WEAKEST-LINK PUBLIC GOODS:

GIVING IN-KIND OR TRANSFERRING MONEY

by

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Abstract

This paper extends the analysis of weakest-link public goods, whose level equals the smallest of the agents' provision levels, by permitting an agent either to increase one's own provision or else augment both one's own provision and that of the other agent(s). Nash equilibria may be either symmetric with agents matching one another's provision or else asymmetric with an in-kind transfer by one agent to another. An equilibrium with cash transfers, but no in-kind transfers, may be Pareto superior to one with only in-kind transfers. If agents differ in their efficiency, then in-kind transfers by the low-cost agent may dominate a cash transfer. The possibility of Pareto-improving transfers is enhanced as the number of agents increases.

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1. Introduction

In recent years, there has been an increased interest in transnational public goods (TPGs) and their cross-border spillovers. These TPGs involve world health (e.g., the spread of contagious diseases), labor practices, the atmosphere (e.g., global warming, acid rain, ozone-shield depletion), cyberspace (e.g., computer viruses), and security (e.g., transnational terrorism, nuclear weapons proliferation) (see, e.g., Kaul et al., 1999; Sandler, 1997, 1998). The increased prevalence of TPGs can be traced to the impact of new technologies, an enhanced ability to monitor the planet, the fragmentation of nations, and increased population pressures on earth's carrying capacity. Recent advances in the theory of the private provision of public goods have much to say about whether or not nations can independently provide these TPGs, and, in cases where they cannot, this theory can indicate what kinds of policies can achieve greater allocative efficiency (Bergstrom et al., 1986; Cornes and Sandler, 1996, 2000). Recent studies have begun to recognize that cross-border public good spillovers between developing and developed countries imply that foreign aid may take the form of in-kind TPG transfers *or* income transfers so as to improve welfare in both the donor and recipient countries (Jayaraman and Kanbur, 1999; Kanbur et al., 1999). An understanding of the proper choice between in-kind giving of a public good or the transferring of income can enlighten the emerging debate about foreign aid and a host of TPGs. The dispatching of peacekeeping forces to quell civil wars or medical teams to contain deadly diseases underscores that some TPGs are given in-kind by the donor country. In the latter case, the Centers for Disease Control (CDC) provides the public good of stemming the spread of an epidemic *directly* in the host country. Income transfers are not typically used for controlling disease outbreaks.

A key ingredient for analyzing the choice between in-kind giving and income transfers is the "aggregation technology," by which individual contributions to the public good determine the overall level of the public good. Most private provision models have assumed a summation technology whereby the level of the public good is a function of the sum of the individual contributions. Hirshleifer (1983, 1985) broadened the technology of aggregation underlying public goods to include other representations. Two important special cases are weakest-link, where the overall level of the public good equals the smallest of the individual provision amounts, and best-shot, where the overall level of the public good equals the largest of the individual provision amounts. Efforts to limit the diffusion of a pest or to contain a disease are weakest-link public goods, because one's success hinges on the least effort. Similarly, the nation with the smallest efforts at immunization determines the overall chances of eradicating a disease. In contrast, best-shot public goods include discovering a research breakthrough (e.g., a cure for a disease) or neutralizing a rogue nation. The underlying aggregation technology of public supply influences the extent of suboptimality, the multiplicity of equilibria, the effects of income inequality on provision, and the efficacy of income redistribution. For example, Vicary (1990) and Jayaraman and Kanbur (1999) demonstrated that income transfers may be Pareto improving when the public good abides by a weakest-link technology.[1] This result shows us that the neutrality theorem, whereby income redistribution among contributors does not alter the level of a pure public good (Cornes and Sandler, 1996; Warr, 1983), may not necessarily hold for non-summation technologies of aggregation. Thus, an income redistribution can engineer an increase in the public good from a suboptimal level for such aggregation technologies. Weakest-link public goods are crucial TPGs, where income *or* in-kind transfers can be anticipated to play a greater role in the form of foreign aid. For example, rich countries may be dissatisfied with prophylactic measures taken by poor countries to prevent the spread of pathogens and step in to augment these measures directly.

The primary purpose of this paper is to extend the weakest-link model to allow an agent (say a country) to provide its own increments to the public good or to bolster the provision efforts of another agent so as to raise the overall level of the public good. Unlike previous studies, this extension allows for a comparison of in-kind transfers versus giving income transfers. When countries are the agents, it is sometimes convenient to speak of these in-kind transfers as providing the public good to another agent's territory. This analysis not only serves to illuminate real-world policy concerns, such as foreign-aid support of the TPGs, but it also furthers our understanding of weakest-link public goods. Complications arise because our extended weakest-link paradigm is associated with kinked budget constraints and reaction path correspondences, leading to nonconvexities, nondifferentiable corners, and multiple equilibria. As a consequence, we must eschew a mathematical representation in key places of the analysis in favor of a graphical representation.

When one can augment one's own contribution or that of another agent, the Nash equilibria for weakest-link public goods may involve asymmetric equilibria in which one agent makes an in-kind transfer. A weakened form of income-redistribution neutrality characterizes an asymmetric scenario if the income transfer from the larger contributor does not result in a matching equilibrium, in which everyone gives equally to the public good. Starting from an asymmetric equilibrium, a Pareto-improvement can, however, result if the cash transfer is sufficiently large to result in matching behavior and an end to in-kind transfers. The desirability of cash transfers over in-kind transfers is a surprising result that need not hold for cost differences among agents.

