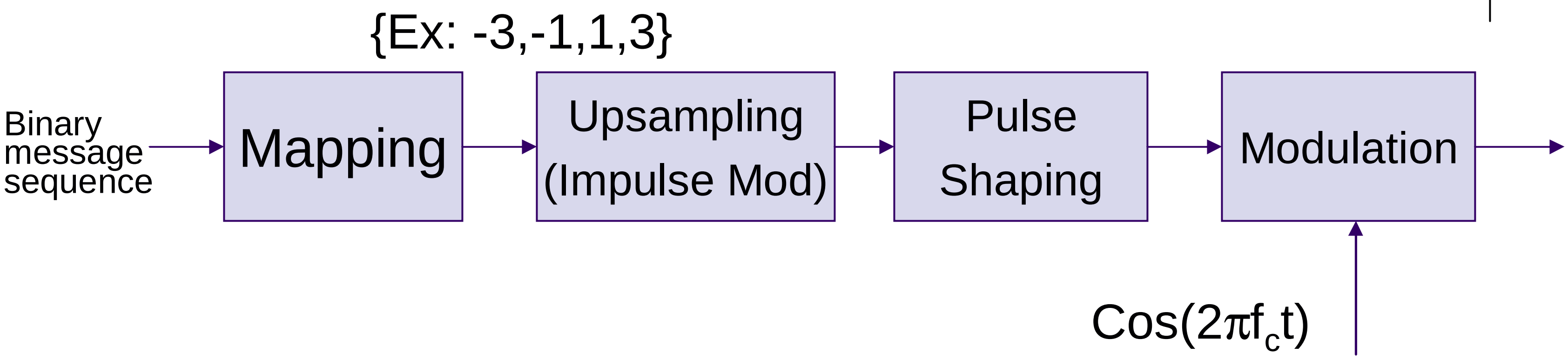
PAM Communication System



- Modulate $M = 2^J$ discrete messages or J bits of information into amplitude of signal
- If amplitude mapping changes at symbol rate of f_{sym} then bit rate is $R_b = Jf_{sym}$
- Conventional mapping of discrete messages to M uniformly space amplitudes

$$a_i = d(2i - 1)$$

$$i = -\frac{M}{2} + 1, \ldots, 0, \ldots, \frac{M}{2}$$

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \, \delta(t - k \, T_{sym})$$

No pulses overlap in time: requires infinite bandwidth

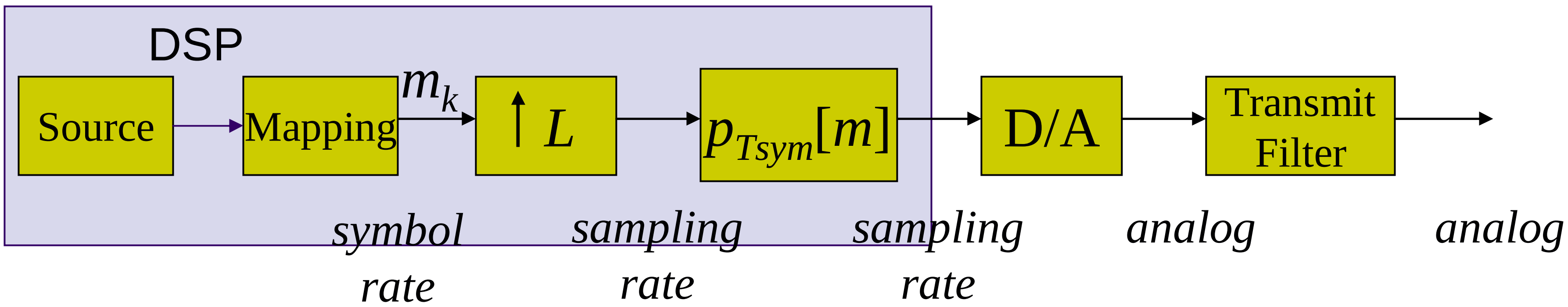
Impulse modulator

- Represent the symbol sequence by the Dirac impulse train

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s)$$

- The impulse modulator block forms this function. This impulse train is applied to a transmit pulse shaping filter so that the signal is band limited to the channel bandwidth.

