We learned that a “down” silver atom is in a superposition of being “left” and “right”. Let us express this using a mathematical formalism in which we express the wavefunctions as column vectors in a 2d spin space.

Define the left/right operator:

$$\hat{LR} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ (1)

This operator has 2 eigenvalues/eigenvectors.

Eigenvalue 1 ("left"), eigenvector $$\phi_L = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Eigenvalue -1 ("right"), eigenvector $$\phi_R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Define the up/down operator:

$$\hat{UD} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ (2)

Eigenvalue 1 ("up"), eigenvector $$\phi_U = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Eigenvalue -1 ("down"), eigenvector $$\phi_D = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Superposition is expressed as:

$$\phi_U = \frac{1}{\sqrt{2}} \phi_L + \frac{1}{\sqrt{2}} \phi_R$$ (3)

$$\phi_D = \frac{1}{\sqrt{2}} \phi_L - \frac{1}{\sqrt{2}} \phi_R$$ (4)

$$\phi_L = \frac{1}{\sqrt{2}} \phi_U + \frac{1}{\sqrt{2}} \phi_D$$ (5)

$$\phi_R = \frac{1}{\sqrt{2}} \phi_U - \frac{1}{\sqrt{2}} \phi_D$$ (6)
Let us examine the state of the initially “down” atom which enters the apparatus. At time $t_1$,

$$\psi_{t_1} = \phi_D; X \rightarrow x_1; Y \rightarrow y_1 \quad (7)$$

$$= \left( \frac{1}{\sqrt{2}} \phi_L - \frac{1}{\sqrt{2}} \phi_R \right); X \rightarrow x_1; Y \rightarrow y_1 \quad (8)$$

$$= \frac{1}{\sqrt{2}} \phi_L; X \rightarrow x_1; Y \rightarrow y_1 - \frac{1}{\sqrt{2}} \phi_R; X \rightarrow x_1; Y \rightarrow y_1 \quad (9)$$

If the state at $t_1$ was just $\phi_{t_1} = a$, then at $t_2$ the state would be $\phi_L; X \rightarrow x_2; Y \rightarrow y_2$

If the state at $t_1$ was just $\phi_{t_1} = b$, then at $t_2$ the state would be $\phi_R; X \rightarrow x_3; Y \rightarrow y_1$

But then it follows from linearity that the actual state is

$$\psi_{t_2} = \frac{1}{\sqrt{2}} \phi_L; X \rightarrow x_2; Y \rightarrow y_2 - \frac{1}{\sqrt{2}} \phi_R; X \rightarrow x_3; Y \rightarrow y_1 \quad (10)$$

There is a non-separable correlation between the spin properties and spatial properties of the atom.

At time $t_3$ we have

$$\psi_{t_3} = \frac{1}{\sqrt{2}} \phi_L; X \rightarrow x_3; Y \rightarrow y_3 - \frac{1}{\sqrt{2}} \phi_R; X \rightarrow x_4; Y \rightarrow y_2 \quad (11)$$

and at time $t_4$

$$\psi_{t_4} = \frac{1}{\sqrt{2}} \phi_L; X \rightarrow x_5; Y \rightarrow y_4 - \frac{1}{\sqrt{2}} \phi_R; X \rightarrow x_5; Y \rightarrow y_4 \quad (12)$$

$$= \left( \frac{1}{\sqrt{2}} \phi_L - \frac{1}{\sqrt{2}} \phi_R \right); X \rightarrow x_5; Y \rightarrow y_4 \quad (13)$$

$$= \phi_D; X \rightarrow x_5; Y \rightarrow y_4 \quad (14)$$

What if we measure the position of the atom at $t_3$? The wavefunction collapses to either $X \rightarrow x_3; Y \rightarrow y_3$ or $X \rightarrow x_4; Y \rightarrow y_2$ in terms of position, so that,

$$\psi_{t_3} = \phi_L; X \rightarrow x_3; Y \rightarrow y_3 \quad \text{or} \quad \psi_{t_3} = \phi_R; X \rightarrow x_4; Y \rightarrow y_2 \quad (15)$$

Thus at $t_4$ we would find either (with equal probability)

$$\psi_{t_4} = \phi_L; X \rightarrow x_5; Y \rightarrow y_4 \quad \text{or} \quad \psi_{t_4} = \phi_R; X \rightarrow x_5; Y \rightarrow y_4 \quad (16)$$
If we put an absorbing wall at $X \rightarrow x_3; Y \rightarrow y_1$ (R path) then

$$\psi_{i_4} = \frac{1}{\sqrt{2}} \phi_L; X \rightarrow x_5; Y \rightarrow y_4 - \frac{1}{\sqrt{2}} \phi_R; X \rightarrow x_3; Y \rightarrow y_1$$ (17)

If we look for the atom at $X \rightarrow x_5; Y \rightarrow y_4$ we will find it there 50\% of the time, and if found its LR property would be “left” and its UD property would be 50/50 up/down.