The remainder of the paper has four sections. Section 2 contains a representation of the basic model for a Nash equilibrium, complete with reaction path correspondences. In section 3, there is an analysis of alternative transfers for various Nash equilibria. Section 4 indicates an extension of the model to *n* agents. Concluding remarks are presented in section 5.

2. Nash model and reaction paths

In the standard representation of weakest-link, the actual level of the public good G is represented by

$$G = \min\{g_1, g_2, \ldots, g_n\}, \tag{1}$$

where g_i is the contribution of agent i to the public good. Ambiguity arises because it is *not* clear who performs the action that yields g_i – is it agent i or is it some other agent who provides g_i for agent i. For Hirshleifer's (1983) dike building on a circular island, it matters not for the good's provision who builds the dike on i's land. Similarly, other agents can provide g_i when controlling a pest, immunizing against a disease, maintaining defenses along a perimeter, or securing fissile material from dismantled nuclear warheads. In fact, the United States dismantled nuclear warheads for Ukraine to ensure an acceptable level of security. Next consider a weakest-link example where a group of individuals planning to go to some event, wish not to be late, but can only start out when the last person shows up. When the "contribution" is turning up in time, the action must by its very nature be performed by the agent in question. Other similar cases where an agent must contribute his/her own g_i include completing a network or keeping a secret. Despite these exceptions, there are numerous important public goods and TPGs for which agent j can contribute for agent i, and in so doing, agent j can directly influence the effective level of the public good through his/her efforts to augment the smallest contribution. In fact, many of the most interesting instances of weakest-link TPGs permit surrogate contributions which can be in the form of foreign-aid, in-kind transfers.

To allow for a graphical analysis while limiting complexity, we initially assume just two agents, i = 1, 2, whose twice differentiable, strictly quasi-concave utility function U_i is also strictly increasing in the private good x and the public good G:

$$U_i = U_i(x_i, G), \qquad i = 1, 2. \qquad (2)$$

Interesting differences arise, compared with the standard analysis, when depicting the budget constraint for the weakest-link representation with permissible proxy contributions. In this scenario, the conventional budget constraint, with both commodity prices normalized to unity is:

$$w_i = x_i + g^i_{\ i} + g^i_{\ j}, \qquad i, j = 1, 2, \qquad (3)$$

where w_i denotes agent i's income, x_i is his/her consumption of the private good, $g^i_{\ i}$ represents agent i's contribution to the public good on his/her own behalf, and $g^i_{\ j}$ is agent i's contribution to the public good on j's behalf. By adding individual j's own public good contributions, $g^j_{\ j}$, and those to the other agent to both sides of (3), we derive

$$w_i + g^j_{\ j} + g^j_{\ i} = x_i + g_i + g_j, \qquad (4)$$

where g_i or g_j denotes the total contributions made on behalf of i or j, or else to either agent's territory. Given that $G = \min\{g_1, g_2\}$, we can write the formal utility maximization of individual i as follows:

$$\max U_i(x_i, G)$$

subject to:

$$w_i + g^j_{\ i} = x_i + G, \qquad g^j_{\ j} \geq G \geq g^j_{\ i} \qquad (5a)$$

$$w_i - (g^j_{\ j} - g^j_{\ i}) + 2g^j_{\ j} = x_i + 2G, \qquad G \geq \max\{g^j_{\ j}, g^j_{\ i}\} = g^j_{\ j}. \qquad (5b)$$

Some thought is required when interpreting the two segments of the budget constraint in (5a)-(5b) and the g terms on the left-hand side of these equations. For (5a), agent i obtains the quantity $g^j_{\ i}$ as a free contribution by agent j. To increase public good provision beyond this free level, agent i need only contribute extra units of the public good on his/her own behalf up until the sum of i's and j's contribution to i equals g^j_j. In the range of G indicated in (5a), the effective price of G for agent i is unity as this agent only contributes to his/her own territory. When G exceeds g^j_j in (5b), agent i can be thought of as being endowed with g^j_j of the public good after paying a fixed amount of $(g^j_j - g^j_i)$. If agent i is to increase G beyond g^j_j, then i must contribute to both territories and, in so doing, faces an effective price of the public good of 2. As such, the endowed value of spillovers is $2g^j_j$ as displayed on the left-hand side of (5b). The two joined segments of the full-income budget constraint indicate that there are two effective prices for G that each agent may face.

The ith agent's demand function φ_i for the public good, associated with the above maximization problem, is dependent on the relevant effective price of G and the full-income expression on the left-hand side of (5a) or (5b). We write the ith agent's reaction functions as

$$g^i = \varphi_i(1, w_i + g^j_i) - g^j_i, \qquad \text{for } g^j_j \geq g_i = g^i_i\,, \quad (6a)$$

and

$$g^i = 2[\varphi_i(2, w_i - (g^j_j - g^j_i) + 2g^j_j) - g^j_j] + g^j_j - g^j_i = 2\varphi_i(2, w_i + g^j) - g^j, \qquad \text{for } G \geq g^j_j, \quad (6b)$$

where we assume that an agent will never contribute less on his/her own behalf (or territory) than on the other agent's behalf (or territory) so that $g^j_j \geq g^j_i$. In these reaction functions, $g^i (= g^i_i + g^i_j)$ denotes agent i's own contributions plus in-kind transfers. In (6a), agent i needs only to contribute on his/her own account to gain extra units of G beyond g^j_i as long as g_i is less than agent j's contribution on his/her behalf. This latter contribution, as compared with g^j_i, does not affect i's full income, but does ensure that agent i faces a lower price of unity for the public good as agent i only contributes on his/her own account. For the second segment of the budget constraint in (6b), agent i can increase G beyond g^j_j by first bringing contributions to oneself up to g^j_j and then contributing equally for both agents so that the effective price is 2. With G on the vertical axis and x on the horizontal axis, the full-income budget constraint has a kink at g^j_j with a slope of minus a half to the left and minus one to the right. Furthermore, the budget constraint ends at the level of free contributions, g^j_i. These kinks means that care must be exercised when constructing the reaction paths and identifying Nash equilibria.