Pulse Shaping Block Diagram



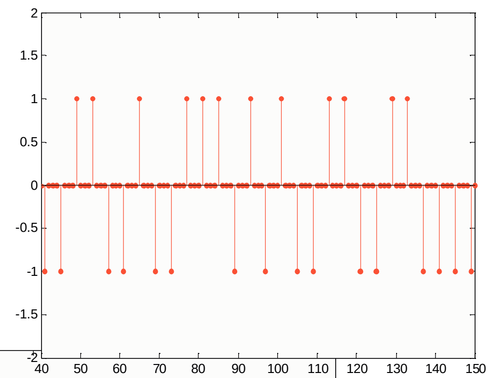
- Upsampling by L denoted as $\uparrow L$
  - Outputs input sample followed by L-1 zeros
  - Upsampling by converts symbol rate to sampling rate
- Pulse shaping (FIR) filter $p_{Tsym}[m]$
  - Fills in zero values generated by upsampler
  - Multiplies by zero most of time (L-1 out of every L times)

DSP Implementation

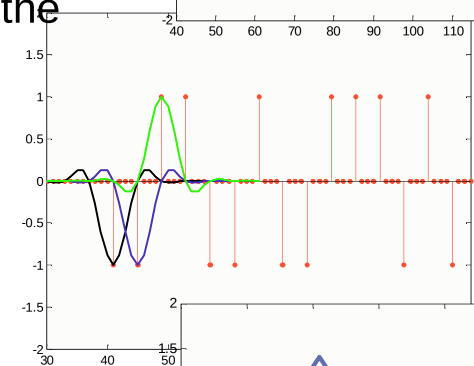
- Random bit generation

…001100100111010….

- Mapping bits onto symbols, 1→1, 0→-1

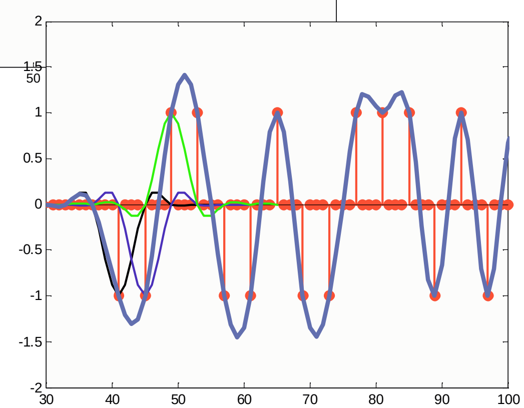


- Upsampling to match the sampling rate



- Pulse shaping filter

- Send the output samples through serial port D/A



Source — *Bit rate*

Mapping — *symbol rate*

$L \uparrow$ — *sampling rate*

$p_{Tsym}[m]$ — *sampling rate*

D/A — *analog*

Intersymbol Interference

- If the analog pulse is wider than the time between adjacent symbols, the outputs from adjacent symbols may overlap
  - A problem called intersymbol interference (ISI)
- What kind of pulses minimize the ISI?
- Choose a shape that is one at time kT and zero at mT for all $m \neq k$
- Then, the analog waveform contains only the value from the desired input symbol and no interference from other nearby input symbols.
- These are called *Nyquist Pulses*

Nyquist Pulses

- Sinc Pulse

$$p_S(t) = \frac{\sin \pi f_0 t}{\pi f_0 t}$$

  - where $f0 = 1/T$. Sinc is Nyquist pulse because $p_S(0) = 1$ and $p_S(kT) = \sin(\pi k)/\pi k = 0$.
  - Sinc envelope decays at $1/t$.
- Raised-cosine pulse:

$$p_{RC}(t) = 2f_0 \left(\frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right) \left[\frac{\cos(2\pi f_\Delta t)}{1 - (4 f_\Delta t)^2} \right]$$

