

**PURE PUBLIC GOODS VERSUS COMMONS: BENEFIT-COST DUALITY**

by

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## Abstract

This paper utilizes benefit-cost duality to differentiate the problems associated with a pure public good from that associated with a commons. For the public good scenario, contributors' benefits are public or available to all, while provision costs impact only the contributor. In a commons, crowding costs are public, while benefits affect only the user. Although both problems possess the same game form for their canonical representations, collective-action implications differ: e.g., the relative positions of the Nash equilibrium and Pareto optimum, the form of the exploitation hypothesis, and the need for selective incentives or punishments. Other essential differences concern policy implications, equity considerations, and strategic aspects (influences on the underlying game). (JEL: H41, D70)

## PURE PUBLIC GOODS VERSUS THE COMMONS: BENEFIT-COST DUALITY

### I. INTRODUCTION

Since Mancur Olson's (1965) *The Logic of Collective Action*, there has been a tendency to lump together collective-action problems, such as contributing to a pure public good or the exploitation of an open-access commons. In their *standard* representation, both problems are characterized as abiding by a Prisoner's Dilemma (PD) game, though other game forms (e.g., chicken, assurance) may apply when such problems are examined in greater detail (Ostrom, Gardner, and Walker 1994; Sandler 1992, 1998). Once the pure public good contribution game (henceforth, called the contribution game) and the open-access commons game (henceforth, called the commons game) are recognized as analogous collective-action problems, both scenarios are then thought to be similarly influenced by group size, group composition (i.e., group symmetry or asymmetry), and standard policy remedies for correcting collective-action failures (i.e., selective incentives and institutional design) (Hardin 1982).

The purpose of this paper is to draw out the collective-action similarities and differences as well as the policy implications for the contribution and commons games. To accomplish this, we identify and apply a *benefit-cost duality*, which has been essentially ignored in the theoretical literature.<sup>1</sup> Much as the price-quantity duality for private and pure public goods, first recognized by Samuelson (1954, 1955), has yielded significant insights,<sup>2</sup> the benefit-cost duality can also inform. Two problems display benefit-cost duality when the roles played by benefits and costs are switched between the problems. For the contribution game, derived benefits from a contribution are public and so received by all, while provision costs are private to the contributor; for the commons game, derived benefits from the commons are private to the user, while the associated costs from the use are public and so imposed on everyone. Even when

analogous games apply to these alternative problems, some collective-action implications differ. For instance, Olson's exploitation hypothesis has the large being exploited by the small for the contribution game,<sup>3</sup> whereas the opposite is true for the commons game, so that equity-based policy differs between the two problems. Additionally, the Nash equilibrium and the Pareto optimum are in opposite positions for the  $n$ -player representation of the two games. The institutional means for changing the underlying game form of the contribution and commons problems differ drastically. Most important, the policy prescription for achieving alternative social goals varies between the two problems owing to benefit-cost duality. Our essential message is that the tendency to treat public goods and commons scenarios as analogous, because their most stylized representation share a Prisoner's Dilemma form, masks essential conceptual and policy differences. The commons problem is rooted in the presence of private benefits, groupwise costs from individual actions, and the *need for inaction*; whereas, the contribution problem is grounded in the presence of private costs, groupwise benefits from individual contributions, and the *need for action*.

In an earlier paper, Haveman (1973) draws interesting similarities and distinctions for the commons and public good problem (in the form of environmental pollution). His primary result is that there are limits to the overexploitation of the commons, but not to the misallocation of resources associated with environmental pollution. Thus, Haveman singles out public goods as being potentially the more difficult of the two situations to address. Our analysis differs from his in three crucial ways: the identification of benefit-cost duality, the focus on collective-action differences, and the identification of strategic concerns. Under many circumstances, benefit-cost duality identifies the commons problem, in contrast to Haveman, as potentially the harder one to rectify owing to the need for inaction rather than action.

The remainder of the paper contains five sections. Section II presents a two-person game

representation of the contribution and the commons problem. This is followed in Section III with a more general  $n$ -player representation where collective-action failures are discussed. In Section IV, additional collective-action and policy remedies are addressed. Intertemporal aspects are then taken up in Section V, followed by concluding remarks in Section VI.

## II. TWO-PERSON REPRESENTATIONS

We begin with the canonical representation of the contribution and the commons game. For the contribution game, two symmetric players are assumed to contribute one or no units to a pure public good, in which each unit contributed adds to the total of the public good, available for collective consumption by contributors and noncontributors alike. The additivity of contributions implies a “*summation*” technology of aggregation, which is responsible for the perfect substitutability of contributions (Cornes and Sandler 1996; Hirshleifer 1983; Sandler 1998). Benefits of the pure public good are “public” in the sense that each unit contributed provides benefits of 5 to *both* agent  $A$  and  $B$ , regardless of the contributor. The per-unit cost of 8 is, however, only incurred by the contributing agent(s).

The game is displayed in normal form in the  $2 \times 2$  matrix in panel  $a$  of Figure 1, where agent  $A$  is the row player and agent  $B$  is the column player. For each strategic combination, agent  $A$ 's net payoff is on the left and agent  $B$ 's net payoff is on the right. If neither agent contributes, then each receives 0, since the status quo of no contributions is initially associated with no penalty. If  $A$  contributes alone, then she receives  $-3 (= 5 - 8)$  as costs exceed benefits, while  $B$  receives the free-riding benefit of 5. The payoffs are reversed when  $B$  is the sole contributor. If both agents contribute a unit, then each receives  $2 (= 2 \times 5 - 8)$ . In this calculation, an agent receives 10 in benefits (5 from each unit contributed minus the costs of 8 from providing a single unit). The summation technology of aggregation is evident, because

each unit adds equally and in a simple additive fashion to the benefits.

