Analog Sinusoidal Modulation

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio

Table of Easy FT Properties

Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j(\frac{\omega}{a}))$$

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

Convolution Property

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

Modulation Property

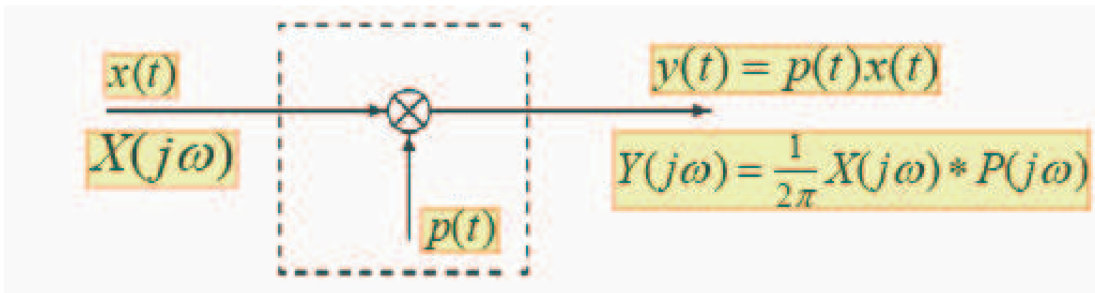
$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Modulation

- Signal Multiplier

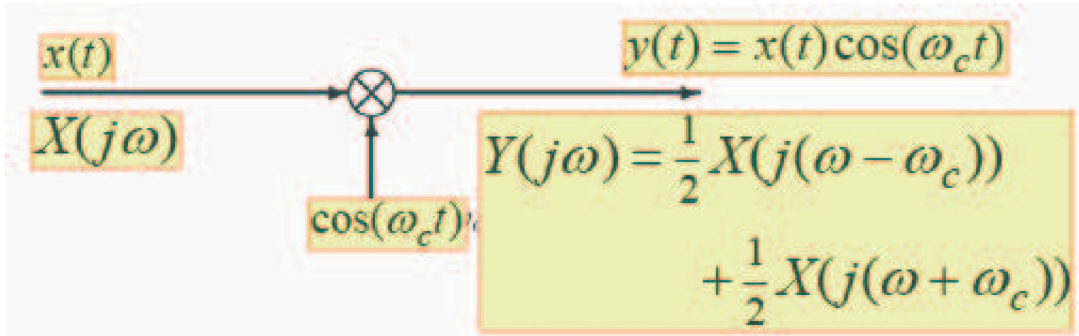


- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta) P(j(\omega - \theta)) d\theta$$

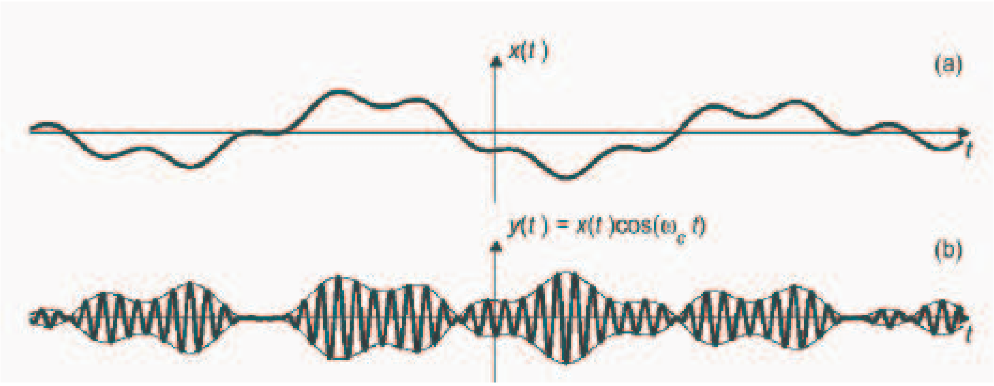
Amplitude Modulation



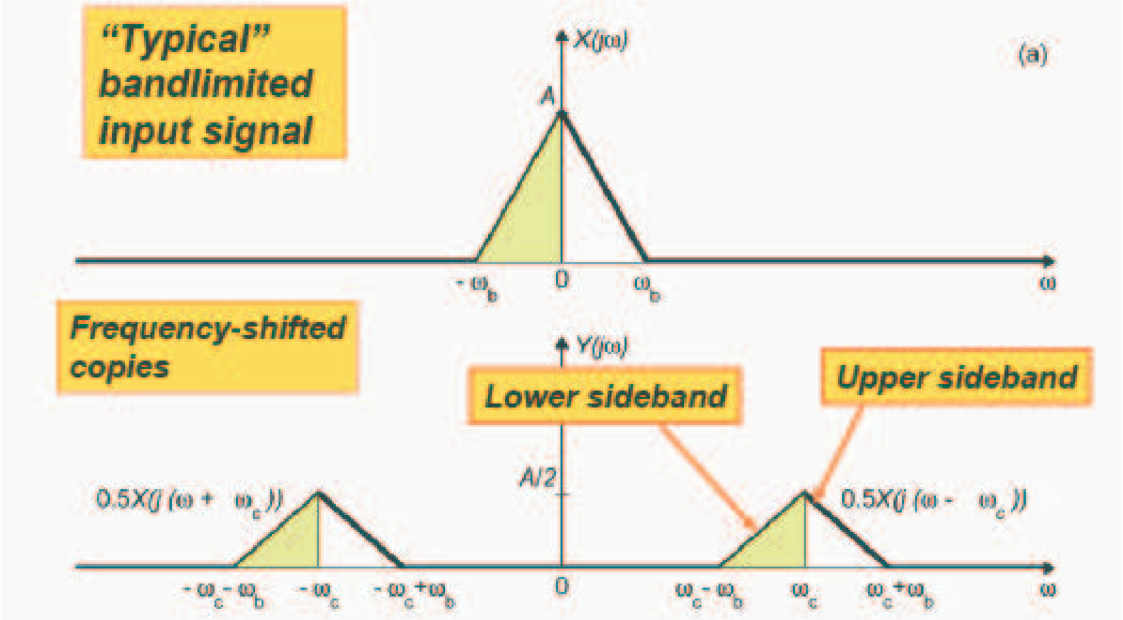
- *x(t)* modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of *X(jω).*
- Double Sideband AM Modulator (DSBAM)
  - If *X(jω)*=0 for $|\omega|>\omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled **exact copies** of *X(jω).*

