

Fundamentals of Microelectronics

- **CH1 Why Microelectronics?**
- **CH2 Basic Physics of Semiconductors**
- **CH3 Diode Circuits**
- **CH4 Physics of Bipolar Transistors**
- **CH5 Bipolar Amplifiers**
- **CH6 Physics of MOS Transistors**
- **CH7 CMOS Amplifiers**
- **CH8 Operational Amplifier As A Black Box**

Chapter 3 Diode Circuits

- **3.1 Ideal Diode**
- **3.2 PN Junction as a Diode**
- **3.3 Applications of Diodes**

Diode Circuits

Diodes as Circuit Elements

- Ideal Diode
- Circuit Characteristics
- Actual Diode

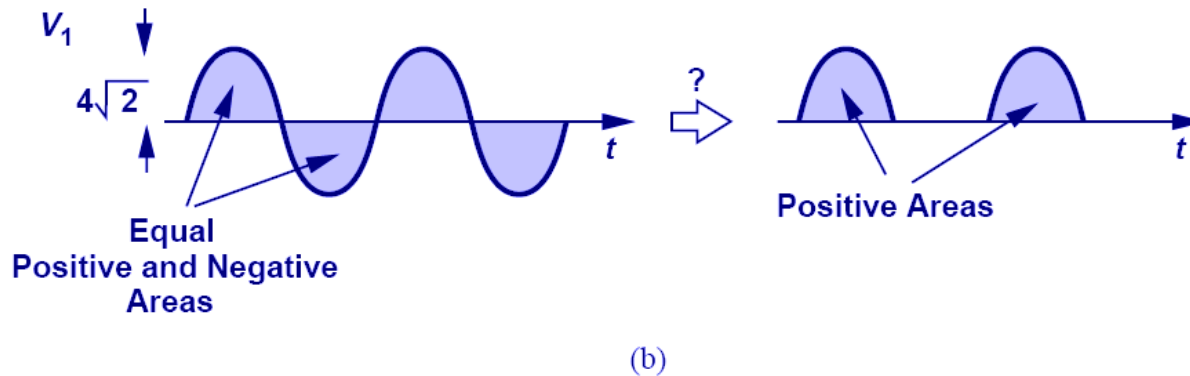
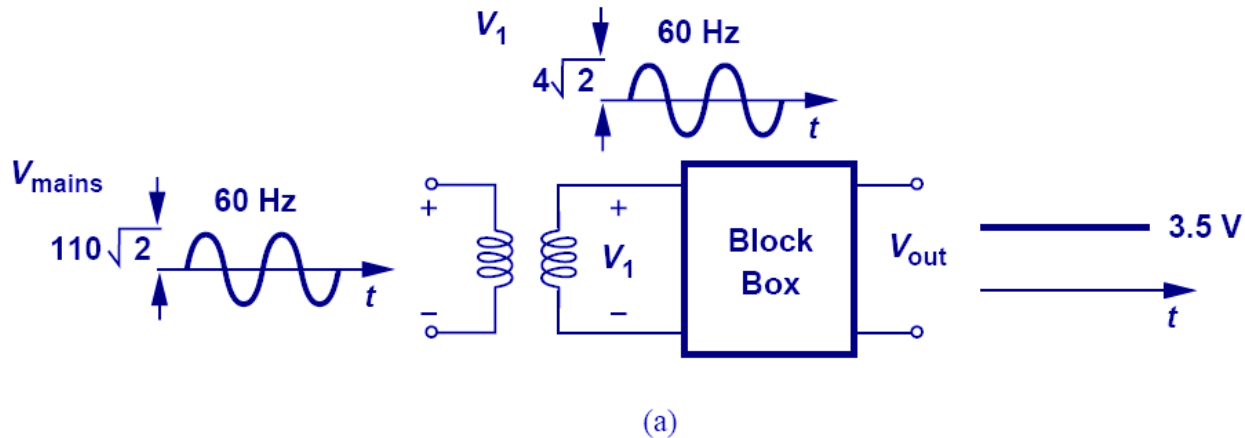