For both agents, the dominant strategy is not to contribute,<sup>4</sup> which results in the Nash equilibrium, highlighted by the asterisk in panel *a*. This game is a PD where both agents are better off if they contribute. However, this mutual-contribution Pareto optimum *cannot* be supported as a Nash equilibrium payoff unless the game is played repeatedly and indefinitely with some threat-based strategy (e.g., tit-for-tat) (Sandler 1992, 79-83).

In panel *b* in Figure 1, the commons game is displayed for two agents who have two strategies: to graze or not to graze their cattle on an open-access field. Costs of grazing are “public,” because each grazer imposes a cost of 5 on *both* agents by degrading the common.<sup>5</sup> Now the benefits are 8 and “private” to just the grazer. In panel *b*, the absence of any grazing gives a payoff of 0 to each agent. If only agent *B* sends his cattle out to graze the commons, then he receives 3 ( $= 8 - 5$ ), while agent *A* just experiences the harm of  $-5$  caused by her counterpart. Payoffs are reversed when just *A*’s cattle are put on the common pasture. If both agents graze their cattle, then each receives a net payoff of  $-2$  ( $= 8 - 2 \times 5$ ), where degradation follows a summation or cumulative technology of degradation.

In the commons game, the dominant strategy is for both agents to graze their cattle, so that the Nash equilibrium (marked with an asterisk) is in the lower right-hand corner. Analogous to the contribution game, this is a PD. The low-level Nash equilibrium, where everyone is harmed, is known as the “tragedy of the commons” (Hardin 1968).

There are a number of differences between the two problems even though they result in the same game in their simplest canonical representation. First, the Nash equilibrium and the Pareto optimum switch position. In the commons game, the former involves everyone acting, while the latter involves no one acting. Inaction is desired. The opposite holds in the contribution game, so that the absence of action is the problem. This difference between the

desirability for inaction or action may have profound policy implications for the two problems as shown in subsequent discussions. Second, the payoff matrices are negative transposes of one another, owing to the transposition of the roles of benefits and costs. Third, the technology of supply aggregation (or aggregation technology) indicates how benefits are accumulated in the contribution game, while the technology of commons degradation indicates how costs are accumulated.<sup>6</sup> In both cases, changes in these underlying technologies can cause the two problems to differ in their identifying game.

If, for example, a weakest-link technology applies, where the minimal contribution determines the benefit payoffs for public good provision, then an assurance game results with matching behavior – i.e., no one contributes or both contribute – as the two pure-strategy Nash equilibria (Sandler 1997, 34, 1998, 228-29). For weakest link, both players must give a unit before the benefits of 5 from each unit are received by the agents. This can be shown in panel *a* of Figure 1 by replacing just the 5s with 0s. As a result, there is no dominant strategy, but there are now two Nash equilibria along the main diagonal of the game box (not shown), where neither contributes or both agents contribute. For the commons game, changes in the degradation technology can affect the payoffs in the lower right-hand cell, thereby leading to alternative game forms. If, for example, further grazing leads to *increasing* marginal costs per grazer, then the negative payoffs for two grazers may be worse than listed. When these net payoffs are worse than  $-5$ , two equilibria result, each with a single grazer (not shown), which is a chicken game. In distinction to the contribution game, altering the underlying technology does not lend itself to generating an assurance game for the commons problem. Instead, some institutional arrangement that coordinates the actions of the agents is required (Runge 1984).

For our purposes, the essential feature to emphasize is that the associated technologies for the two problems may influence the payoffs differently. This is best brought out by consulting

panels *a* and *b* in Figure 2, which lists the payoffs in terms of per-unit benefits,  $b_i$ , and per-unit costs,  $c_i$ , for the two games. Recall that  $b_i = 5$  and  $c_i = 8$  for the contribution game, while  $b_i = 8$  and  $c_i = 5$  in the commons game. The aggregate technology in the former game influences the  $b_i$ s and the 2 in front of the  $b_i$ s, whereas the degradation technology affects the  $c_i$ s and the 2 in front of the  $c_i$ s. The 2 follows from the summation technology and constant marginal cost associated with the standard representations of the respective games. Technologies relevant for the contribution (commons) game – e.g., weakest link (increasing marginal costs) may have no analogous counterpart in the other game. Thus, even a small degree of added complexity can cause the associated game forms for the two problems to differ greatly. The *shared* underlying game representation for the canonical statement of the contribution and commons game is *not robust* to small alterations in analogous factors.

The recognition of the generating mechanism for alternative game forms for the two problems has public policy importance, because institutional and policy decisions can sometimes determine the aggregator or degradation technology (Sandler 1998). For a weakest-link public good, policy may necessitate supporting a poorer agent's contributions, so that the Pareto-optimal matching equilibrium results (Vicary and Sandler 2002). Additionally, public policy that regulates utilization in a commons (e.g., the length of the fishing season) can influence the rate of diminishing returns. Policy that alters a PD to an assurance game holds out greater hope for efficiency, since one of the Nash equilibria for assurance is a Pareto optimum. The need for policy also hinges on the underlying game and its implied strategic incentives to act or not.

Another difference involves how to transform the alternative games in Figure 2 (or 1) into a *chicken* representation. For the contribution problem, a chicken game results if the status quo of doing nothing implies a sufficient loss greater in absolute value than  $b_i - c_i$ .<sup>7</sup> For the commons, the net payoffs from mutual grazing must exceed  $c_i$  in absolute value, which implies



the introduction of increasing marginal costs. In panel *a*, mutual inaction must have dire consequences (e.g., no one retaliating against a state sponsor of terrorism leads to further attacks, so that inaction does not result in a 0 payoff), while in panel *b*, mutual action must have increasingly bad consequences (e.g., everyone grazing their cattle creates harm greater than the sum of their individual actions).

