Quadrature Amplitude Modulation (QAM)

- PAM signals occupy twice the bandwidth required for the baseband
- Transmit two PAM signals using carriers of the same frequency but in phase and quadrature

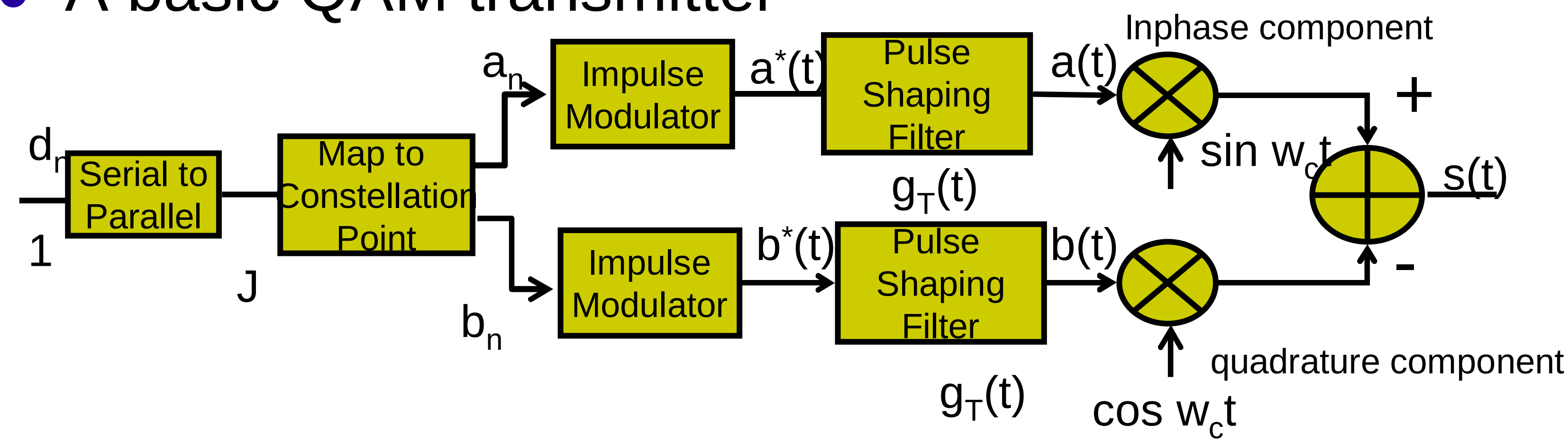
$$\varphi_{QAM}(t) = m_1(t)\cos(w_c t) - m_2(t)\sin(w_c t)$$

- Demodulation:

$$\begin{aligned} x_1(t) &= 2\varphi_{QAM}(t)\cos(w_c t) = 2(m_1(t)\cos(w_c t) - m_2(t)\sin(w_c t))\cos(w_c t) \\ x_1(t) &= m_1(t) + m_1(t)\cos(2w_c t) + m_2(t)\sin(2w_c t) \end{aligned}$$

QAM Transmitter Implementation

- QAM is widely used method for transmitting digital data over bandpass channels
- A basic QAM transmitter