Applications

- Regulators
- Rectifiers
- Limiting and Clamping Circuits

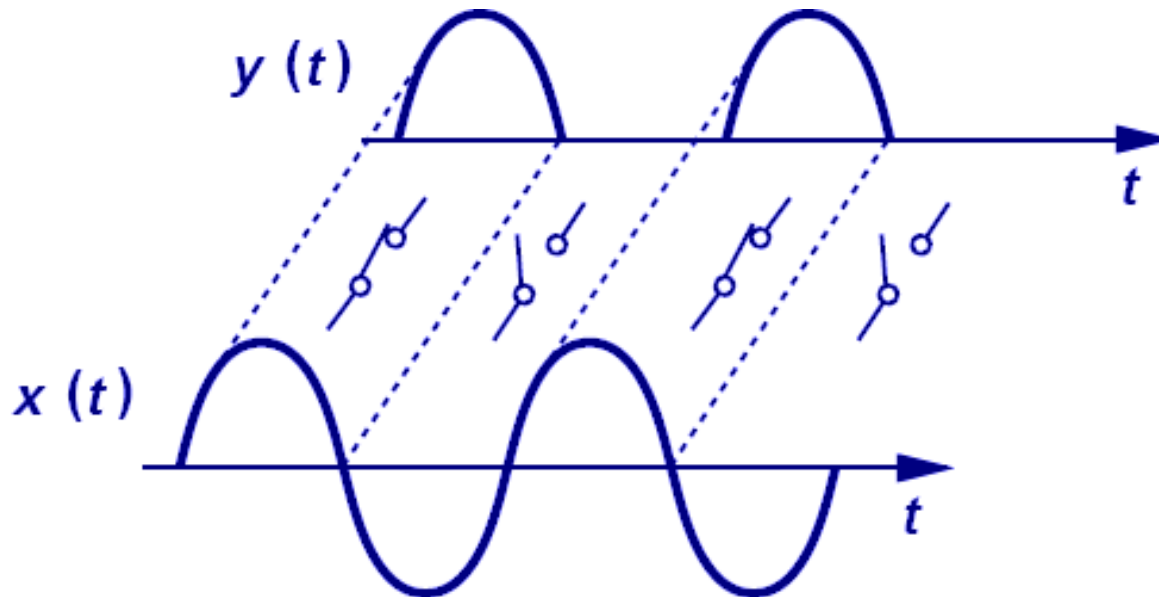
- **You know the physics of a diode from EE3310, in EE3311 we will study its behavior as a circuit element and its many applications.**

Diode's Application: Cell Phone Charger



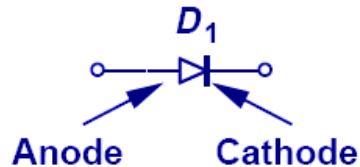
- An important application of diode is chargers.
- Diode acts as the black box (after transformer) that passes only the positive half of the stepped-down sinusoid.

Diode's Action in The Black Box (Ideal Diode)



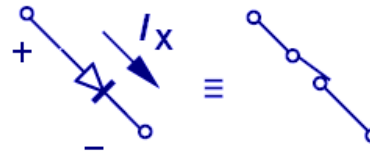
- The diode behaves as a short circuit during the positive half cycle (voltage across it tends to exceed zero), and an open circuit during the negative half cycle (voltage across it is less than zero).

Ideal Diode

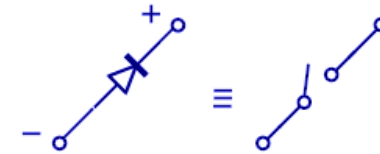


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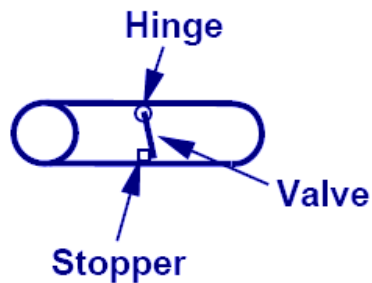
Forward Bias
 $V_{\text{anode}} > V_{\text{cathode}}$



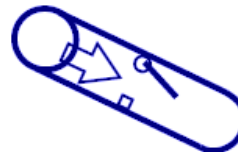
Reverse Bias
 $V_{\text{anode}} < V_{\text{cathode}}$



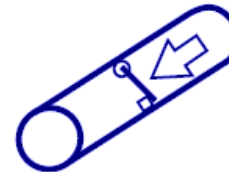
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Forward Bias



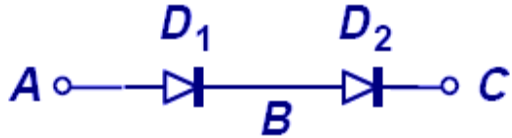
Reverse Bias



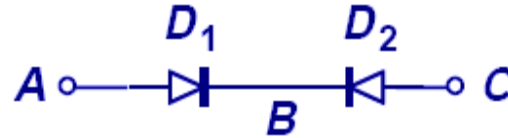
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- In an ideal diode, if the voltage across it tends to exceed zero, current flows.
- It is analogous to a water pipe that allows water to flow in only one direction.