### III. MULTI-AGENT REPRESENTATIONS: COLLECTIVE ACTION

To draw out the collective-action differences between the two games, we turn to multi-agent symmetric versions of each. These representations are based on the same payoff arrangements as those in Figure 1. For illustrative purposes, five agents are assumed, though the analysis generalizes directly to  $n$  identical agents.

In panel *a* of Figure 3, the contribution game is displayed for five identical agents, where representative agent  $i$ 's payoffs are indicated in the cells. The columns refer to the number of contributors other than  $i$  who contributes a unit, which provides benefits of 5 to all at a cost of 8 to a contributor. If, for example,  $i$  does not contribute, then he gets a free-rider benefit of 5 times the number of other contributors – e.g., 15 for three other contributors. If, in contrast,  $i$  contributes, then his payoff is  $(nb_i - c_i)$ , where  $n$  is the number of contributors including  $i$ , and  $c_i = 8$ . As displayed, the dominant strategy (indicated by the arrow) is not to contribute, given that each payoff in the top row is higher than the corresponding payoff in the bottom row. Because everyone views the game in the same fashion, everyone plays this dominant strategy and so no one contributes, which is the Nash equilibrium from which no agent would unilaterally change his strategy. The Pareto optimum is, however, for everyone to contribute with a net per-person payoff of 17. As  $n$  increases, the extent of suboptimality increases, as measured by the *difference* in per-agent payoffs between the Pareto optimum and the Nash equilibrium. This

difference is  $nb_i - c_i$  for  $n$  agents. For an  $n$ -agent contribution game, Olson's collective-action prediction holds, because an increase in group size worsens suboptimality. This clear-cut result hinges on the Nash behavioral assumption, the summation technology, and other structural factors (Sandler 1992).

Next, we turn to the multi-agent standard representation of the commons in panel *b* of Figure 3. When  $i$  does not graze her cattle on the commons, her payoffs equal the number of others who graze their cattle times the cost of 5 per grazer. Thus,  $i$  loses 15 if three others chose to graze their herds. If  $i$  also grazes her herd, then her payoff is  $(b_i - nc_i)$ , where  $n$  is the number of grazers including  $i$ , and  $b_i = 8$ . The dominant strategy for  $i$ , and thus everyone, is to graze the cattle, resulting in the Nash equilibrium with a per-person payoff of  $-17$ . The Pareto optimum with payoffs of 0 is for no one to send out her herd to graze. Analogous to the contribution game, the suboptimality increases as measured by  $nc_i - b_i$ . The role reversal between the benefits and costs is readily apparent. Once again, Olson's prediction of the influence of group size holds for this stylized commons game owing to the Nash behavioral assumption, the constant marginal cost of degradation, and other structural factors. Thus far, the two games mirror one another except for the switch in the position of the Nash equilibrium and the Pareto optimum.

Again, we see that action is desirable for the contribution game, while inaction is desirable for the commons game. This difference may hold out greater hope for public goods than for the commons, insofar as action is often viewed as easier to accomplish than inaction. Studies have shown that the marginal willingness to pay to take an action is often smaller in practice than the marginal willingness to accept in giving up the right to an action (e.g., Bishop and Heberlein 1979) even though these two measures should have the same value theoretically for an identical action. Individuals appear to place a premium on inaction which requires

sacrificing a right,<sup>8</sup> so that the commons problem may pose a greater policy dilemma in practice. Moreover, the *enforcement* of inaction raises a collective-action concern that has been characterized by Heckathorn (1989) as a “second-order free-rider problem” more difficult to address than encouraging public-good contributions.

### *Group Composition and the Exploitation Hypothesis*

To address the impact of the group composition on the contribution and commons game, we must use representations that do not rely on identical contributors. In the case of a pure public good, we shall utilize results from the literature to establish Olson’s exploitation hypothesis. Consider the private provision of public good model where individual  $i$  possesses a quasi-concave, strictly increasing utility function,

$$U_i = U_i(y_i, G), \quad [1]$$

where  $y_i$  is the individual’s consumption of a composite private good and  $G$  is the total amount of the pure public good available for consumption, so that

$$G = \sum_{i \in C} g_i. \quad [2]$$

In equation [2],  $g_i$  is the  $i$ th person’s contribution to the public good and  $C$  denotes the set of contributors ( $g_i > 0$ ). Total contributions can also be written as  $g_i + \tilde{G}$ , where  $\tilde{G}$  is the sum of the contributions from contributors other than  $i$ .

By normalizing the price of the private good to equal one, we can write the  $i$ th individual’s budget constraint as:

$$w_i = y_i + pg_i, \quad [3]$$

where  $w_i$  denotes  $i$ ’s income and  $p$  is the unit price of the public good. To derive  $i$ ’s demand for the total amount of the public good, we depict  $i$  as solving:

$$\max_{y_i, G} \{U_i(y_i, G) \mid w_i + p\tilde{G} = y + pG; G \geq \tilde{G}\}. \quad [4]$$

In equation [4], we have added  $p\tilde{G}$  to both sides of [3] to give a full-income representation of the budget constraint where total contributions,  $G$ , becomes the choice variable (Bergstrom, Blume, and Varian 1986). The inequality constraint indicates that the  $i$ th individual is a contributor and, hence, a member of  $C$  only when  $G > \tilde{G}$ , so that  $g_i > 0$ . Noncontributors choose a  $G$  that equals  $\tilde{G}$  and contributes nothing. Based on the Kuhn-Tucker first-order conditions, the following continuous demand function,

$$G = \max \{f_i(w_i + p\tilde{G}, p), \tilde{G}\}, \quad [5]$$

for the public good follows from [4]. If  $i \in C$ , then  $i$ 's demand for  $G$  is  $f_i(\cdot)$ ; otherwise, it is  $\tilde{G}$ . If everyone's tastes are such that both goods are normal with positive income elasticities, then the underlying Nash equilibrium is unique (Cornes, Hartley, and Sandler 1999). This equilibrium corresponds to the vector of *individual* contributions,  $g_i$ , that satisfies equation [4] for the best-response level of spillins,  $\tilde{G}^*$ .