2.1. Constructing the reaction paths

In constructing the reaction correspondences, we limit the diagram from four dimensions to two by relating the total contributions, regardless of location, to the public good by any agent, g^i, to that of the other agent. The resulting correspondences are still sufficient for analyzing Nash equilibria. The reaction functions in (6a)-(6b) possess the same mathematical form as those of the standard summation case with g^i representing total contributions G. Given that both goods are henceforth assumed to be normal, i's reaction functions are downward sloping with a slope algebraically greater than –1, when the other agent's total contributions are on the horizontal axis. This follows because $0 < P_G\varphi_{iI} < 1$ where I_i is given by the left-hand side of eq. (5a) or (5b) as appropriate and represents i's full income, P_G(= 1,2) is the implied price of G, and φ_{iI} is the partial derivative of i's demand for the public good with respect to the relevant full income. The reaction path for (6a) is above that for (6b) due to the lower implied price of the public good in (6a).

To derive these reaction correspondences, we first examine the budget constrained optimization of an agent where in-kind transfers are taken into account. Figure 1 reproduces (implicitly) the preference map of agent i, together with the (thickened) income expansion paths IEP(1) and IEP(2), corresponding to an effective price for G of P_G = 1, 2, respectively. On the horizontal axis, w_i denotes the maximum consumption of the private good for agent *i* where his/her entire budget is spent on the private good. The line w_iN (also thickened) indicates *i*'s budget line when agent *j* provides for him- or herself as much as *i*'s public good contributions on *i*'s own behalf (territory), while line w_iC′ with slope –0.5 indicates *i*'s budget line if the other agent fails to contribute at all so that *i* has to contribute on both agents' behalf to raise *G*. Consider a small total contribution to the public good by agent *j* that places agent *i* somewhere on the section AD of the income expansion path IEP(2). Agent *j* could equally divide his/her contribution of, say, w_iF between the two territories, so that the budget line facing agent *i* would be BC. Alternatively, *j* could set $g^j_i = 0$, so that the budget line is w_iEC with a kink at point E, where *i* must henceforth contribute for both agents if *G* is to be increased. Up until point E, agent *i* merely matches the contribution of agent *j* along w_iE. Naturally, agent *j* could contribute to both agents' domains, while making a greater donation to his/her own domain. For any small contribution below w_iJ by agent *j*, agent *i*'s optimum is located at a tangency point such as H between his/her indifference curve (not shown) and the relevant budget constraint. Agent *i*'s expansion path IEP(2) with $P_G = 2$ is relevant for these small contributions by *j*, because *i* will have to supplement *j*'s contribution if *G* is to increase. In summary, if, for small contributions, *j* divides equally (w_iB = BF) his/her public good contribution between *i*'s and *j*'s domains, then *i*'s budget line is BC; if, however, *j* contributes w_iF entirely to his/her own territory, then *i*'s budget line will be kinked w_iEC.

Next suppose that agent *j* contributes more than w_iJ, say w_iM, which may allow agent *i* to be on IEP(1) where $P_G = 1$ because *i* may not need to give in-kind to *j*. If agent *j* devotes this entire contribution to his/her territory, agent *i* faces the budget constraint w_iNP, where point L is chosen on income expansion path IEP(1). If, however, agent *j* divides his/her contribution between the two territories, agent *i* then faces the budget constraint KNP in Figure 1 and selects point R on IEP(2). This follows because to increase the provision of *G* beyond the height of K, agent *i* will have to contribute to *both* territories. Other divisions of *j*'s contribution between the two territories can be pictured by selecting an appropriate point, say S, along the line RN, and taking a 45° line TS back to segment w_iM on the right-hand axis, so that w_iT equals g^j_i and w_iU equals g^j_j. Given the location of the income expansion paths, *i*'s indifference curve at point S must have a slope between those of line SP and TS along *i*'s (solid) budget constraint TSP. On this kinked constraint, agent *i* will choose point S, a corner point. Line segment KS is dashed to highlight kinked TSP. If agent *j* increases the public good contribution to his/her own territory at the expense of that to *i*'s territory, the corner point moves towards V and then along line VN, although *i* would then choose a tangency points along VL. Thus, when agent *j* chooses to contribute w_iM to the public good, agent *i* could choose a point on IEP(2), IEP(1), or some intermediate point S.[2] When agent *j* contributes twice w_iW, *i* cannot be on IEP(2) so that the lower bound to *i*'s total contribution is zero. If, similarly, contributions by *j* are twice w_iY, then *i* will never contribute to the public good.

