$\boxed{\text{metaphysics handout}}$

We learned that a "down" silver atom is in a superposition of being "left" and "right". Let us express this using a mathematical formalism in which we express the wavefunctions as column vectors in a 2d spin space.

Define the left/right operator:

$$\hat{LR} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{1}$$

This operator has 2 eigenvalues/eigenvectors.

Eigenvalue 1 ("left"), eigenvector $\phi_L = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Eigenvalue -1 ("right"), eigenvector $\phi_R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Define the up/down operator:

$$\hat{UD} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

Eigenvalue 1 ("up"), eigenvector $\phi_U = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

Eigenvalue -1 ("down"), eigenvector $\phi_D = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

Superposition is expressed as:

$$\phi_U = \frac{1}{\sqrt{2}}\phi_L + \frac{1}{\sqrt{2}}\phi_R \tag{3}$$

$$\phi_D = \frac{1}{\sqrt{2}}\phi_L - \frac{1}{\sqrt{2}}\phi_R \tag{4}$$

$$\phi_L = \frac{1}{\sqrt{2}}\phi_U + \frac{1}{\sqrt{2}}\phi_D \tag{5}$$

$$\phi_R = \frac{1}{\sqrt{2}}\phi_U - \frac{1}{\sqrt{2}}\phi_D \tag{6}$$

Let us examine the state of the initially "down" atom which enters the apparatus. At time t_1,

$$\begin{aligned}
\psi_{t_1} &= \phi_D; X \to x_1; Y \to y_1 && (7)\\
&= \left(\frac{1}{\sqrt{2}}\phi_L - \frac{1}{\sqrt{2}}\phi_R\right); X \to x_1; Y \to y_1 && (8)\\
&= \frac{1}{\sqrt{2}}\underbrace{\phi_L; X \to x_1; Y \to y_1}_{a} - \frac{1}{\sqrt{2}}\underbrace{\phi_R; X \to x_1; Y \to y_1}_{b} && (9)
\end{aligned}$$

If the state at t_1 was just $\phi_{t_1} = a$, then at t_2 the state would be $\phi_L; X \to x_2; Y \to y_2$

If the state at t_1 was just $\phi_{t_1} = b$, then at t_2 the state would be $\phi_R; X \to x_3; Y \to y_1$

But then it follows from linearity that the actual state is

$$\psi_{t_2} = \frac{1}{\sqrt{2}}\phi_L; X \to x_2; Y \to y_2 - \frac{1}{\sqrt{2}}\phi_R; X \to x_3; Y \to y_1 \qquad (10)$$

There is a non-separable correlation between the spin properties and spatial properties of the atom.

At time t_3 we have

$$\psi_{t_3} = \frac{1}{\sqrt{2}}\phi_L; X \to x_3; Y \to y_3 - \frac{1}{\sqrt{2}}\phi_R; X \to x_4; Y \to y_2 \qquad (11)$$

and at time t_4

$$\begin{aligned}
\psi_{t_4} &= \frac{1}{\sqrt{2}}\phi_L; X \to x_5; Y \to y_4 - \frac{1}{\sqrt{2}}\phi_R; X \to x_5; Y \to y_4 && (12)\\
&= \left(\frac{1}{\sqrt{2}}\phi_L - \frac{1}{\sqrt{2}}\phi_R\right); X \to x_5; Y \to y_4 && (13)\\
&= \phi_D; X \to x_5; Y \to y_4 && (14)
\end{aligned}$$

What if we measure the position of the atom at t_3? The wavefunction collapses to either $X \to x_3; Y \to y_3$ or $X \to x_4; Y \to y_2$ in terms of position, so that,

$$\psi_{t_3} = \phi_L; X \to x_3; Y \to y_3 \qquad \text{or} \qquad \psi_{\mathrm{t}_3} = \phi_{\mathrm{R}}; \mathrm{X} \to \mathrm{x}_4; \mathrm{Y} \to \mathrm{y}_2 \qquad (15)$$

Thus at t_4 we would find either (with equal probability)

$$\psi_{t_4} = \phi_L; X \to x_5; Y \to y_4 \qquad \text{or} \qquad \psi_{\mathrm{t}_4} = \phi_{\mathrm{R}}; \mathrm{X} \to \mathrm{x}_5; \mathrm{Y} \to \mathrm{y}_4 \qquad (16)$$

If we put an absorbing wall at $X \to x_3; Y \to y_1$ (R path) then

$$\psi_{t_4} = \frac{1}{\sqrt{2}}\phi_L; X \to x_5; Y \to y_4 - \frac{1}{\sqrt{2}}\phi_R; X \to x_3; Y \to y_1 \tag{17}$$

If we look for the atom at $X \to x_5; Y \to y_4$ we will find it there 50% of the time, and if found its LR property would be "left" and its UD property would be 50/50 up/down.