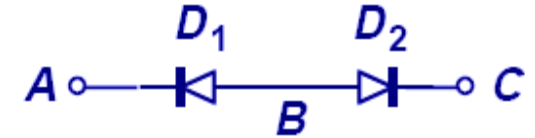
Diodes in Series



(a)



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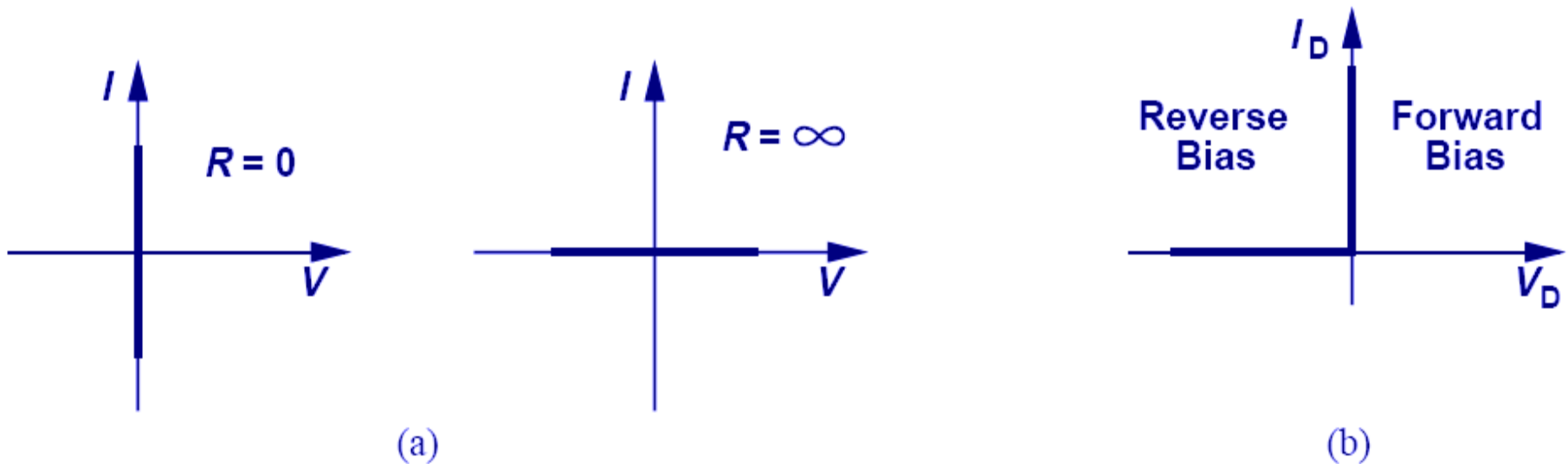


(c)

- Diodes cannot be connected in series randomly. For the circuits above, only a) can conduct current from A to C .

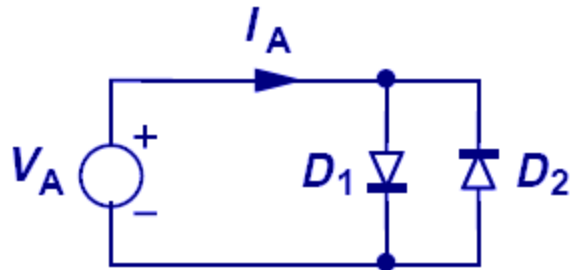
IV Characteristics of an Ideal Diode

$$R = 0 \Rightarrow I = \frac{V}{R} = \infty \quad R = \infty \Rightarrow I = \frac{V}{R} = 0$$

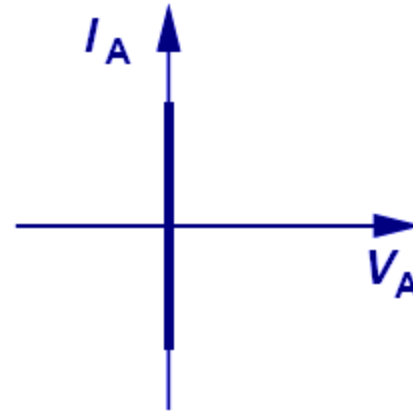


- If the voltage across anode and cathode is greater than zero, the resistance of an ideal diode is zero and current becomes infinite. However, if the voltage is less than zero, the resistance becomes infinite and current is zero.

