**Problem 1, 20 marks – classical wave equation**

a) Show that \( u(x, t) = \sin(kx - \omega t) \) satisfies the classical wave equation by directly using the function \( \sin(kx - \omega t) \) in the wave equation.

b) Show that \( u(x, t) = \sin(kx - \omega t) \) satisfies the classical wave equation by using the trigonometric identity \( \sin(A - B) = \sin A \cos B - \cos A \sin B \).

c) Show that \( u(x, t) = e^{i(kx - \omega t)} \) satisfies the classical wave equation by using the Euler identity \( e^{i\theta} = \cos \theta + i \sin \theta \).

d) Show that \( u(x, t) = e^{i(kx - \omega t)} \) satisfies the classical wave equation by directly differentiating the function \( e^{i(kx - \omega t)} \).

**Problem 2, 10 marks – different wavelength components**

a) Show that \( u(x, t) = \sin(k_1 x) \cos(\omega_1 t) - \cos(k_2 x) \sin(\omega_2 t) \) is not a classical wave if \( k_2 = 2k_1 \) and \( \omega_1 = \omega_2 \).

b) Show that \( u(x, t) = \sin(k_1 x) \cos(\omega_1 t) - \cos(k_2 x) \sin(\omega_2 t) \) is a classical wave if \( k_2 = 2k_1 \) and \( \omega_2 = 2\omega_1 \). What is the propagation speed of this wave?

**Problem 3, 10 marks – Taylor series**

For (a), (b), and (c) you can look up the answers using any resource.

a) Write down, up to (and including) 7th powers of \( x \), the Taylor series for \( \sin x \).

b) Write down, up to 7th powers of \( x \), the Taylor series for \( \cos x \).

c) Write down, up to 7th powers of \( x \), the Taylor series for \( e^x \).

d) Write down, up to 7th powers of \( x \), the Taylor series for \( e^{ix} \) by using your answer (c).

e) By comparing your answer (d) to the Euler formula \( e^{i\theta} = \cos \theta + i \sin \theta \) show how you could identify the \( \sin x \) and \( \cos x \) Taylor series (assuming you didn’t know them).