UNIVERSITY OF TEXAS AT DALLAS
Telecommunications Engineering

TE3302 Signals & Systems
Problem Set #F: Fourier Transform and Sampling

This is a practice problem set for the final exam. Study these problems carefully!

Problem F.1 Fourier Transform Properties

First evaluate the Fourier transform of a continuous-time signal $x(t) = e^{-bt}u(t)$ where b is a positive constant. Then, determine the Fourier transform $V(jw)$ of the following signals.

(a) $v(t) = x(5t - 4)$

(b) $v(t) = t^2 x(t)$

(c) $v(t) = x(t)e^{jt}$

(d) $v(t) = x(t)\cos 4t$

(e) $v(t) = \frac{d^2x(t)}{dt^2}$

(f) $v(t) = x(t) * x(t)$

(g) $v(t) = \frac{1}{jt-b}$ Use duality property.

Problem F.2 Inverse Fourier Transform

Compute the inverse Fourier transform of the following functions

(a) $X(jw) = 3\delta(w-1) + j2\delta(w-2) + 3\delta(w+1) - j2\delta(w+2)$

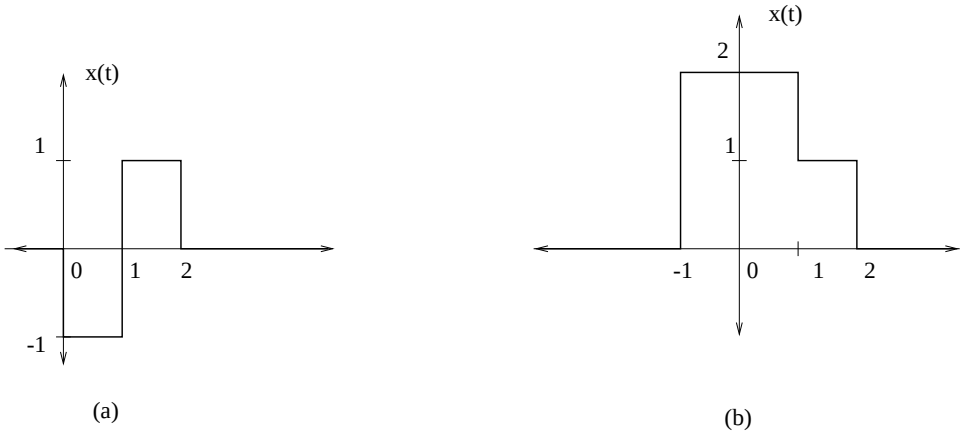
(b) $X(jw) = \cos 4w$

(c) $X(jw) = \frac{\sin w/2}{jw+2}e^{-jw2}$

(d) $X(jw) = \frac{3(3+jw)}{(4+jw)(2+jw)}$

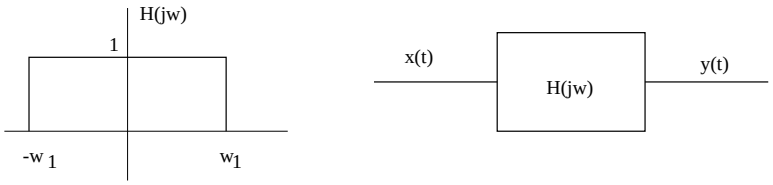
Problem F.3 Fourier Transform

By first expressing $x(t)$ in terms of rectangular pulse functions, compute the Fourier transform of the signals below.



Problem F.4 Convolution Property of Fourier Transform and Filtering

Consider an LTI system with frequency response $H(jw)$ shown below



Determine the system response $y(t)$ to the input $x(t) = 2\cos w_0 t + 3\sin 2w_0 t + 4\cos 4w_0 t$ where $2w_0 < w_1 < 4w_0$.

Problem F.5

The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the *Nyquist rate*. Determine the Nyquist rate corresponding to each of the following signals.

(a) $x(t) = 1 + \cos 2,000\pi t + \sin 4,000\pi t$

(b) $X(jw) = \begin{cases} 1 & \text{for} |w| \leq 1000\pi \\ 0 & otherwise \end{cases}$