Anti-Parallel Ideal Diodes



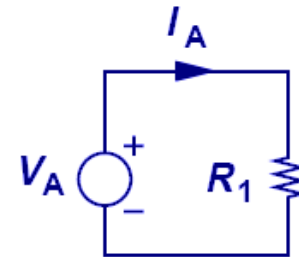
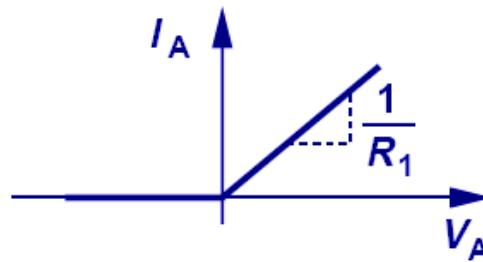
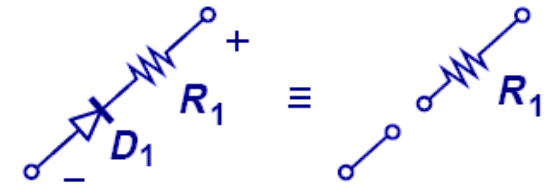
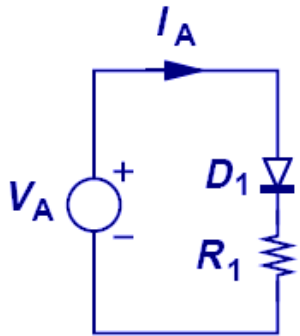
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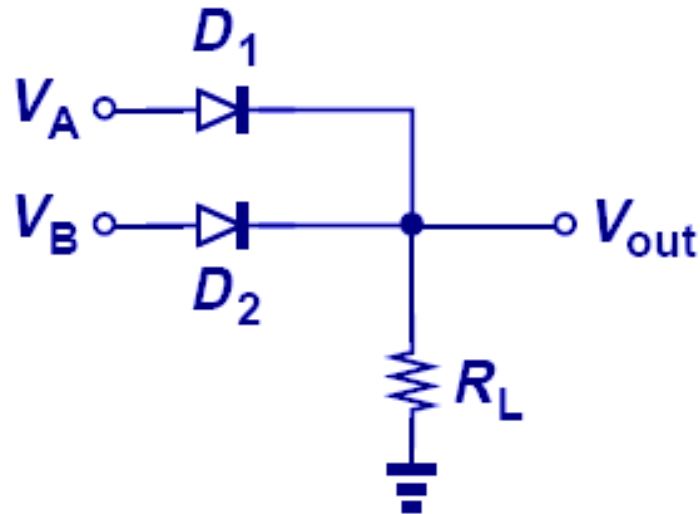
- **If two diodes are connected in anti-parallel, it acts as a short for all voltages.**