DSBAM Waveform

- In the time-domain, the "envelope" of sinewave peaks follows $|x(t)|$

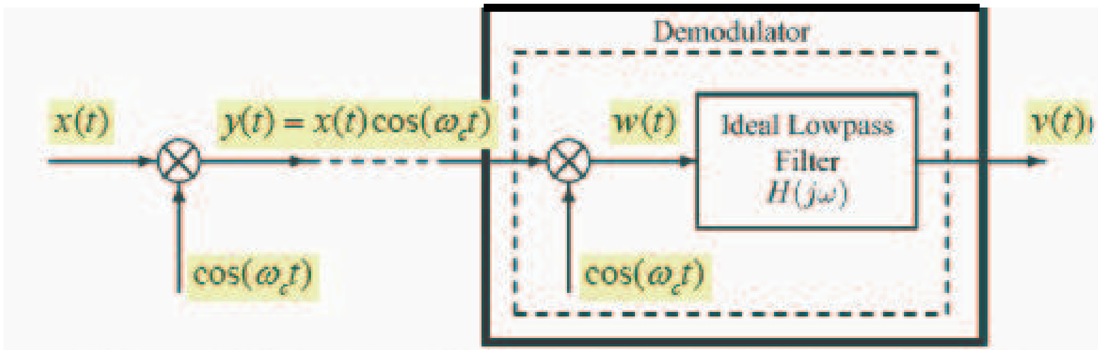


- Frequency domain definitions



DSBAM Demodulator

Time domain

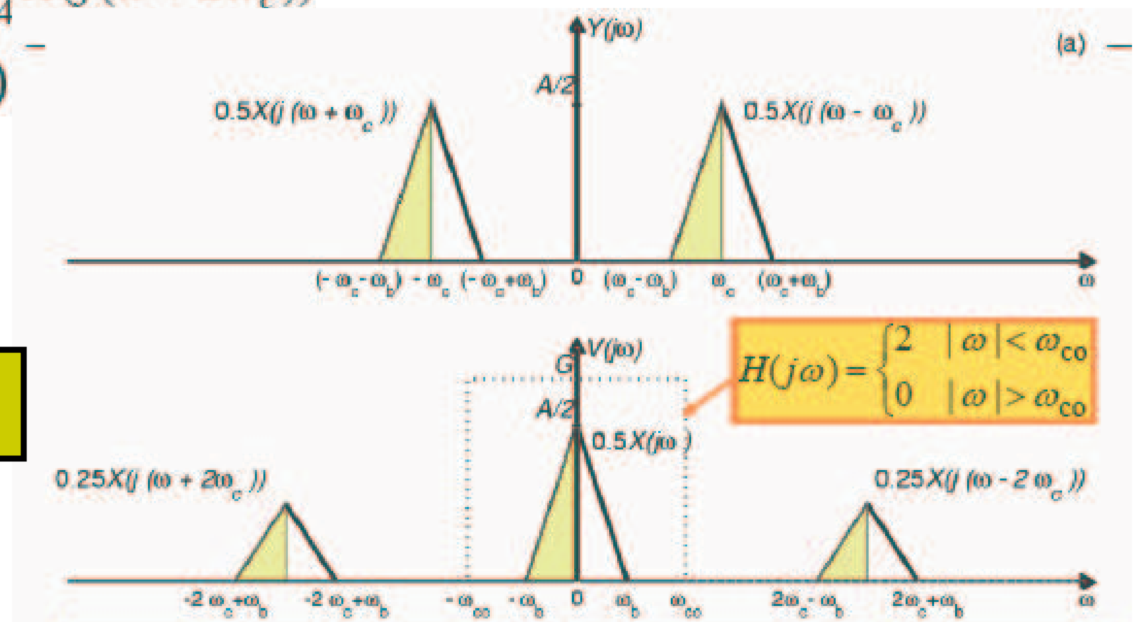


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

Frequency domain



$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$