The analysis of Figure 1 can be applied for two agents, denoted by 1(male) and 2(female), to justify the reaction *correspondences* in Figure 2, where $R_i(P_G)$ denotes agent *i*'s reaction paths for a given P_G. $R_i(2)$ is derived from eq. (6b) and $R_i(1)$ from to eq.(6a). Given normal goods, the slopes of these reaction paths for agent 2 (1) will be algebraically greater (less) than –1.[3] Because an agent may end up on either income expansion path or even between the two expansion paths, as shown in Figure 1, the reaction correspondences must include a path for both P_G = 1 and P_G = 2, as well the shaded points in-between these reaction paths. For g^1 in the range zero to J (0J in Figure 2 equals w_iJ in Figure 1), agent 2's g^2 is found on $R_2(2)$ and relates to points on IEP(2) in

Figure 1. Agent 2's aggregate contribution is bounded by $R_2(2)$ and the 45° line for g^1 between J and L in Figure 2 (with 0L being the height of L in Figure 1). If g^1 lies between L and 2W in Figure 2, where 0(2W) is twice w_iW in Figure 1, agent 2 can contribute any g^2 between her two reaction curves. Finally, if g^1 is above 2W, agent 2's contribution is on $R_2(1)$ or in the shaded area below it. Her contribution is clearly zero if g^1 exceeds 2Y, which equals twice w_iY in Figure 1. The Nash reaction correspondences for agent 1 is found analogously and has a slope less than –1 in value, because g^2 is on the vertical axis. These correspondences can now be used to find the set of Nash equilibria.

2.2. Constructing the set of Nash equilibria

In Figure 2, the set of Nash asymmetric equilibria lies on line AB, where the aggregate public good contribution of each agent is a best reply to the aggregate contribution of the other agent. Line AB is defined by the intersection of the two reaction correspondences. Along AB, agent 2 contributes more in aggregate than agent 1, and will always contribute to both territories. Agent 1 contributes to just his own domain, except at point A on $R_1(2)$. For any point between A and B, agent 1's marginal rate of substitution between the public good and the private good (MRS^1_{Gx}) is less than two and it cannot be optimal for agent 1 to contribute for himself and in-kind to agent 2.

Next consider the utility of the two agents along AB. Agent 2's utility increases as the equilibrium moves from A to B, since she gets more public good spillins from agent 1, resulting in a positive income effect. For agent 1, he is located at an intermediate point, such as S in Figure 1. When the agent experiences a fall in spillins, the equilibrium moves to B along AB in Figure 2. Provision of G rises, since the slope of reaction path $R_2(2)$ is less than one, so that point S in Figure 1 will be higher than previously. Although 2's contributions fall, agent 1's contribution to the public good increases by more than this fall. This then implies that the NS section of 1's budget constraint in Figure 1 will be lower owing to reduced spillins, so that he/she is worse off. Hence, the two agents' interests are diametrically opposed along the set of equilibria of AB.

An alternative set of equilibria to that displayed could occur if agent 1's reaction curves were located further to the right, so that the shaded areas for each agent intersect along the 45° line from the origin in Figure 2. These intersection points along the 45° line are the standard weakest-link matching equilibria (Hirshleifer, 1983) where each agent matches the other's contribution and solely on his/her domain. There will be a continuum of such equilibria with a maximum provision of G, where the upper $R_i(1)$ is reached at which point $MRS^{i}_{Gx} = 1$ for some i. A movement upwards toward this maximal equilibrium improves both agents well-being. Given this mutual gain, it is reasonable to assume that this maximal outcome will be the final equilibrium, if the choice of Nash outcomes is solely from matching outcomes. Thus, the set of Nash equilibria may be asymmetric where one agent must supplement the contributions of another, or it may be symmetric with just matching contributions on one's own domain. Moreover, these equilibria may include both asymmetric and symmetric outcome as shown in Section 3.

3. The analysis of transfers

Much work on the private provision of public goods has concerned itself with the neutrality theorem of income transfers (Cornes and Sandler, 1996; Vicary, 1990; Warr, 1983), and this paper is no exception. As there will always be a *set* of Nash equilibria in our model, neutrality is unlikely to involve every equilibrium, so that neutrality is not strict. However, in the restricted sense of being possible, neutrality is a general property as we now demonstrate for points along AB in Figure 2 where agent 2 supports both her own provision of g and that of agent 1. Suppose that money is transferred from agent 2 to agent 1, so that $\Delta w_1 = -\Delta w_2 > 0$. Further suppose that agent 2 reduces $g^2{}_1$ or her in-kind support by the exact amount of the fall in her income ($\Delta w_1 + \Delta g^2{}_1 = 0$). From budget constraint eq. (5a), it follows that as long as the new level of $g^2{}_1$ is positive, as assumed, agent 1's budget constraint is essentially unchanged and the optimal response is for him to select the same level of G as before at the kink on his budget constraint where P_G switches value from 1 to 2. This then requires him to set $\Delta g^1{}_1 = -\Delta g^2{}_1$, so that $\Delta w_1 = \Delta g^1{}_1 = \Delta g^1$. If $\Delta w_2 + \Delta g^1 = 0$, the optimal response for agent 2 is to set $\Delta G = \Delta \varphi_2 = 0$ by eq. (6b), and this is accomplished by cutting g^2 by the precise amount of the rise in 1's contribution. This cut falls solely on her in-kind support of 1's provision of the public good ($\Delta g^2{}_1 + \Delta g^1{}_1 = 0$). Thus, given that one agent adjusts total contributions by the exact amount of the change in income, it is optimal for the other to do the same. Neutrality results as the overall provision of G, each agent's consumption of the private good, and both utilities remain unchanged. In Figure 2, the impact of an income transfer from agent 2 to agent 1 is to shift the set of equilibria AB downwards and rightward along a 45° line (not drawn), since the aggregate expenditure on G is unaffected. Consequently, the set of equilibria moves toward the matching line and, for a sufficiently large transfer, would reach the 45° matching line through the origin.