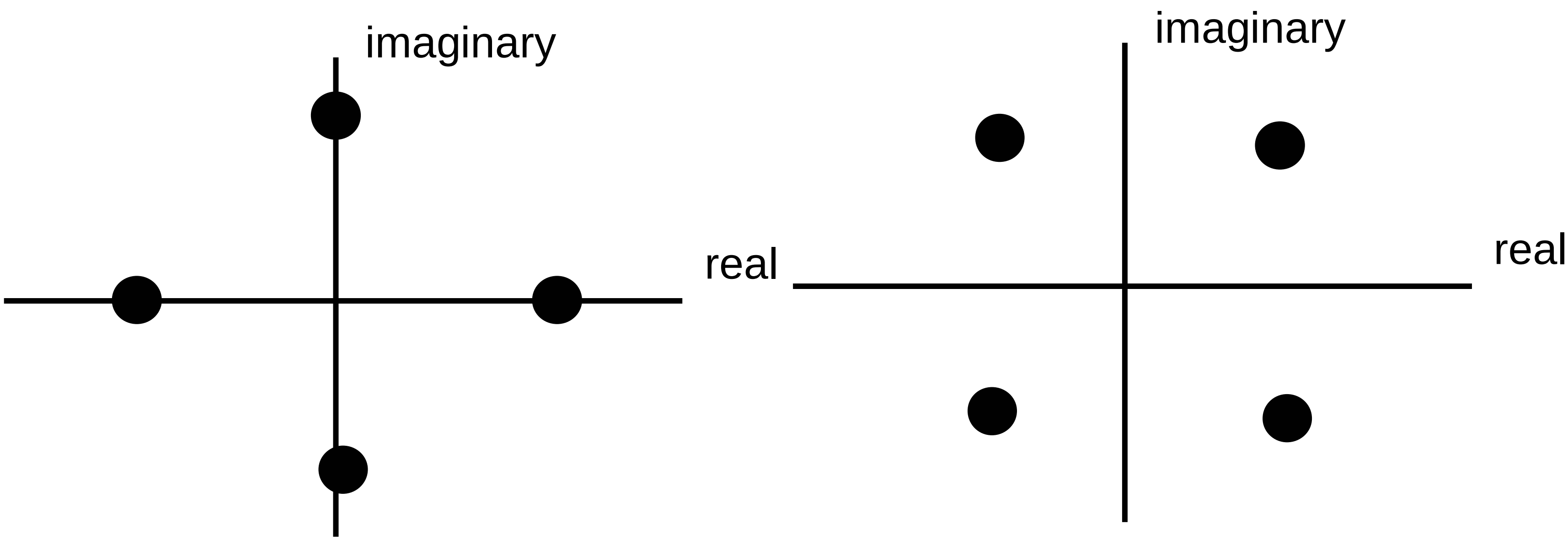
- Using complex notation

$$s(t) = a(t)\cos w_c t - b(t)\sin w_c t \quad s(t) = \Re\{s_+(t)\} \text{ which is the preenvelope of QAM}$$

$$s_+(t) = \sum_{-\infty}^{\infty} (a_k + jb_k) g_T(t-kT) e^{jw_c t} \quad s_+(t) = \sum_{-\infty}^{\infty} (c_k e^{jw_c kT}) g_T(t-kT) e^{jw_c(t-kT)}$$

Complex Symbols

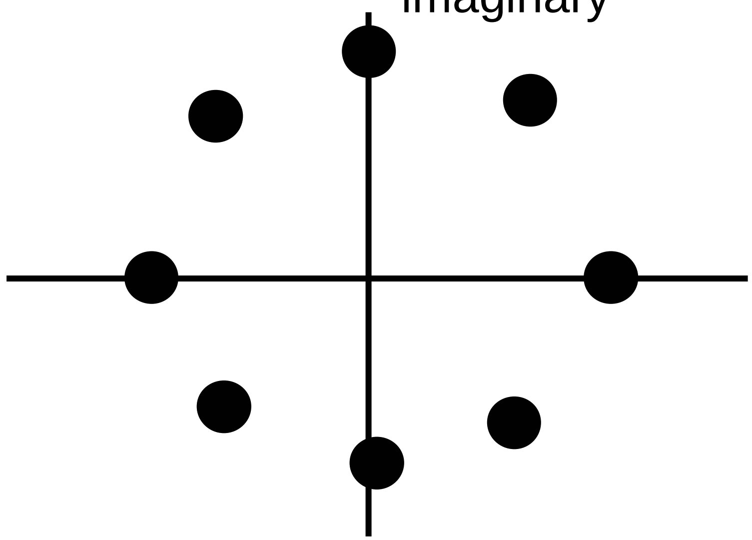
- Quadriphase-Shift Keying (QPSK)
  - 4-QAM
  - Transmitted signal is contained in the phase



$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{\pi}{4}(2i-1)\right), \quad i = 1, 2, 3, 4$$

M-ary PSK

- QPSK is a special case of M-ary PSK, where the phase of the carrier takes on one of M possible values