The simplest means for establishing the exploitation hypothesis, where "the large is exploited by the small," is to assume identical taste but different incomes. For this case,  $i$ 's equilibrium contribution,  $g_i^*$  is:

$$g_i^* = \begin{cases} w_i/p - w^*(p)/p, & \text{if } w_i > w^*, \\ 0, & \text{if } w_i \leq w^*, \end{cases} \quad [6]$$

where the critical income  $w^*$  is the level below which an individual does not contribute (Andreoni and McGuire 1993; Bergstrom, Blume, and Varian 1986).<sup>9</sup> Equation [6] is consistent with the exploitation hypothesis, because low-income individuals – those whose income is below  $w^*$  – free ride on higher income individuals. Moreover, the level of contributions is directly related to income, since  $g_i = (w_i - w^*)/p$ . The higher is an individual's income above the cut-

off income, the greater is the individual's contribution and, hence, the larger are spillovers that the individual's contribution provides to others. As the rich contributor equates his marginal rate of substitution of  $G$  for  $y$  (i.e.,  $MRS_{Gy}$ ) to  $p$ , he will supply much, if not all, of the needs of the poor, provided that tastes are identical.

If tastes are not identical, then the individual's preference for the public good is also a determinant of  $g_i^*$  and exploitation is not so clear-cut. For example, a low-income individual may be a large contributor if his preferences for the public good are very strong. In essence, large income effects can overcome the standard exploitative tendencies when the private good is inferior. Exploitation may also not hold in the contribution game when the Nash assumption is replaced with leader-follower and the rich is the leader (Sandler 1992, 57-58).<sup>10</sup>

In the case of the commons, the benefit-cost duality turns the exploitation hypothesis on its head, thereby leading to the exploitation of the *small by the large*. The intuition behind this result is straightforward. A larger firm receives a greater share of the output (the private benefit) that follows from its proportionally larger overall effort. This larger relative effort means that the bigger firms impose more of the public costs on others. Thus, the large firms are more responsible for diminishing returns, crowding, and the reduced profits that results from their use of the open-access common property resource.

This can be displayed mathematically by deriving the externality imposed by one firm on the other firms. For the Pareto optimum, we consider the commons – say, a fishery – as though it is utilized by a single firm selling its output and buying its input in competitive markets. Total catch,  $\hat{C}$ , from the commons is dependent on the neoclassical production function,  $F(R)$ , where

$$\hat{C} = F(R), \quad F'(R) > 0, \quad \text{and} \quad F''(R) < 0. \quad [7]$$

$R$  represents the total number of vessels that plies the commons. The production function displays a positive but diminishing marginal product,  $F'(R)$ . The Pareto optimum follows when

$R$  is chosen to maximize aggregate profit,  $\Pi$ :

$$\Pi = [F(R) - pR], \quad [8]$$

where  $p$  is the rental rate per vessel in the fleet and the price of fish is one. The Pareto optimum follows from the first-order condition associated with [8] when the *marginal product equals the rental price*:  $F'(R) = p$ .

For the Nash equilibrium, each fishing firm in the commons must choose its fleet size,  $r$ , to maximize its profit,

$$\pi = \left\{ \left[ \frac{r}{r + \tilde{R}} \right] F(r + \tilde{R}) - pr \right\}, \quad [9]$$

while taking the aggregate fleet size,  $\tilde{R}$ , of other firms in the commons as given. The total fleet  $R$  equals  $r + \tilde{R}$ . In [9], each firm in the commons receives a proportion of the output equal to its share of total effort, consistent with the fish being equally spread throughout the commons. The first-order conditions associated with [9] can be written as:

$$\left( \frac{r}{R} \right) F'(R) + \left( \frac{\tilde{R}}{R} \right) \left[ \frac{F(R)}{R} \right] = p. \quad [10]$$

In [10], a firm equates the weighted sum of its marginal and average product to the price. If there is just one firm in the commons, then equation [10] implies optimality as  $F'(R) = p$ , because  $\tilde{R} = 0$  implies  $r = R$ . As the number of firms in the commons grows, more weight is placed on the average product until  $n \rightarrow \infty$ , where  $\tilde{R}/R = 1$  and profits are driven to zero. At such a scenario, the tragedy of the commons is fully experienced. Insofar as the average product exceeds the marginal product for a neoclassical production function, [10] implies overutilization of the commons for  $n > 1$ .

To address the exploitation hypothesis, we first derive an expression for the negative marginal externality that the  $j$ th firm imposes on the  $i$ th firm's profit,  $\pi^i$ . This profit can be expressed in terms of  $r^j$  as follows:

$$\pi^i = \frac{r^i}{R} F(R) - pr^i = \frac{r^i}{r^j + \tilde{R}^j} F(r^j + \tilde{R}^j) - pr^i. \quad [11]$$

Taking the partial derivative of  $\pi^i$  with respect to  $r^j$  and simplifying the result, we get

$$\partial\pi^i/\partial r^j = (r^i/R)\{F'(R) - [F(R)/R]\} < 0. \quad [12]$$

The *aggregate* externality imposed by firm  $j$  on the other firms in the commons equals:

$$\sum_{i \neq j} (\partial\pi^i/\partial r^j) r^j = \sum_{i \neq j} (r^i/R)\{F'(R) - [F(R)/R]\} r^j, \quad [13]$$

which increases in absolute value with the size of the firm as  $r^j$  increases and, hence, the *difference* between average and marginal product widens. For the case of two firms, [13] indicates that the larger firm imposes a greater total externality on the smaller firm.<sup>11</sup>

Thus, the large firms impose more crowding costs on the small firms and, in so doing, “exploit” them. This collective-action reversal in who gains the advantage for asymmetric participants arises from the benefit-cost duality associated with the contribution and commons problems. This difference in who is exploiting whom has equity considerations. If policy is intended to offset the exploitation, then compensation must flow to the smaller agents in the commons and to the larger agents in the public good scenario. Such income transfers are for equity rather than efficiency purposes. So, again, we find that problems sharing the same underlying game structure may require different policy actions because of benefit-cost duality.