We can now ask whether voluntary cash transfers could ever be observed in the model. Without cash transfers, there are two types of initial equilibria: (i) asymmetric equilibria where one agent contributes towards the public good in both territories; and (ii) symmetric equilibria where agents contribute just to their own domain, while matching the other agent's contributions.

In the first, we have seen that the transfer from the greater contributor (agent 2) takes the economy towards the matching line. Agent 2 must ensure that the post-transfer set of Nash equilibria includes some matching outcomes, otherwise nothing will have been gained owing to neutrality. We take the neutral outcome to be focal unless a Pareto-superior Nash equilibrium results from the transfer.

Figure 3 illustrates what might happen for the first case. The economy is initially at E, with Nash equilibria along segment AB. Reaction functions are now denoted by $R^k_i(P_G)$ where superscript k refers to the initial pre-transfer (a) or final post-transfer (b) state. Suppose that agent 2 transfers income to agent 1 in such a way that point E moves onto the 45° line at C. As a result of the transfer, the set of Nash equilibria is now A′CD. Agent 2 can gain from this transfer since along segment CD agents′ interests are complementary, analogous to the conventional weakest-link case (Hirshleifer, 1983). Thus, both agents are motivated to move to outcome D, where $R^b_1(1)$ intersects the 45° matching line, so that there is not only an incentive to give but also to receive. At D, both agents are only contributing to their own territory at a level greater than C – there are no in-kind transfers. Given that both agents are indifferent as between E and C, and both prefer D to C, the cash transfer can achieve a Pareto improvement.[4]

In the second kind of initial equilibrium, the pre-transfer equilibrium starts on the matching line where there are no in-kind contributions. The equilibrium will be at the maximal matching contribution corresponding to the lower of the two agents’ reaction paths for $P_G = 1$. We shall show next that there can be no gains in the two-agent case from transfers once such a maximal match is reached.

Having established that agent 2 can gain from a transfer of money to agent 1, our next task is to establish the nature of the trade-off that she faces and to characterize the optimal transfer from her point of view. Again consider Figure 3 where a transfer can alter the set of Nash equilibria from AEB to A′CD, with D as the final maximal matching equilibrium. At the post-transfer equilibrium D, agent 2 would like to move to an equilibrium involving a higher matching rate, but agent 1 limits this move, given the position of his post-transfer reaction path for $P_G = 1$. By increasing transfers beyond the level implied by A′CD, agent 2 can increase the provision of G as point D moves up the matching line, but it is never in her interest to do so. This follows by realizing that raising provision by ΔG makes two demands on agent 2. First, she must increase agent 1's income to induce him to increase provision by this amount. As $\Delta G = \varphi_{1I}\Delta T$, where ΔT represents the required extra transfer, this involves agent 2 sacrificing an amount of the private good equal to $\Delta G/\varphi_{1I}$. Second, she must sacrifice an amount of x equal to ΔG, the amount she must contribute to her own territory to ensure the increased overall provision that agent 1 now desires. In total, agent 2 must sacrifice $\Delta G/\varphi_{1I} + \Delta G$ units of x, so that the trade-off between the two commodities that she faces in making an increased transfer is: $\Delta x_2/\Delta G = (\varphi_{1I} + 1)/\varphi_{1I} > 2$. Given the possibility of contributing extra units to both territories, we, however, know that $MRS^{2}_{Gx} \leq 2$ always holds, so that the additional transfer makes her worse off. Indeed, this suggests that agent 2 should in fact reduce the transfers for the case illustrated in Figure 3. As this argument can be repeated for any such point as D, suggesting that transfers should always be decreased, we appear to have a paradox in that we also know that transfers can produce a Pareto improvement.

The resolution of this paradox lies in the nature of the trade-off involved. As found by Buchholz et al. (1997) in a different context, the trade-off that agent 2 faces when transferring is "discontinuous" at a critical point.[5] If, in Figure 3, the initial equilibrium is at point E, the recipient of an intended transfer (agent 1) will not move from this point unless he does not lose.

To simplify our reasoning, we assume that in a case of any agent's indifference as between an asymmetric equilibrium and a matching equilibrium, the latter is focal. The argument for understanding the limits to transfers is illustrated in Figure 4, which reproduces Figure 1 for agent 1. Initially, agent 1's income is $0w_1$, and his optimum is at E on budget line KEL. At this equilibrium, agent 2 provides w_1K of the public good to 1's territory and $0G_n$ units to her own territory, where $0G_n$ is the initial provision of G. Suppose that agent 2 now transfers w_1w_1' to agent 1, which will turn out to be the critical level of the transfer. There are two possible outcomes to focus upon: the neutral outcome for which 1's optimum remains at E, with $g^2{}_1$ falling by the exact amount of the rise in 1's income (budget line MEL); alternatively, the matching equilibrium at D, for which neither agent contributes to the other's territory. This matching equilibrium delivers a higher quantity of the public good. Agent 1 is indifferent as between points E and D, but agent 2 prefers D. Since D is assumed focal, the resulting transfer in Figure 4 is Pareto improving – the transferrer gains while the transferee does not lose. In summary, agent 2 will want to transfer the amount w_1w_1': anything less produces neutrality and no one gains, and anything greater causes agent 1's utility to rise at the expense of agent 2. The transfer illustrated is the optimum from the transferrer's point of view.