$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \quad i = 1, 2, \ldots, M$$

Pulse Shaping Filters

- The real pulse shaping filter response $\Rightarrow$ $g_T(t)$
- Let $\Rightarrow$ $h(t) = g_T(t)e^{jw_c t} = h_I(t) + jh_Q(t)$

where

$$h_I(t) = g_T(t)\cos w_c t$$
$$h_Q(t) = g_T(t)\sin w_c t$$

- This filter is a bandpass filter with the frequency response $\Rightarrow$ $H(w) = G_T(w - w_c)$

Transmitted Sequences

- Modulated sequences before passband shaping

$$a_k^{'} = \Re\{c_k e^{jw_c kT}\} = a_k \cos w_c kT - b_k \sin w_c kT$$
$$b_k^{'} = \Im\{c_k e^{jw_c kT}\} = a_k \sin w_c kT + b_k \cos w_c kT$$

- Using these definitions

$$s(t) = \Re\{s_+(t)\} = \sum_{k=-\infty}^{\infty} a_k^{'} h_I(t-kT) - b_k^{'} h_Q(t-kT)$$

- Advantage: computational savings

Ideal QAM Demodulation

- Exact knowledge of the carrier and symbol clock phases and frequencies
- Apply the Hilbert Transform of the received signal to generate the pre-envelope $s_+(t)$

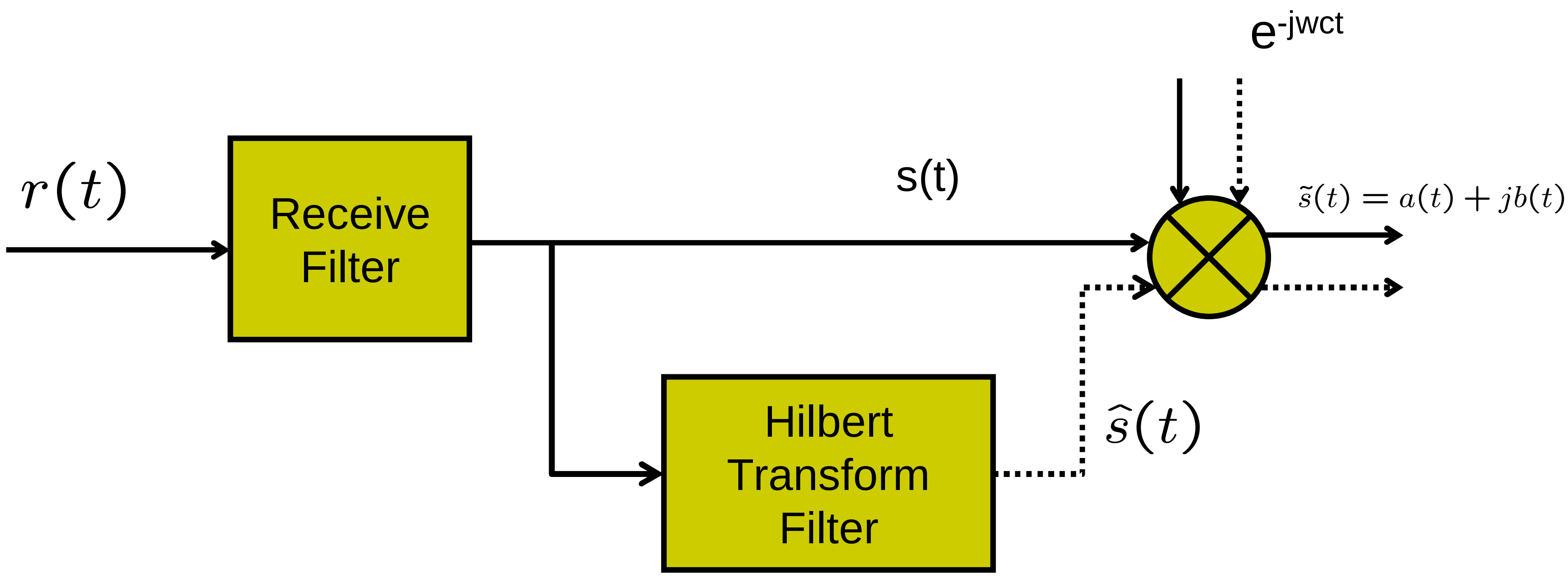
$$s(t) = s_+(t)e^{-jw_c t} = \sum_{k=-\infty}^{\infty}(a_k + jb_k)g_T(t-kT)$$