### *Constrained Isoutilility Curves Versus Isoprofit Curves*

Duality is always associated with a change in convexity between dual problems. Thus, a similar reversal of convexity should characterize the contribution and commons problem owing to benefit-cost duality. In panel *a* of Figure 4, standard constrained isoutilility curves are drawn<sup>12</sup> for the contribution game (see Cornes and Sandler 1996, 145-47). In Figure 4, isoutilility curve  $U_2$  is a higher level of well-being than  $U_1$ , because each level of individual contribution is

associated with a higher level of spillovers,  $\tilde{G}$ , or public good benefits received from others.

Thus, a shift of the isoutility curve in the direction of the arrow marks an improvement in utility.

In panel *b* of Figure 4, three isoprofit curves are drawn for  $\pi$  in equation [9]. The shapes of these contours are justified in Cornes and Sandler (1983). Contour  $\pi_2$  represents a higher constant profit level than contour  $\pi_1$ , because each level of  $r$  on the horizontal axis is associated with a smaller aggregate fleet size of others,  $\tilde{R}$ , so that the cost externality is smaller. The arrow again indicates the direction of improvement. The greatest level of profit,  $\pi^*$ , is associated with a single firm plying the commons, so that point *C* along the horizontal axis denotes the Pareto optimum.

In contrast, the Pareto optimum in panel *a* for the public good contribution lies somewhere above  $U_2$  and often involves widespread participation by benefit recipients. Benefit-cost duality leads to the need to exclude in the commons owing to public costs and the need to include for public goods owing to public benefits. As anticipated, benefit-cost duality reverses the convexity of the welfare contours and, in so doing, it also reverses the direction of welfare improvement in terms of the actions of others.

A similar benefit-cost duality characterizes congestible public goods when compared with purely public goods. As such, the convexity of the isoutility contours for congestible public goods (where congestion costs are public) is the same as the convexity of isoprofit contours of the commons (Cornes and Sandler 1996, 272-77). The degree of exclusion becomes a relevant policy concern for congestible public goods.

#### *Asymmetry: A Further Implication*

Olson (1965) also saw some good coming from asymmetry in the contribution game. In particular, asymmetry bolsters the likelihood that some agent would gain sufficiently to provide



the public good on her own, thus giving the group “privilege” status. Asymmetry may also be welfare enhancing in a commons despite the exploitation of the small. As asymmetry increases, a firm in the commons imposes more of the crowding on itself and this serves to internalize the externality to a greater extent. In the limit, asymmetry results in a single firm and full internalization. In terms of internalizing externalities, asymmetry has the same positive efficiency influence for both problems.

#### IV. FURTHER COLLECTIVE-ACTION AND POLICY REMEDIES

In the public good contribution context, Olson (1965) introduced the notion of selective incentives as a means for circumventing the free-rider problem. Selective incentives consist of tying contributor-specific benefits (e.g., recognition, added contributor-specific benefits, the proximity of the public good) to the provision of the public good, so as to augment the private motives for supplying the public good (Sandler 1992).<sup>13</sup> When contributing to a charity, these selective incentives may, for example, involve free gifts (e.g., season tickets given to supporters of a ballet company). In an insurgency, successful revolutionary leaders can later form a new government with themselves in key positions as the private inducement for risking their lives for the common good. By augmenting the private benefits, selective incentives are meant to give a contributor a greater stake in the public benefits, so as to overcome the relatively high private costs and the devalued public good spillovers.

Because of benefit-cost duality, the private inducement for a commons must now come on the cost side in terms of selective punishments. In the absence of these punishments, an individual focuses on comparing derived private benefits and his *share* of the public costs, thereby ignoring public costs imposed on others. Because the cost side is undervalued, *selective punishments* must be introduced to push individual actions in the desired direction of reduced

exploitation of the commons. The presence of private benefits is the root of the commons problem, while the presence of private costs is the root of the contribution problem. Selective punishments (incentives) are aimed at offsetting the imbalance between public costs (benefits) and private benefits (costs).

This distinction between the need for selective inducements to encourage action versus selective punishments to induce inaction also has implications for the application of incentive-compatible mechanisms. For public goods, incentive-compatible bribes can be applied to motivate agents to view honest preference revelation as a dominant strategy. Although a first best does not result owing to the revenue requirements of the incentive payments, greater efficiency is nevertheless achieved. An analogous mechanism to induce inaction in a commons is more problematic, because the penalties must be large to reflect the associated public costs. If these costs have intertemporal components (see the next section), then such penalties are still larger and also difficult to calculate. Moreover, the optimum often involves a corner solution (see Figure 4) with a single exploiter. In a commons, the need for inaction restricts the application of incentive-compatible mechanism, so that some other regime must evolve to limit use. Thus, the two problems require different corrective institutional arrangements.

