The Postulates of Quantum Mechanics

(from *Quantum Mechanics* by Claude Cohen-Tannoudji, Bernard Diu, and Franck Laloë)

<u>Introduction</u> The postulates of quantum mechanics *are* the theory. Their physical content and their consequences are what we take time to develop in the course. The postulates provide us with an answer to the following questions:

(i) How is the state of a quantum mechanical system described mathematically?
(ii) How can we predict the results of the measurement of physical quantities?
(iii) How can the state of the system at an arbitrary time t be found when the state at time t_0 is known?

<u>First Postulate</u> (mathematical representation)

(McQuarrie & Simon: #1, page 116)

At a fixed time t_0, the state of a physical system is defined by specifying a wavefunction $\psi(x, y, z, t_0)$.

<u>Second Postulate</u> (measurement)

(McQuarrie & Simon: #2, page 118)

Every measurable physical quantity Q is described by an operator $\hat{Q}$; this operator is called an observable.

Examples of observables Q: energy, dipole moment, quadrapole moment, field gradient at a nucleus, diamagnetic susceptibility, ...

<u>Third Postulate</u> (measurement)

(McQuarrie & Simon: #3, page 122)

The only possible result of the measurement of a physical quantity Q is one of the eigenvalues of the corresponding observable $\hat{Q}$.

Fourth Postulate (measurement, non-degenerate case)

When the physical quantity Q is measured on a system in the normalized state ψ, the probability $\mathcal{P}(q_n)$ of obtaining the non-degenerate eigenvalue q_n of the corresponding observable $\hat{Q}$ is

$$\mathcal{P}(q_n) = \left(\int \phi_n^* \psi \right)^2 \tag{1}$$

where ϕ_n is the normalized eigenvector of $\hat{Q}$ associated with the eigenvalue q_n.

Fifth Postulate (collapse)

If the measurement of the physical quantity Q on the system in the state ψ gives the result q_n, the state of the system immediately after the measurement is ϕ_n.

Sixth Postulate (time evolution)

(McQuarrie & Simon: #5, page 125)

The time evolution of the wavefunction $\psi(x, y, z, t)$ is governed by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{2}$$

where $\hat{H}$ is the observable associated with the total energy of the system.

Seventh Postulate (symmetrization)

(McQuarrie & Simon: #6, page 287)

When a system includes several identical particles, only certain wavefunctions can describe its physical states (leads to the concept of *bosons* and *fermions*). For electrons (which are fermions), the wavefunction must *change sign* whenever the coordinates of two electrons are interchanged.