Chem 3322 homework #1, due January 26, 2024

Problem 1, 16 marks – classical wave equation

a) Show that $u(x,t) = \sin(kx - \omega t)$ satisfies the classical wave equation by directly using the function $\sin(kx - \omega t)$ in the wave equation.

b) Show that $u(x,t) = \sin(kx - \omega t)$ satisfies the classical wave equation by using the trigonometric identity $\sin(A - B) = \sin A \cos B - \cos A \sin B$.

c) Show that $u(x,t) = e^{i(kx-\omega t)}$ satisfies the classical wave equation by using the Euler identity $e^{i\theta} = \cos\theta + i \sin\theta$.

d) Show that $u(x,t) = e^{i(kx-\omega t)}$ satisfies the classical wave equation by directly differentiating the function $e^{i(kx-\omega t)}$.

Problem 2, 10 marks – different wavelength components

a) Show that $u(x,t) = \sin(k_1 x)\cos(\omega_1 t) - \cos(k_2 x)\sin(\omega_2 t)$ is not a classical wave if $k_2 = 2k_1$ and $\omega_1 = \omega_2$.

b) Show that $u(x,t) = \sin(k_1 x)\cos(\omega_1 t) - \cos(k_2 x)\sin(\omega_2 t)$ is a classical wave if $k_2 = 2k_1$ and $\omega_2 = 2\omega_1$. What is the propagation speed of this wave?

Problem 3, 10 marks – Taylor series

For (a), (b), and (c) you can look up the answers using any resource.

a) Write down, up to (and including) 7th powers of x, the Taylor series for $\sin x$.

b) Write down, up to 7th powers of x, the Taylor series for $\cos x$.

c) Write down, up to 7th powers of x, the Taylor series for e^x.

d) Write down, up to 7th powers of x, the Taylor series for e^{ix} by using your answer (c).

e) By comparing your answer (d) to the Euler formula $e^{i\theta} = \cos\theta + i\sin\theta$ show how you could identify the $\sin x$ and $\cos x$ Taylor series (assuming you didn't know them).

Problem 4, 10 marks – operators

a) We usually denote an operator by a capital letter with a carat over it, *e.g.*, $\hat{A}$. Thus, we write

$$\hat{A} f(x) = g(x) \tag{1}$$

to indicate that the operator $\hat{A}$ operates on $f(x)$ to give a new function $g(x)$.

Evaluate (see page 75) $\hat{A}f(x)$ where $f(x) = 2x^2$ and where

$$\hat{A} = \frac{d^2}{dx^2} + 2\frac{d}{dx} + 3 \tag{2}$$

b) Consider the operator (see page 79)

$$\hat{C} = \hat{A}\hat{B} - \hat{B}\hat{A} \tag{3}$$

Specifically, take $\hat{A} = x$ and $\hat{B} = d/dx$. What does this operator $\hat{C}$ do to a function $f(x)$? Based on your answer, express this operator in a simpler form.