Since agents have an aversion to imposing punishments, Olson's selective incentives may be more problematic to remedy the commons problem. For the Montreal Protocol on preserving the stratospheric ozone layer, positive inducements resulted in widespread participation. The treaty's inducements allowed developing (Article 5) nations to augment their chlorofluorocarbons (CFCs) use in the short run, followed by pledged assistance during their later phasing out of CFCs. Similar positive inducements have not characterized the Kyoto Protocol on limiting greenhouse gases (GHGs) that warm the atmosphere.

When it comes to public goods versus commons problems, there is also a qualitative

difference in the effects of *exit*, given the target of the exploitation – i.e., large or small agent. In a public good regime, the remaining signatories may make up the difference of an exiting country (especially if the underlying aggregator is summation). By contrast, the consequence of exit from a commons regime may not be tempered as large nonparticipants can (potentially) undo the actions of the participants.

This exit concern also helps distinguish the relative success to date of the Montreal and Kyoto Protocols. For the Montreal Protocol, the initial inclusion of all major CFC producers and consumers effectively created a public good: maintenance of the global ozone layer and avoidance of health and environment impacts of a thinning layer. Although the large CFC producers and users were exploited by the small in creating the public benefit, the small nonsignatories could not limit the public good by much. In contrast, the Kyoto Protocol is concerned with GHG emissions in which the small are exploited by the large, which treat the atmosphere as a common pool resource for emissions. Nations that did not pledge GHG cutbacks included not only large rich nations (e.g., the United States), but also large developing nations. The latter asked to be exempted on equity grounds. As a consequence, countries with no cutback responsibilities can significantly offset the actions of treaty participants. This is particularly true when the growth of GHG emissions of developing countries are considered.

Thus far, we have made much of action and inaction when distinguishing the contribution and commons game, respectively. At times, a commons problem may be transformed into a contribution scenario if action can be achieved to address the problem. In the case of global warming, the atmosphere is a commons where inaction has been the norm. If, however, countries were to agree to limit future emissions, then a contribution game becomes relevant, with efforts to limit GHGs as the pure public good. As inaction gives way to action, benefits become public and costs become private. Thus, the trick is somehow to induce the action that

yields the public good and the more favorable prognosis. Even though a commons problem may be transformed into a public good, the benefit-cost duality distinction is essential, because the collective action implications differ depending on the problem's current form.

## V. INTERTEMPORAL ASPECTS

To consider the intertemporal aspects, we again return to the symmetric representations of the problems displayed in Sections II and III for two and  $n$  agents, respectively. The repeated plays of the contribution game can be associated with rounds of mutual contributions when the game goes on indefinitely and a tit-for-tat strategy is employed. With such a strategy, one agent's noncooperative response (i.e., failing to contribute) stimulates the other agent(s) to not contribute for future rounds as a punishment to the defector. These punishments end after the defector again begins to contribute. That is, the threat to *stop* contributing can achieve cooperation. In the commons, the threat to *start* grazing one's cattle serves the same role in an analogous tit-for-tat strategy. When distinguishing the working of tit-for-tat for the two problems, threatened inaction is replaced by threatened action. Owing to the benefit-cost duality, the threat of *ending* the public benefits in the contribution game serves the analogous function to the threat of *beginning* the public costs in the commons.

An important difference that can intertemporally distinguish the two problems concerns the extinction problem when a renewal resource is the common property resource. Overexploitation of the commons can therefore carry an intertemporal consequence not generally tied to the provision of pure public goods. This contingency limits the rational application of a tit-for-tat commons strategy of further exploitation owing to the long-term negative consequences that extinction implies. In short, the tit-for-tat strategy may lose its credibility or desirability in the commons. If, for example, an Annex I country emits more than its allotted

amount of GHGs during a given commitment period, then point 3 of Article 13 of the Kyoto Protocol requires that the violator makes up the difference in subsequent periods. Clearly this treaty provision recognizes that punishment à la tit-for-tat would only exacerbate the commons problem. Such a provision is more akin to a *penance* strategy, for which the violator must take an action that simultaneously punishes itself while rewarding all others before all parties can return to a phase of mutual cooperation.<sup>14</sup>

## VI. CONCLUDING REMARKS

Duality is known to turn problems inside out, so that convexity changes, positive signs turn negative or vice versa, and the Nash equilibrium and the Pareto optimum exchanges positions. In this paper, we use benefit-cost duality, previously unexplored, to distinguish the pure public good contribution game from the commons game. Although both problems are PD games in their most stylized form, collective-action implications, nevertheless, differ owing to benefit-cost duality. For the contribution game, the large is exploited by the small, while, for the commons game, the small is exploited by the large. This means that equity-motivated policy to balance burdens among different size agents must be in the opposite directions for the contribution and commons scenarios. When applying selective incentives to promote more optimal results, a third-party must use positive inducements for the contribution game and punishments for the commons game. If incentives are easier to institute than punishments, as agents are more inclined to reward than to punish, inefficiencies in the contribution game may be easier to address.

The underlying technologies that aggregates benefits for the contribution game and costs for the commons game stem from alternative considerations. The aggregator technology for public goods involves how individual contributions relate to the overall level of the good and its

benefits, whereas the degradation technology for the commons indicates how individual use impacts diminishing returns and common costs. These technologies affect the underlying game form for more complex representations of the contribution and commons game. As such, these technologies determine strategic interactions and the design of institutions and corrective policies to circumvent collective-action difficulties.

For policy considerations and institutional design, the profound influence that benefit-cost duality has in distinguishing the contribution and commons problem must be taken into account. The practice of treating the two scenarios the same must be avoided.