This result is a central one in our analysis, and is in some ways rather curious. Because there is no altruism in the model, the sole purpose of agent 2 in giving money to agent 1 is to engineer an increase in the provision in the public good. Given that agent 2 enjoys no cost advantage over agent 1, one must wonder why she transfers resources to get him to increase his own contributions, when these contributions can be made directly. The explanation lies in the fact that there is a relative price effect in operation. With a matching equilibrium, both agents face a price of the public good of unity up to the amount provided; however, this is not the case at any initial asymmetric Nash equilibrium along AB in Figure 2. At these equilibria, agent 2's price of increasing provision of the public good is two. By transferring resources to agent 1, agent 2 can effectively (at the initial G level) lower this price to agent 1, and given agent 2's MRS_{Gx} of 2, she is willing to match any increase in contributions that the recipient might want to make as a result. Because of neutrality, it costs agent 2 nothing to get agent 1 to the stage where he is willing, owing to the effective fall in P_G, to increase contributions.[6]

In the case of Nash behavior and no cost advantage, equilibria involving in-kind transfers of a weakest-link public good by one agent to another can be Pareto dominated by a cash transfer, leading each agent to provide his/her own public good while matching the other agent's public good amount. Thus, in the case of a weakest-link public good, such as containing the spread of disease, our model indicates that the richer country is better off giving general aid rather than going in and providing the good itself in the poorer country. However, efforts by the CDC do not fit this model's policy recommendation, since CDC does provide the good directly. To explain why in-kind transfers exist in such real-world situations, we must now allow for cost differences.

3.1 Cost differences

There are two ways to abandon the assumption of identical costs. First, the unit price of the weakest-link public good may be cheaper in one *location* regardless of the contributor. In the case of dike building, there may be features of the terrain that makes it cheaper to build sea defenses on some parts of the coast than on others. Second, the unit price of the public good may vary according to the agent making the contribution, regardless of the recipient's location – e.g., the United States may possess technology for controlling a disease superior to that of other countries.

With location-based cost differences and two agents, we assume that the (constant) unit cost of providing G in agent 1's territory is p, compared with unity in 2's territory. Figure 2 can be reconstructed along the same lines, except for the labelling of reaction paths. An agent contributing to both territories faces an implied price of $1 + p$ for G, so that the inner reaction curve is a function of $1 + p$ rather than 2. The outer reaction curve for agent 1 is now a function of p, while agent 2's outer reaction curve remains a function of unity. The resulting picture and background scenario is the same as Figure 2 with its asymmetric set of Nash equilibria.

While maintaining that agent 2 contributes more in the initial set of Nash equilibria, we suppose that she makes a cash transfer to agent 1 of $\Delta w_1 = -\Delta w_2$, while reducing her in-kind transfers to him by $-p\Delta g^2{}_1 = -\Delta w_2$. Throughout, transfers are small, so that $g^2{}_1 > 0$ and $g^2{}_2 > g^1{}_1$. The budget constraint in eq. (5a) for $i = 1$ and $j = 2$ is

$$w_1 + pg^2{}_1 = x_1 + pG, \qquad g_2 \geq G \geq g^2{}_1, \quad (5a')$$

so that $\Delta w_1 + p\Delta g^2{}_1 = 0$. Insofar as 1's full income remains unchanged, he chooses the same G by using his extra income to offset the fall in in-kind spillins from agent 2.

For agent 2, eq. (5b) is

$$w_2 + pg^1{}_1 = x_2 + (1 + p)G, \qquad G \geq g^1{}_1. \quad (5b')$$

Because agent 1 only contributes to his own territory in contrast to agent 2, 1's contributions to his own domain acts like a gift of $pg^1{}_1$ to agent 2. It follows that $w_2 + p\Delta g^1{}_1 = 0$, so that 2's full income is unchanged and she will maintain G's provision at its original level. Reducing in-kind transfers by the fall in her income is the best response for agent 2 in light of 1's actions. Both agents maintain the consumption of the private good, and neutrality results.

As with identical costs, a cash transfer from agent 2 to 1 moves the set of equilibria along a 45° line in Figure 2 towards the line of matching allocations, which will be reached for sufficiently large transfers. Thus, a Pareto improvement can be achieved by moving to a maximal matching equilibrium. As before, there is no incentive for agent 2 to make further cash transfers once the economy is at this maximal match.[7] In all essential aspects, our analysis extends to location-based cost differences.

We next consider agent-based cost differences, when one agent sacrifices a lower quantity of the private good than another when providing a unit of the public good. The unit price of G is p for agent 1 regardless of where he provides the good, while it is unity for agent 2. Agent 2's reaction correspondence is unchanged in Figure 2, whereas the lower path for agent 1 has a price of $2p$ and the higher path has a price of p. Once again, AB denotes the pre-transfer equilibria.

