

Notes on distinguishability versus indistinguishability

The following comments are from the book *Quantum Mechanics* by Claude Cohen-Tannoudji, Bernard Diu, and Franck Lalœ.

Identical particles: definition

Two particles are said to be identical if all their intrinsic properties (mass, spin, charge, etc.) are exactly the same: no experiment can distinguish one from the other. Thus, all electrons in the universe are identical, as are all the protons and all the hydrogen atoms. Note that this definition is independent of the experimental conditions. Even if, in a given experiment, the charges of the particles are not measured, an electron and a positron can never be treated as identical particles.

An important consequence can be deduced from this definition: when a physical system contains two identical particles, there is no change in its properties or its evolution if the roles of these two particles are exchanged.

In classical mechanics, the presence of identical particles in a system poses no particular problems. Each particle moves along a well-defined trajectory, which enables us to distinguish it from the others and *follow* it throughout the evolution of the system.

It is immediately apparent that the situation is radically different in quantum mechanics, since the particles no longer have definite trajectories, but rather are treated in a probabilistic manner. For example, in figure 2 two identical particles approach one another. When the two particles are still far away from each other, they are distinguishable due to their spatial separation: we can label them “1” and “2”. But when they interact with each other (when they collide), we lose track of which is which, so, looking at figure 3, we are not sure which particle hits the detector (labeled “D”). Nothing in the theory of quantum mechanics enables us to determine which particle hits the detector.

The Symmetrization Postulate

We add a new postulate to the theory of quantum mechanics to deal with this.

Statement of the Postulate

When a system includes several identical particles, only certain wavefunctions can describe its physical states. Physical wavefunctions are, depending of the nature of the identical particles, either completely symmetric or completely antisymmetric with respect to permutation of these particles. Those particles for which the physical wavefunctions are

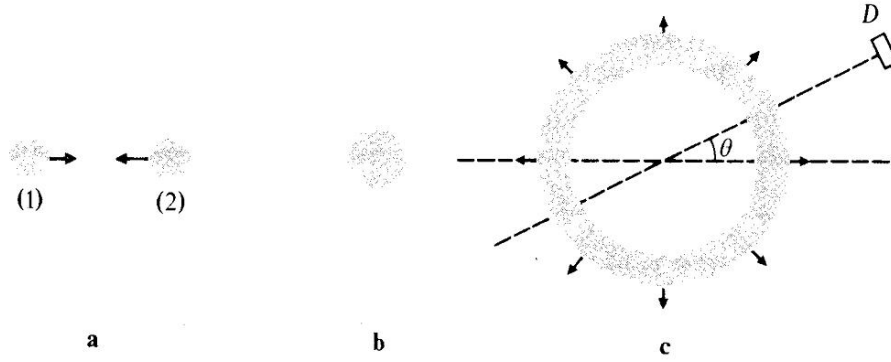


FIGURE 2

Collision between two identical particles in the center of mass frame : schematic representation of the probability density of the two particles.

Before the collision (fig. a), the two wave packets are clearly separated and can be labeled. During the collision (fig. b), the two wave packets overlap. After the collision (fig. c), the probability density is non-zero in a region shaped like a spherical shell whose radius increases over time. Because the two particles are identical, it is impossible, when a particle is detected at D , to know with which wave packet, (1) or (2), it was associated before the collision.

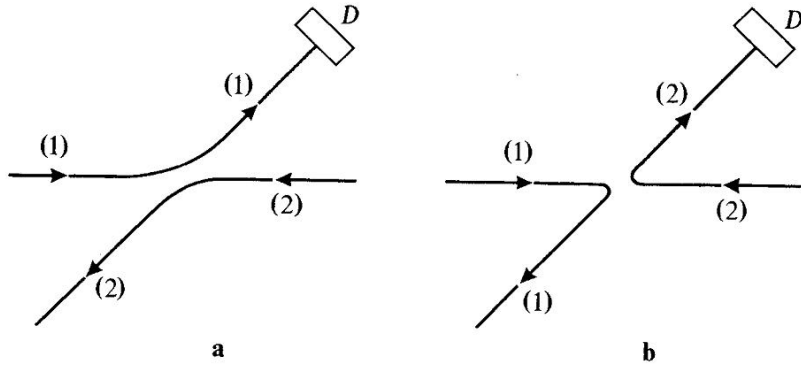


FIGURE 3

Schematic representation of two types of “paths” which the system could have followed in going from the initial state to the state found in the measurement. Because the two particles are identical, we cannot determine the path that was actually followed.

symmetric are called *bosons*, and those for which they are antisymmetric, *fermions*.

The symmetrization postulate thus limits the possible wavefunctions for a system of identical particles. From the point of view of this postulate, particles existing in nature are divided into two categories. All currently known particles obey the following empirical rule: particles of half-integral spin (electrons, positrons, protons, neutrons, muons, etc) are fermions, and particles of integral spin (photons, mesons, etc) are bosons.

Once this rule has been verified for all the particles which are called “elementary”, it holds for all other particles as well, inasmuch as they are composed of these elementary particles. Consequently, nuclei whose mass number (the total number of nucleons) is even

are bosons, and those whose mass number is odd are fermions. Thus, the nucleus of the ^3He isotope of helium is a fermion, and that of the ^4He isotope, a boson.

Predictions based on this principle, which are often spectacular, have always been confirmed experimentally.

Quantum Statistics

The object of statistical mechanics is to study systems composed of a very large number of particles. In numerous cases, the mutual interactions between these particles are weak enough to be neglected in a first approximation. Since we do not know the microscopic state of the system exactly, we content ourselves with describing it globally by its macroscopic properties (pressure, temperature, density, etc). A particular macroscopic state corresponds to a whole set of microscopic states. We then use probabilities: the statistical weight of a macroscopic state is proportional to the number of distinct microscopic states which correspond to it, and the system, at thermodynamic equilibrium, is in its most probable macroscopic state (with any constraints that may be imposed taken into account). To study the macroscopic properties of the system, it is therefore essential to determine how many different microscopic states possess certain characteristics and, in particular, a given energy.

In classical statistical mechanics (Maxwell-Boltzmann statistics), the N particles of the system are treated as if they were of different natures, even if they are actually identical. Such a microscopic state is defined by specifying the individual state of each of the N particles. Two microscopic states are considered to be distinct when these N individual states are the same but the permutation of the particles is different.

In quantum statistical mechanics, the symmetrization postulate must be taken into account. A microscopic state of a system of identical particles is characterized by the enumeration of the N individual states which form it, the order of these states being of no importance since their overall wavefunction must be symmetrized or anti-symmetrized. The numbering of the microscopic states therefore does not lead to the same result as in classical statistical mechanics. In addition, Pauli's principle (which is a consequence of the fermion symmetry) radically differentiates systems of identical bosons and systems of identical fermions: the number of particles occupying a given individual state cannot exceed one for fermions, while it can take on any value for bosons. Different statistical properties result: bosons obey *Bose-Einstein statistics* and fermions, *Fermi-Dirac statistics*. This is the origin of the terms "bosons" and "fermions".

The physical properties of systems of identical fermions and systems of identical bosons are very different. These differences can be observed, for example, at low temperatures. The particles then tend to accumulate in the lowest-energy individual states, as is possible for identical bosons (this phenomenon is called *Bose condensation*), while identical fermions are subject to the restrictions of Pauli's principle. Bose condensation is at the origin of the remarkable properties (superfluidity) of the ^4He isotope of helium, while the ^3He isotope, which is a fermion, does not possess the same properties.