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### Footnotes

1. Benefit-cost duality is *implicit* in some of the experimental literature (e.g., Goetze 1994), but this literature has not elucidated the theoretical aspects of this duality.
2. Cornes and Sandler (1996, 540) have shown that by switching the role of the indirect and direct utility functions that findings for the price of private goods can be readily established for the quantity of a pure public good.
3. Olson's exploitation hypothesis indicates that, when a large (wealthy) contributor equates his or her marginal willingness to pay (MWTP) for the public good to the good's marginal cost, he or she will supply much or all of the public good needs for the smaller (poorer) individuals. As a consequence, the burden for the public good is anticipated to be borne by the large, wealthier individual(s). Olson (1965) used the value-laden term, exploitation, which is maintained here in keeping with the literature.
4. A dominant strategy provides an agent with a higher payoff regardless of the other agent's strategy. Not to contribute is dominant, since its associated payoffs of 0 and 5 in the top row are greater than their respective -3 and 2 counterparts in the bottom row.
5. This representation is based, in part, on Bruce (2001, 96-97).
6. The aggregator technology relates to how individual contributions determine the overall level of the pure public good available for consumption. In contrast, the degradation technology indicates how individual uses reduce the productivity of the fixed common resource.
7. See Sandler (1997, 36) for a discussion of chicken in the context of contribution games. In terms of ordinal payoffs, chicken differs from the PD by switching the 1s and 2s.
8. The distinction between the right to exclude and the right to be included is addressed in Macpherson's (1971) study of democracy and property rights. These rights can be related to the commons and the contribution games.

9. This equation follows by inverting  $G^* \geq f(w_i + p\tilde{G}^*, p)$  for  $g_i^* > 0$  and adding  $pg_i^*$  to both sides. Rearranging terms gives [6], where  $w^*(p) = f^{-1}(G^*, p) - pG^*$ .

10. Thus far, the problem assumes a summation technology of aggregation, which is clear from equation [2]. The form of equation [6] and its implications are crucially dependent on the summation technology. For example, exploitation is much less of a concern for weakest link, where matching behavior severely limits or even reverses exploitation.

11. For  $r^2 > r^1$ , we have  $\left| \frac{(r^1 r^2)}{R} \right| \{ \bullet \} > \left| \frac{(r^2 r^1)}{R} \right| \{ \bullet \}'$ , where  $\{ \bullet \}$  and  $\{ \bullet \}'$  denote the difference between the marginal and average product associated with  $r^2$  and  $r^1$ , respectively. This difference in absolute value is greater for the larger firm (producer).

12. The equation for these contours is:

$$\bar{U}^i = U^i(w^j - pg, g + \tilde{G}) = U(g, \tilde{G}; p, w^j),$$

where terms have been defined earlier, except for  $\bar{U}^i$ , which denotes a constant level of utility.

13. In the literature, these selective incentives are often referred to as jointly produced private benefits (Sandler and Hartley 2001).

14. In terms of the PD game, the penance stage has the violator receiving the “sucker’s” payoff and the remaining players receiving the “temptation” payoff. The violator and the other players resume cooperation thereafter. Penance implies a greater level of credibility than tit-for-tat, because the remaining players are not required to undertake a punishment that hurts them.

		<i>Agent B</i>	
		Does not contribute	Contribute
<i>Agent A</i>	Does not contribute	* 0, 0	5, -3
	Contributes	-3, 5	2, 2

a.  $2 \times 2$  Pure public good contribution PD game

		<i>Agent B</i>	
		Does not Graze	Grazes
<i>Agent A</i>	Does not graze	0, 0	-5, 3
	Grazes	3, -5	* -2, -2

b.  $2 \times 2$  Commons PD game

**FIGURE 1: Two-Person Contribution and Commons Games**

		<i>Agent B</i>	
		Does not contribute	Contribute
<i>Agent A</i>	Does not contribute	* $0, 0$	$b_i, b_i - c_i$
	Contributes	$b_i - c_i, b_i$	$2b_i - c_i, 2b_i - c_i$

a. Pure public good contribution game

		<i>Agent B</i>	
		Does not Graze	Grazes
<i>Agent A</i>	Does not graze	$0, 0$	$c_i, b_i - c_i$
	Grazes	$c_i, b_i - c_i$	* $b_i - 2c_i, b_i - 2c_i$