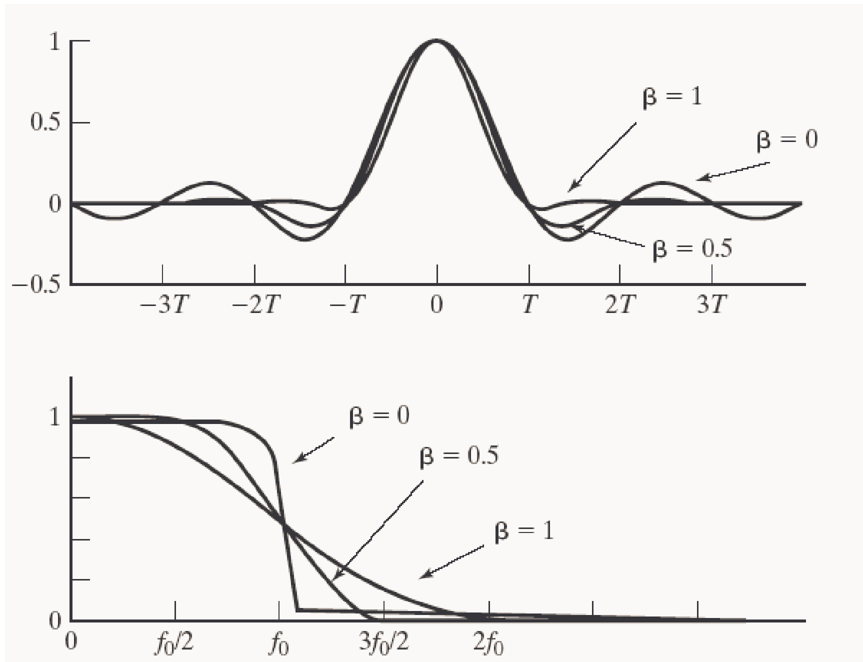
  - with roll-off factor $\beta = f_\Delta / f_0$.
  - $T = 1/2f_0$ because p_{RC} has a sinc factor
  - $\sin(\pi k)/\pi k$ which is zero for all nonzero integers k.
  - Raised-cosine envelope decays at $1/|t^3|$.
  - As $\beta \to 0$, raised-cosine $\to$ sinc.

Frequency Domain

- Fourier transform

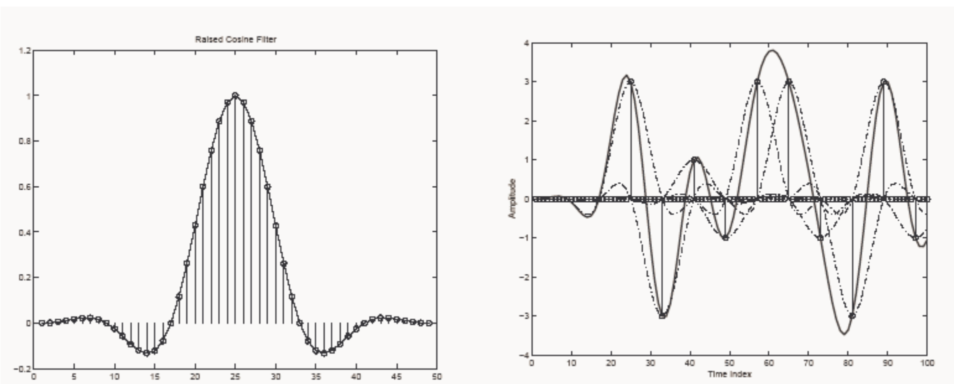
$$P_{RC}(f) = \begin{cases} 1, & |f| < f_1 \\ \frac{1+\cos(\alpha)}{2}, & f_1 < |f| < B \\ 0, & |f| > B \end{cases}$$

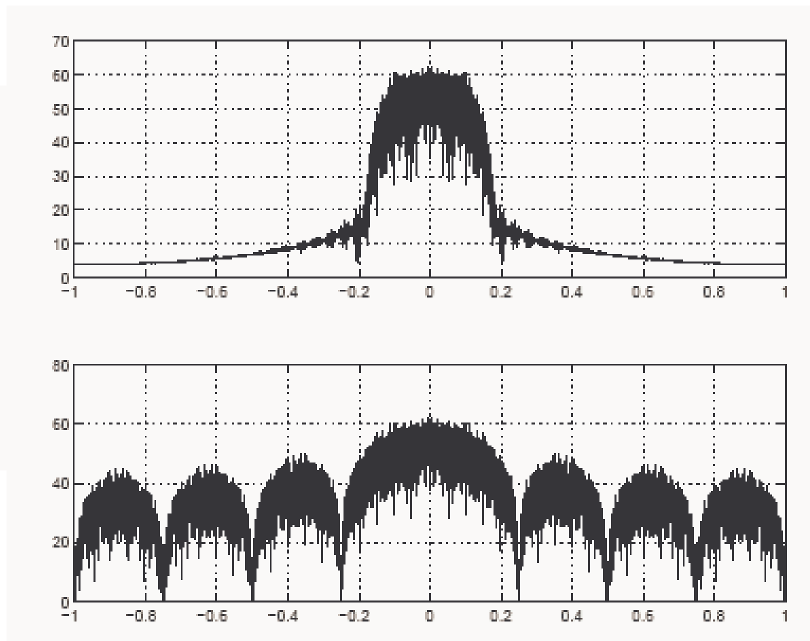
- where
  - B is the absolute bandwidth,
  - f_0 is the 6db bandwidth,
  - $f_\Delta = B - f_0$,
  - $f_1 = f_0 - f_\Delta$, and
  - $\alpha = \pi(|f| - f_1)/2f_\Delta$