To investigate the impact of cash transfers from 2 to 1, we employ the reasoning of Buchholz and Konrad (1995) and add together the relevant budget constraints of the two agents:

$$w_1 + w_2 = x_1 + x_2 + pg^1{}_1 + g^2{}_2 + g^2{}_1 = x_1 + x_2 + 2pG + g^2{}_1(1 - p), \tag{7}$$

where we used the fact that $g^2{}_2 = g^2{}_1 + g^1{}_1 = G$ at the relevant equilibria. Now suppose that a cash transfer leaves the provision of G unchanged and consider just the equilibria where agent 2 continues to make in-kind transfers. As the relative price she faces does not change, we require $\Delta G = 0$ and $\Delta x_2 = 0$, so that it follows that $\Delta g^2{}_1 < 0$. From (7), if $p < 1$, then agent 1's consumption of the private good must rise and he is better off, so that there is a Pareto-improving Nash equilibrium available to the community. If this equilibrium were focal, agent 2 would be willing to transfer income to agent 1 until a matching equilibrium occurs, at which point further Pareto improvement can be achieved by reaching the maximal matching equilibrium.

When, however, $p > 1$, eq. (7) indicates that agent 1's consumption of x must fall with the cash transfer, making him worse off. Agent 1 might be made better off by moving to a new Nash equilibrium towards the equivalent of point A in Figure 2, but this has the effect of lowering 2's welfare.[8] For $p > 1$, neither Pareto-improving nor Pareto-indifferent cash transfers are possible for an asymmetric equilibrium. This does not mean that it is never in 2's interests to make a cash transfer when at an asymmetric equilibrium. If, for example, costs do not differ by much and the economy begins near a matching equilibrium, then such a transfer may be beneficial. When costs differ substantially, this is likely impossible, and the economy will remain at an asymmetric equilibrium with in-kind transfers from a low-cost producer to the high-cost producer. Thus, actions by the CDC and US peacekeepers have a theoretical basis.

4. Symmetric equilibrium with many agents

We now consider transfers in a world of n (> 2) agents and the absence of cost differences, while focusing on the issue of transfers between agents when allocation is determined by a Nash process. A complete characterization of equilibria in such a model would be tedious. For purposes here, it is sufficient to demonstrate that most of the main features of the two-agent model carry through to n agents. With n agents as with two, we must first determine whether there is a set of neutral transfers that produce a matching outcome. If this is the case, we must then ascertain whether further Pareto-improving transfers are feasible after an initial maximal matching equilibrium has been realized.

We first consider transfers when agents contribute in-kind to others. Each agent solves the following n-agent generalization of the two-agent problem:

Max $U_i(x_i, G)$

subject to:

$$w_i + \sum_{k \neq i}^{n} \sum_{j=1}^{n} g_j^k = x_i + \sum_{k=1}^{n} \sum_{j=1}^{n} g_j^k. \tag{8}$$

Non-uniqueness again means that there is no true neutrality result, so that only the *possibility* of neutrality must be examined. Assume that we start at an initial Nash equilibrium, in which all agents contribute something to the public good. Furthermore, the contribution for oneself is again maintained to be less than in-kind transfers to others ($0 < g^i_i \geq g^i_j$, for all i, j with $i \neq j$). In this case, the second right-hand side term in eq. (8) equals nG^*, where G^* represents the initial provision of the public good, as no contributions will be wasted by exceeding the minimal one required. If neutrality is to hold after the transfer, the provision of G must remain the same.

Consider the set of transfers given by

$$T_i = \sum_{k \neq i}^{n} \sum_{j=1}^{n} g^k_j - (n-1)G^*, \tag{9}$$

where T_i represents the total amount of transfers to agent i. Suppose that these transfers are put in effect. Since T_i summed over all agents equals zero, these transfers are clearly feasible. The initial equilibrium provision, G^*, is also feasible, with all agents contributing exactly G^* on their own territory and nowhere else, as follows from inspection of the budget constraint. Effectively, agents abandon in-kind transfers, and make lump-sum transfers of equal value to appropriate recipients. As all agents initially contributed a nonzero amount to the public good, we know that $n \geq MRS^i_{Gx} \geq 1$ holds originally for all agents, where the first inequality follows given that at the initial equilibrium every agent has the option to contribute to all territories, and the second inequality follows from the nonzero contribution assumption. With such a matching equilibrium, the consumption of the private good by all agents remains the same along with their utility. Given the responses of others, the utility-maximizing nature of this response for each individual follows by noting that each agent is at the *kink* of his/her budget constraint with slopes of minus unity and $-n^{-1}$ on either side (with G on the vertical axis). No agent would, thus, wish to move from this outcome given the initial MRS value. If, however, the initial equilibrium is not a matching one, it is then always possible (as in the two-agent case) to achieve through transfers a matching equilibrium, equivalent in all relevant respects. Analogous to the two-agent case, transfers can remove the economy from asymmetric equilibria, where one or more agents contribute in-kind to the others' territories when there is no cost advantage to supplying the weakest-link public good.

We next assume that a matching equilibrium has been reached, and inquire whether any further transfers might be observed. As with two agents, the economy can achieve a maximal matching equilibrium, characterized by the minimal MRS^i_{Gx} across all agents being unity. If this were not the case, then individuals would have incentives to raise contributions on their own territory until it is true. As shown earlier with two agents, no further transfer can take place once this position is attained, because of the upper bound of 2 on the *MRS* for the agent who might be interested in increasing provision. However, with *n* agents the upper bound on the MRS is now *n*, and so further transfers are now feasible. The condition for a Pareto-improving transfer to take place is[9]

$$\max_i \{MRS^i_{Gx}\} > 1 + \sum_{j \in R} (\varphi_{jI})^{-1} \tag{10}$$

where $R = \{i \mid MRS^i_{Gx} = 1\}$ denotes the set of minimal demanders. With as few as three agents, inequality (10) can be met. This can be seen by imagining the maximal *MRS* to be three, with just a single individual in set *R* whose marginal propensity to consume the public good exceeds 0.5. With many agents, and only one or two individuals with an *MRS* of unity, the presumption that transfers might take place increases.[10] Naturally, there may exist a large number of individuals interested in transferring to increase provision of *G*, in which case the transfers themselves take on the character of a summation public good. With the exceptions of the bounds on the *MRS* resulting from potential in-kind transfers, the problem corresponds to that of Vicary (1990), and the reader is referred there for further discussion.

