}\Delta x \Delta p &\geq h \\ \Delta p &\geq \frac{h}{\Delta x} = \frac{6.626 \times 10^{-34}\ \mathrm{J\cdot s}}{10 \times 10^{-12}\ \mathrm{m}} \\ &\geq 6.6 \times 10^{-23}\ \mathrm{kg\cdot m\cdot s^{-1}}\end{aligned}$$

We can calculate the momentum of an electron in the first Bohr radius by using v from Problem 1–25(c):

$$p = m_e v = \left(9.109 \times 10^{-31}\ \mathrm{kg}\right)\left(2.188 \times 10^{6}\ \mathrm{m\cdot s^{-1}}\right) = 1.993 \times 10^{-24}\ \mathrm{kg\cdot m\cdot s^{-1}}$$

The uncertainty of the momentum of an electron somewhere in a 10 pm interval is about thirty times greater than the momentum of an electron in the first Bohr radius.

1–37. There is also an uncertainty principle for energy and time:

$$\Delta E \Delta t \geq h$$

Show that both sides of this expression have the same units.

The units of ΔE are J and those of Δt are seconds, and Planck's constant has units of J·s.

1–38. The relationship introduced in Problem 1–37 has been interpreted to mean that a particle of mass m ($E = mc^2$) can materialize from nothing provided that it returns to nothing within a time $\Delta t \leq h/mc^2$. Particles that last for time Δt or more are called *real particles*; particles that last less than time Δt are called *virtual particles*. The mass of the charged pion, a subatomic particle, is 2.5×10^{-28} kg. What is the minimum lifetime if the pion is to be considered a real particle?

Use the condition for real particles defined above:

$$\begin{aligned}\Delta t &\geq \frac{h}{mc^2} = \frac{6.626 \times 10^{-34}\ \mathrm{J\cdot s}}{(2.5 \times 10^{-28}\ \mathrm{kg})(2.998 \times 10^{8}\ \mathrm{m\cdot s^{-1}})^2} \\ &\geq 2.9 \times 10^{-23}\ \mathrm{s}\end{aligned}$$

1–39. Another application of the relationship given in Problem 1–37 has to do with the excited state energies and lifetimes of atoms and molecules. If we know that the lifetime of an excited state is 10^{-9} s, then what is the uncertainty in the energy of this state?

Using the relationship in Problem 1–37 gives

$$\Delta E \geq \frac{h}{\Delta t} = \frac{6.626 \times 10^{-34}\ \mathrm{J\cdot s}}{1 \times 10^{-9}\ \mathrm{s}} = 7 \times 10^{-25}\ \mathrm{J}$$

1–40. When an excited nucleus decays, it emits a γ-ray. The lifetime of an excited state of a nucleus is of the order of 10^{-12} s. What is the uncertainty in the energy of the γ-ray produced? (See Problem 1–37.)

$$\Delta E \geq \frac{h}{\Delta t} = \frac{6.626 \times 10^{-34}\ \mathrm{J\cdot s}}{1 \times 10^{-12}\ \mathrm{s}} = 7 \times 10^{-22}\ \mathrm{J}$$

1–41. In this problem, we will prove that the inward force required to keep a mass revolving around a fixed center is $f = mv^2/r$. To prove this, let us look at the velocity and the acceleration of a revolving mass. Referring to Figure 1.12, we see that

$$|\Delta \mathbf{r}| \approx \Delta s = r\Delta\theta \tag{1.27}$$

if $\Delta\theta$ is small enough that the arc length Δs and the vector difference $|\Delta\mathbf{r}| = |\mathbf{r}_1 - \mathbf{r}_2|$ are essentially the same. In this case, then

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = r\omega \tag{1.28}$$

where $\omega = d\theta/dt = v/r$.

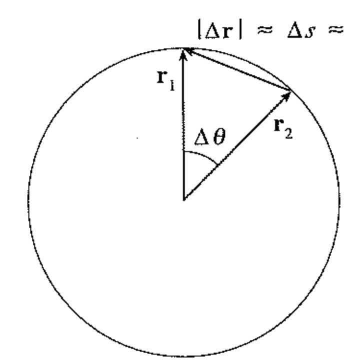


FIGURE 1.12
Diagram for defining angular speed.

If ω and r are constant, then $v = r\omega$ is constant, and because acceleration is $\lim_{t\to 0}(\Delta v/\Delta t)$, we might wonder if there is any acceleration. The answer is most definitely *yes* because velocity is a vector quantity and the direction of $\mathbf{v}$, which is the same as $\Delta\mathbf{r}$, is constantly changing even though its magnitude is not. To calculate this acceleration, draw a figure like Figure 1.12 but expressed in terms of v instead of r. From your figure, show that

$$\Delta v = |\Delta \mathbf{v}| = v\Delta\theta \tag{1.29}$$