Spectrum

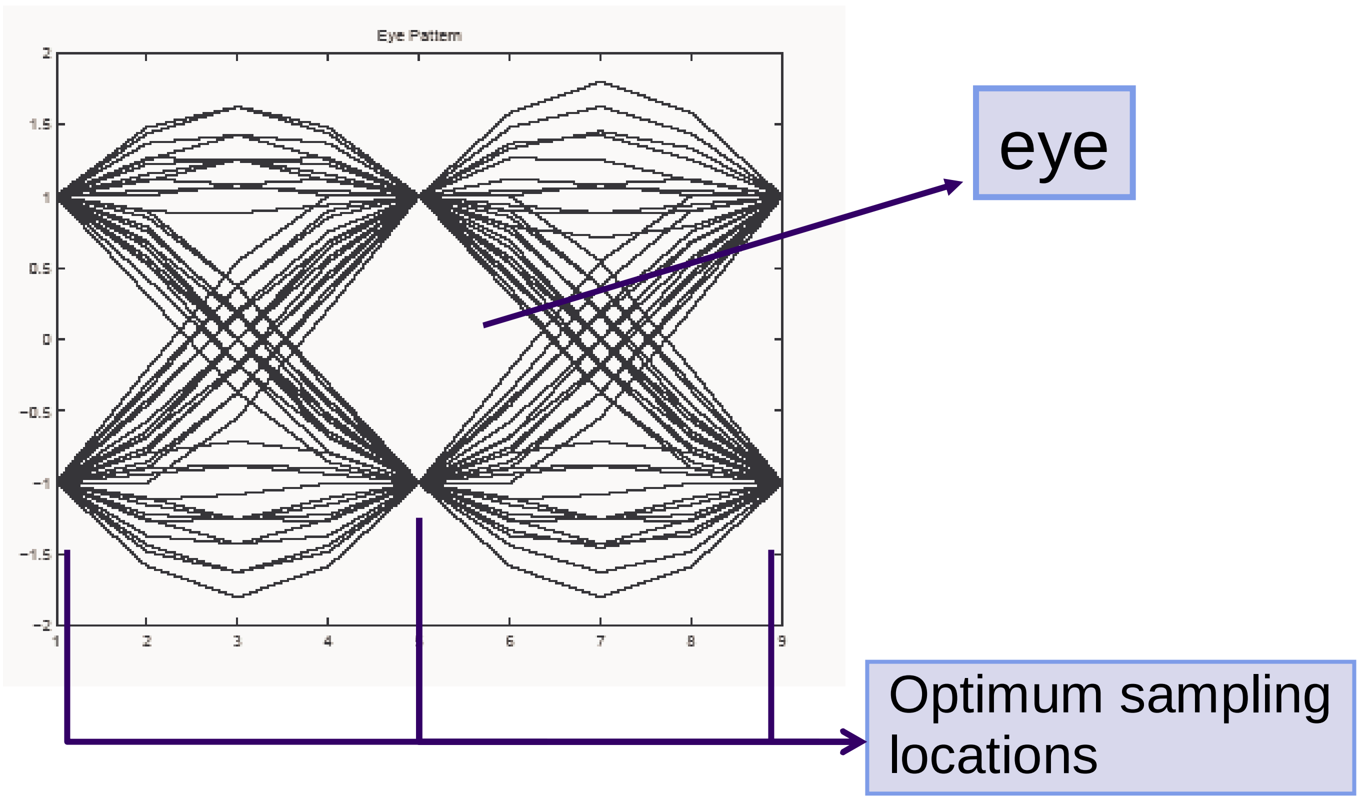
- Spectral comparison of rectangular and raised-cosine pulses
  - Note the band-limitation of raised-cosine shaping





Eye Diagram

- *Eye diagram is a popular robustness evaluation tool.*
- For 4-PAM, single-baud-wide Hamming blip with additive broadband channel noise, retriggering oscilloscope after every 2 baud intervals produces



Eye Diagrams

- Eye diagrams with raised-cosine pulse shaping with 2-PAM and 4-PAM systems