5. Concluding remarks

This paper extends the analysis of weakest-link public goods to situations where an agent can both increase one's own provision of the public good *and* provide in-kind transfers of the good to the other agent(s). In the two-agent case, this can lead to two kinds of Nash equilibria: a set of asymmetric equilibria where one of the two agents provides in-kind transfers to the other; and a set of symmetric equilibria where each agent matches the other's provision on one's own territory. The appearance of these asymmetric equilibria distinguishes our model from the standard representation of weakest-link public good for which Nash equilibria are symmetric. Starting from an asymmetric Nash equilibrium and no cost differences, a Pareto improvement can occur if a cash transfer from the agent originally making in-kind transfers results in matching behavior. This follows because the cash transfer effectively lowers the price of the public good leading to a provision increase and welfare gains for both agents. If, however, the cash transfer is insufficient to result in matching behavior, then neutrality applies and the equilibrium is unchanged. In the case of a symmetric equilibrium, as the number of agents increases, the possibility of Pareto-improving transfers becomes more likely.

When cost differences between locations are allowed, the results are quite similar, provided that the price of donating to an agent's territory is independent of who contributes. If, however, *costs differ as between agents*, then *in-kind transfers* from the low-cost agent to the high-cost agent can be welfare-improving. This outcome is consistent with the way that the United States and other low-cost providers have transferred medical aid and other public goods with in-kind transfers (Sandler, 1998).

A logical extension is to investigate Stackelberg leader-follower behavior. A cash transfer from a follower (agent 2) to a leader (agent 1) can, under some circumstances, be Pareto improving. Following the cash transfer, either the leader-recipient will be the sole contributor to the public good or there will be a matching equilibrium. As in the Nash case, cash transfers may influence the effective price of weakest-link public goods, thus justifying such transfers for an initial asymmetric equilibrium.[11] Unlike the Nash case, in-kind contributions may persist with cash transfers; the analysis is rather complex and is left for our future research.

Another consideration involves how the in-kind option will affect the Nash equilibrium for a best-shot public good with two agents. With best-shot, the Nash equilibrium involves provision by just the agent with the greatest home-territory provision, because smaller provision levels on the other's territory is unproductive. Thus, there is no rationale to provide the good on both territories. Cash transfers and/or in-kind transfers from a nonprovider may improve welfare if such transfers are solely directed to the best-shot agent. The identity of the best-shot agent hinges on income and relative cost considerations.

Notes

*The authors have profited from the comments of two anonymous reviewers and Francois Bourguignon. Sandler is the Robert R. and Katheryn A. Dockson Professor of International Relations and Economics.

1. For a summation technology, Buchholz and Konrad (1995) and Konrad and Lommerud (1995) showed that, when differences in productive efficiency exists among contributors, an income redistribution to the more efficient producer can be Pareto improving.

2. Should agent *j*'s contribution lie between w_iJ and the height of point L in Figure 1, the left-most kink in *i*'s budget constraint will now lie on DL. Agent *i* will locate at the kink, so that each agent contributes only to his/her territory.

3. A mathematical demonstration of these limits on the slope of the reaction paths is available from the authors upon request.

4. This improvement is generally possible unless the cash transfer removes the equilibrium from point B to B′, and B′ is where $R^b{}_1(1)$ (not drawn) cuts the 45° matching line. In this case, B is equivalent to B′, which is already at the maximal matching equilibrium so that nothing further can be gained.

5. Our model has some parallels with that of Buchholz et al. (1997) which analyzed a Stackelberg equilibrium with a summation public good. Transfers come about in their paper due to a discontinuity in the relationship between Nash equilibria and the distribution of income, with one equilibrium Pareto superior to another at the point of discontinuity. Our model has a correspondence between Nash equilibria and the distribution of income, with at some point a change in the nature of the equilibria (to a matching type) enabling a Pareto improvement to take place.

6. At equilibrium B on Figure 2, $MRS^1_{Gx} = 1$. Presented with a price of unity for G, agent 1 would not want to increase his provision. This provides an intuitive explanation as to why no transfers take place in this case.

7. The condition for no further transfers is $MRS^2_{GX} < [(\varphi_{1I} + 1)/ \varphi_{1I}]$. As the left-hand side is $1 + p$, this inequality still holds owing to normality where $(\varphi_{1I})^{-1} > p$.

8. Along AB, the two agents' interests are diametrically opposed.

9. The right-hand side of this inequality is the same as found in Vicary (1990, p. 381), and is justified by the same argument.

10. The increased possibilities to make Pareto-optimal transfers afforded by more than two agents is akin to findings in Cornes and Sandler (2000) for a summation technology of aggregation.

11. Some of this leader-follower analysis is in an earlier draft, available from the authors.

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