b. Commons game

**FIGURE 2: Benefit/Cost Representation of the Contribution and Commons Games**

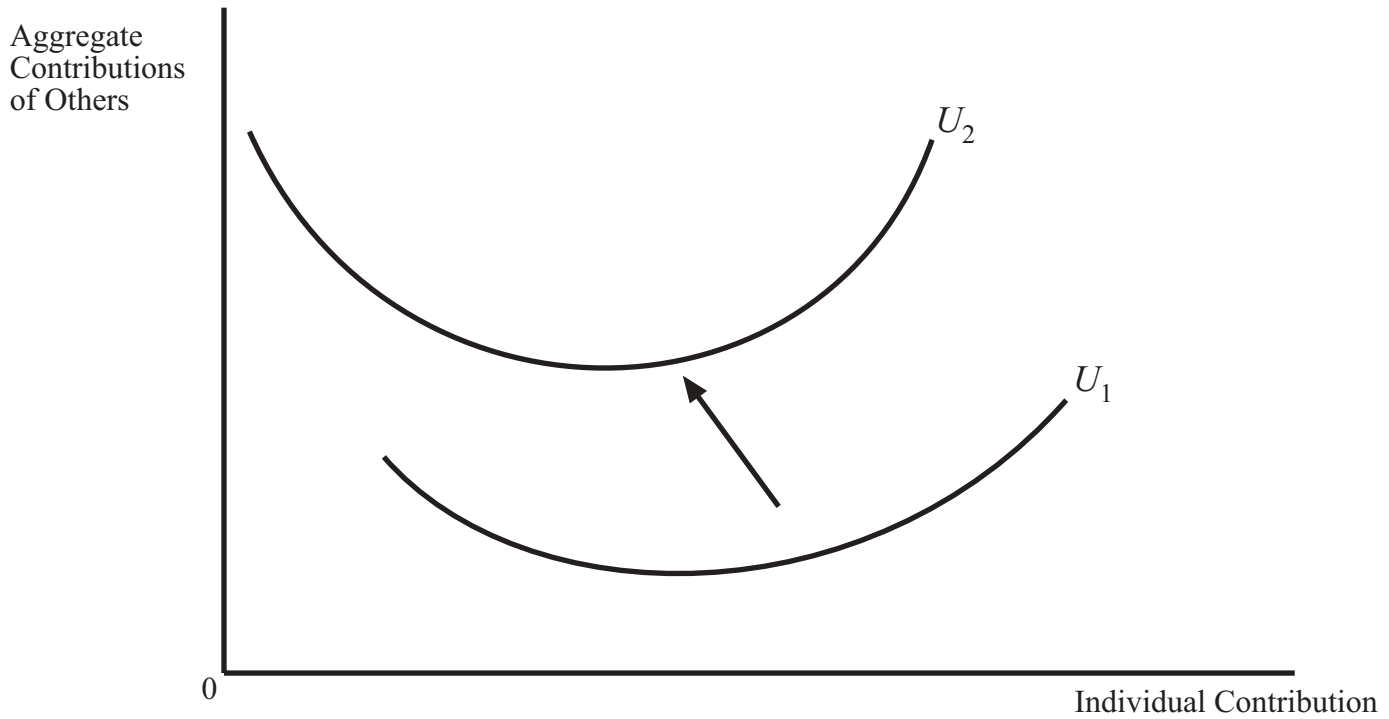
		<i>Number of contributors other than i</i>				
		0	1	2	3	4
→	<i>i</i> does not contribute	<i>Nash</i> 0	5	10	15	20
	<i>i</i> contributes	-3	2	7	12	<i>Pareto Optimum</i> 17

a. Five-person contribution game

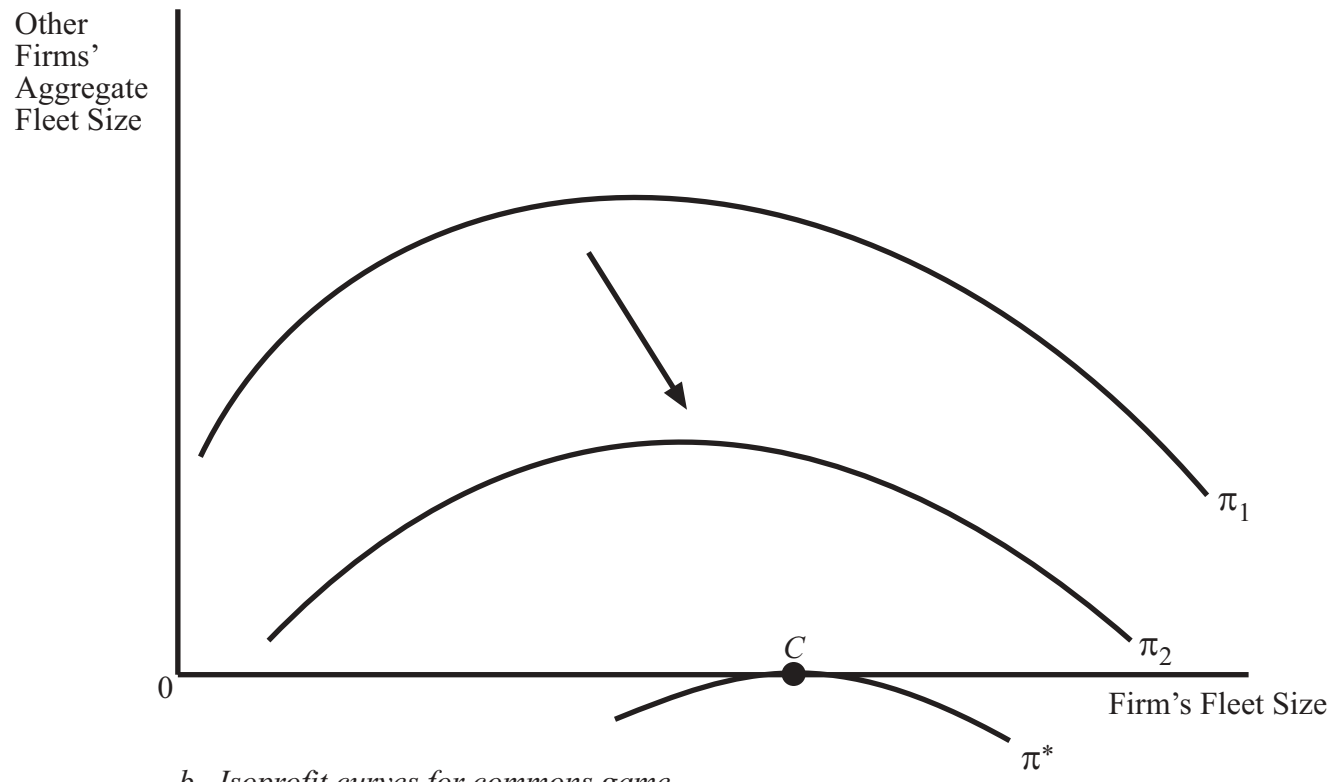
		<i>Number of farmers other than i whose cattle graze in commons</i>				
		0	1	2	3	4
→	<i>i</i> does not graze	<i>Pareto Optimum</i> 0	-5	-10	-15	-20
	<i>i</i> grazes	3	-2	-7	-12	<i>Nash</i> -17

b. Five-person commons game

**FIGURE 3: Five-Person Symmetric Representations**



a. Constrained isoutility curves for contribution game



b. Isoprofit curves for commons game

**FIGURE 4: Isoutility and Isoprofit Curves**