- If $g_T(t)$ has no intersymbol interference, we get exactly the transmitted symbol

$$\tilde{s}(nT) = a_n + jb_n$$

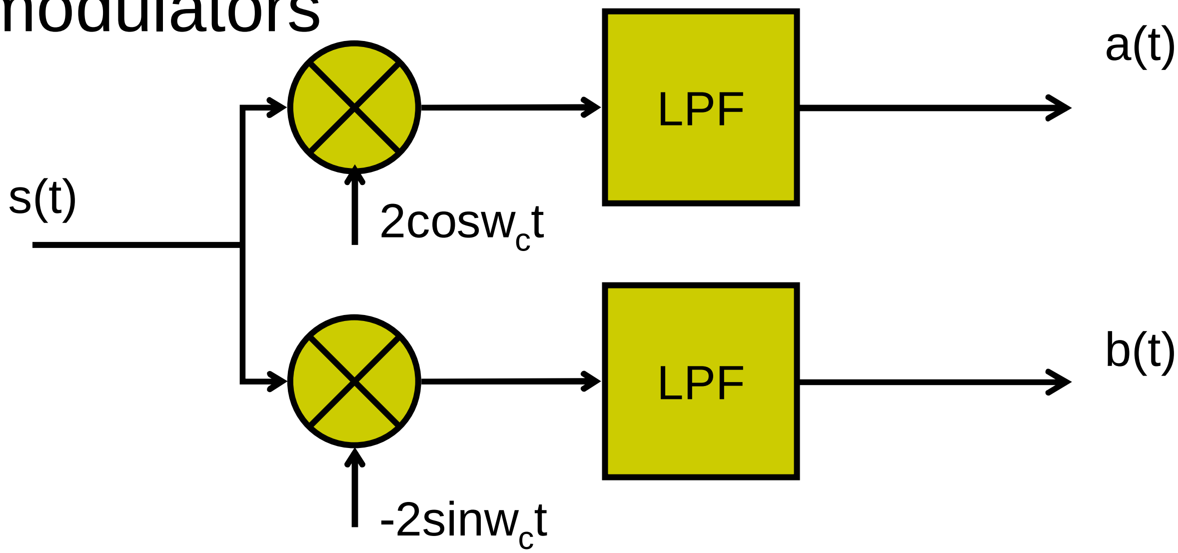
QAM Demodulation

- QAM demodulator using the complex envelope



Second Method

- Based on a pair of DSBSC-AM coherent demodulators



- In DSP implementations, Hilbert transform method is popular because
  - It only requires single filter
  - $2w_c$ terms are automatically cancelled

