a.

$$6i = r\cos\theta + ir\sin\theta$$
$$r = 6$$
$$\theta = \tan^{-1}\left(\frac{6}{0}\right)$$
$$\theta = \frac{\pi}{2}$$

so

$$6i = 6e^{i\pi/2}$$

b.

$$4 - \sqrt{2}i = r\cos\theta + ir\sin\theta$$
$$r = \sqrt{16+2} = 3\sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{4}\right) = -0.340$$

so

$$4 - \sqrt{2}i = 3\sqrt{2}\,e^{-0.340i}$$

c.

$$-1 - 2i = r\cos\theta + ir\sin\theta$$
$$r = \sqrt{1+4} = \sqrt{5}$$
$$\theta = \tan^{-1}\left(\frac{-2}{-1}\right) = 1.11$$

so

$$-1 - 2i = \sqrt{5}\,e^{1.11i}$$

d.

$$\pi + ei = r\cos\theta + ir\sin\theta$$
$$r = \sqrt{\pi^2 + e^2}$$
$$\theta = \tan^{-1}\left(\frac{e}{\pi}\right) = 0.7130$$

so

$$\pi + ei = \sqrt{\pi^2 + e^2}\,e^{0.713i}$$

A–4. Express the following complex numbers in the form $x + iy$:

a. $e^{\pi/4i}$ **b.** $6e^{2\pi i/3}$ **c.** $e^{-(\pi/4)i+\ln 2}$ **d.** $e^{-2\pi i}+e^{4\pi i}$

---

a.

$$e^{\pi/4i} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$$
$$= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$$

b.

$$6e^{2\pi i/3} = 6\cos\left(\frac{2\pi}{3}\right) + 6i\sin\left(\frac{2\pi}{3}\right)$$
$$= -3 + 3\sqrt{3}i$$

c.

$$e^{-(\pi/4)i+\ln 2} = 2e^{-\pi i/4} = 2\left[\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right] = 2\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$
$$= \sqrt{2} - \sqrt{2}i$$

d.

$$e^{-2\pi i} + e^{4\pi i} = \cos(-2\pi) + i\sin(-2\pi) + \cos(4\pi) + i\sin(4\pi)$$
$$= 2$$

A–5. Prove that $e^{i\pi} = -1$. Comment on the nature of the numbers in this relation.

$$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$$

This is an amazing equation. It shows that a transcendental number (e), raised to the product of an imaginary number (i) and another transcendental number (π), is equivalent to an integer.

A–6. Show that

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

and that

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Using Equation A.6,

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$e^{-i\theta} = \cos\theta - i\sin\theta$$

Adding these two expressions gives

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$
$$\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$$

and subtracting the first two expressions gives

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$
$$\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta$$

A–7. Use Equation A.7 to derive

$$z^n = r^n(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$$

and from this, the formula of De Moivre:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Beginning with Equation A.7,

$$z = r\,(\cos\theta + i\sin\theta)$$
$$z^n = r^n\,(\cos\theta + i\sin\theta)^n \qquad (1)$$