Diode-Resistor Combination



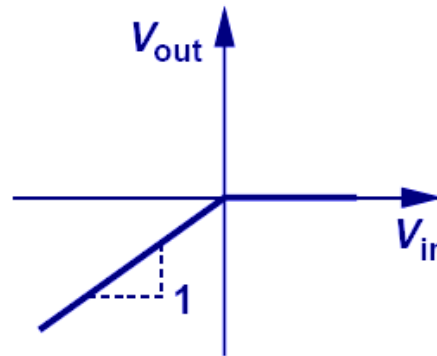
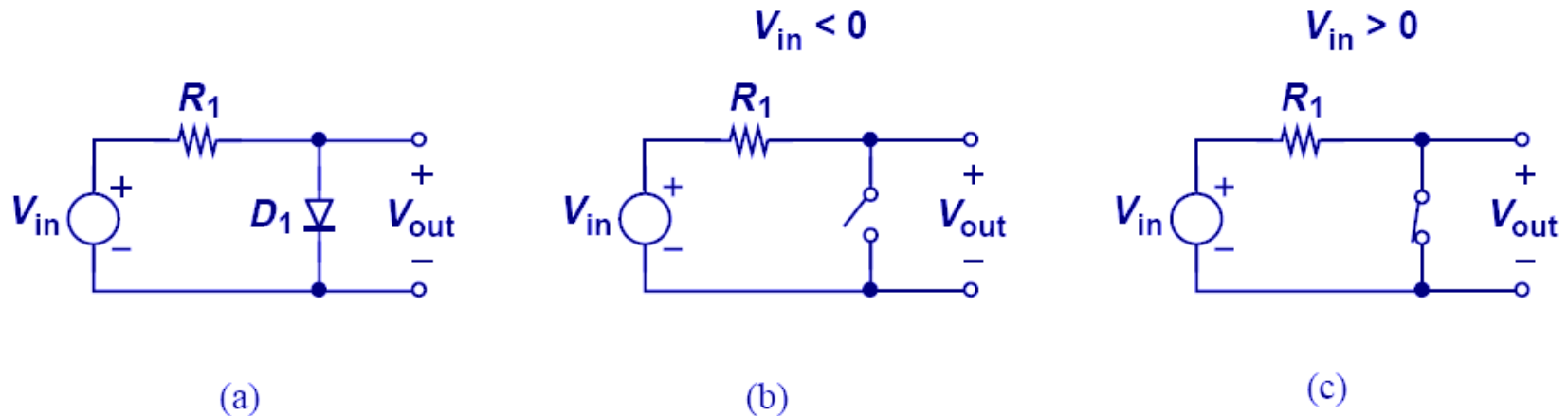
- The IV characteristic of this diode-resistor combination is zero for negative voltages and Ohm's law for positive voltages.

Diode Implementation of OR Gate



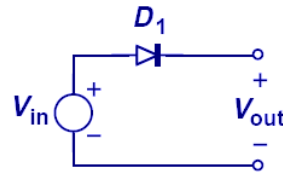
- The circuit above shows an example of diode-implemented OR gate.
- V_{out} can only be either V_A or V_B , not both.

Input/Output Characteristics

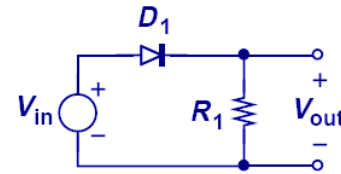


- When V_{in} is less than zero, the diode opens, so $V_{out} = V_{in}$.
- When V_{in} is greater than zero, the diode shorts, so $V_{out} = 0$.

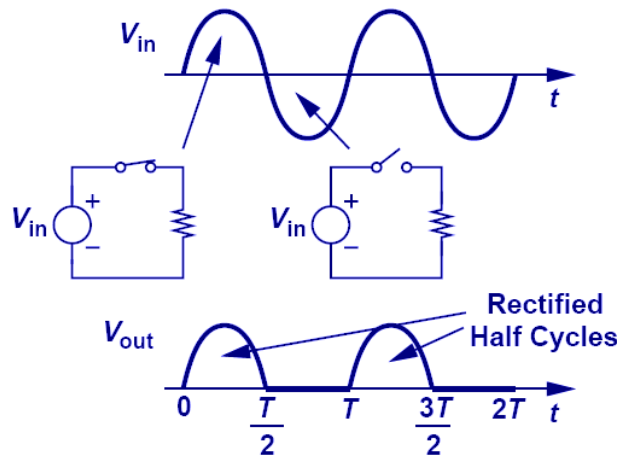
Diode's Application: Rectifier



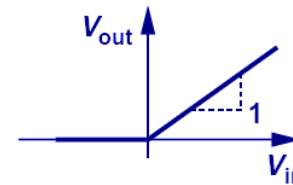
(a)



(b)



(c)



(d)

- A rectifier is a device that passes positive-half cycle of a sinusoid and blocks the negative half-cycle or vice versa.
- When V_{in} is greater than 0, diode shorts, so $V_{out} = V_{in}$; however, when V_{in} is less than 0, diode opens, no current flows thru R_1 , $V_{out} = I \times R_1 = 0$.