QAM Front-End Subsystems

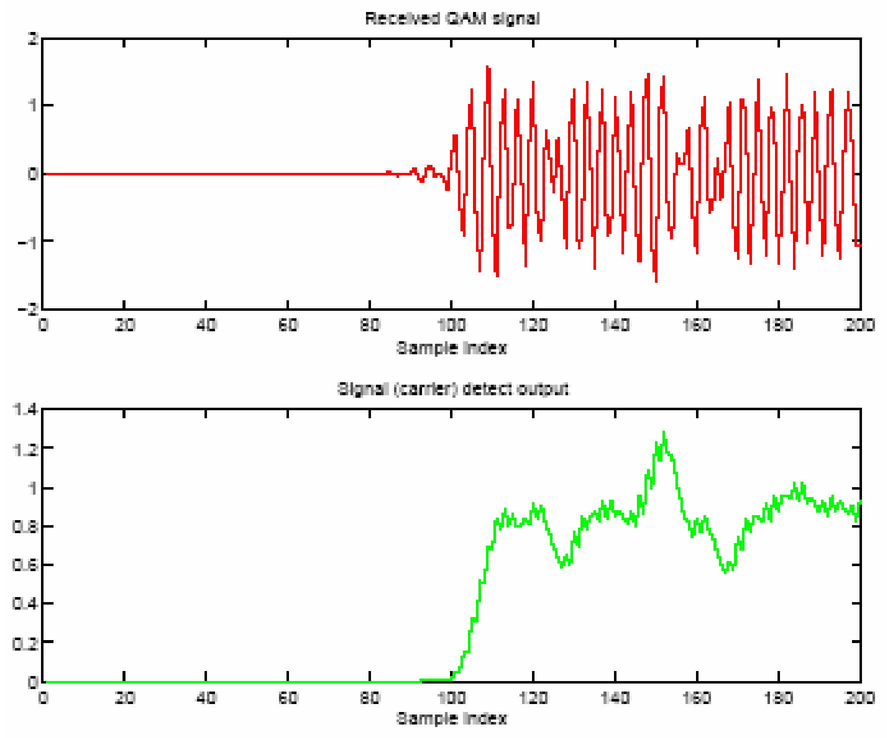
- Automatic gain control
  - Full scale usage of A/D and avoid clipping
  - No need to implement AGC in your project
- The Carrier detect subsystem
  - Determine if a QAM signal is present at the receiver
  - This power estimate can be easily formed by averaging the squared ADC output samples $r(n)$:

$$p(n) = (1 - c)r^2(n) + cp(n-1)$$

  - You can select c as a number slightly less than 1, i.e, 0.9.
  - When the power estimate $p(n)$ exceeds a predetermined threshold for a period of time, a received QAM signal is declared to be present.

Carrier Detect

- Determine a threshold value by inspecting your power estimate.
- The following figure shows the portion of received QAM signal and the power estimate average.



Frame Synchronization

- Assume that the receiver knows marker symbols (say, 10 complex symbols, m(n))
- Assume that $s(k)$ is the output of your QAM demodulator. Note that $s(k)$ and $m(n)$ are complex.
- Of course, in DSP you define complex numbers by keeping real and imaginary parts in different buffers.

Frame Synchronization

- Correlation,
- Peak=

$$\sum_{n=0}^{10} (m(n)^* s(4n+k))$$

- Peakindex=max(abs(Peak))
- Note that Peak is complex quantity

Initial Phase Correction

- Defining $e^{j\theta}$=peak/abs(peak),
- then, the phase shift on s(n) which is the output of QAM demodulator can be corrected as
- $x(k)=s(index+k-1)e^{-j\theta}$

Switch to QAM Modulation

- You can proceed with hilbert transform filtering and complex downconversion by setting your receiver pointer to the beginning of your input buffer.
- First the receiver is on and transmitter is off
- Transmitter starts with marker and PN sequence, both complex
- When transmitter starts, carrier detect will switch to QAM downconversion

Carrier Offset Impairment for QAM

- Received QAM signal

$$s_+(t) = m_1(t)\cos(2\pi f_c t + \theta) - m_2(t)\sin(2\pi f_c t + \theta)$$

- After Hilbert transform, we can form

$$s(t) = s_+(t) + j\tilde{s}(t) = (m_1(t) + jm_2(t))e^{(j2\pi f_c t + j\theta)}$$

- Complex demodulator $e^{j2\pi fct+j\phi)}$

$$x(t) = s(t)e^{(-j2\pi f_c t - j\phi)} = (m_1(t) + jm_2(t))e^{j(\theta-\phi)}$$

- Obtain $m_1(t)$ as the real part

$$\hat{m}_1(t) = \mathcal{R}\{(m_1(t) + jm_2(t))e^{j(\theta-\phi)}\}$$

$$\hat{m}_1(t) = m_1(t)\cos(\theta - \phi) - m_2(t)\sin(\theta - \phi)$$

Carrier Offset Impairment for QAM

- For PAM signaling carrier offset causes scaling with cosine of the phase offset
- For QAM signaling carrier offset causes rotation of the constellation by the carrier offset