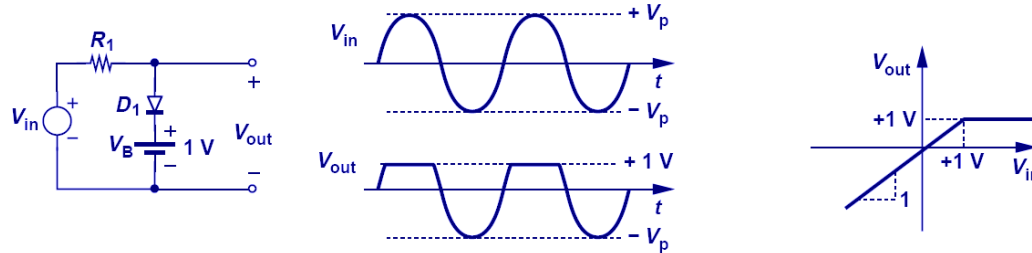
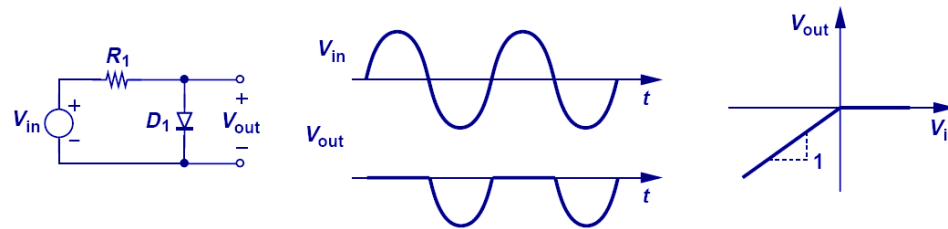
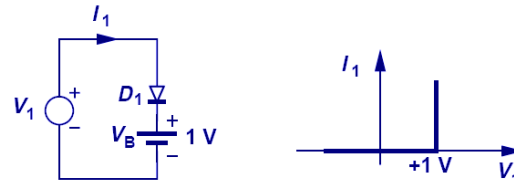
Signal Strength Indicator

$$V_{out} = V_p \sin \omega t = 0 \quad \text{for } \frac{T}{2} \leq t \leq T$$

$$\begin{aligned} V_{out,avg} &= \frac{1}{T} \int_0^T V_{out}(t) dt = \frac{1}{T} \int_0^{T/2} V_p \sin \omega t dt \\ &= \frac{1}{T} \frac{V_p}{\omega} [-\cos \omega t]_0^{T/2} = \frac{V_p}{\pi} \quad \text{for } 0 \leq t \leq \frac{T}{2} \end{aligned}$$

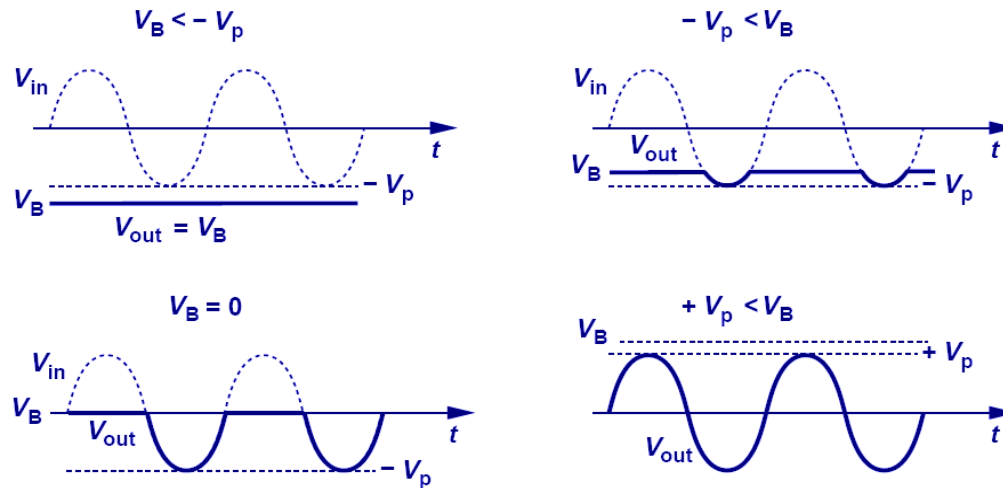
- The averaged value of a rectifier output can be used as a signal strength indicator for the input, since $V_{out,avg}$ is proportional to V_p , the input signal's amplitude.
- RSSI in WiFi receivers

Diode's application: Limiter

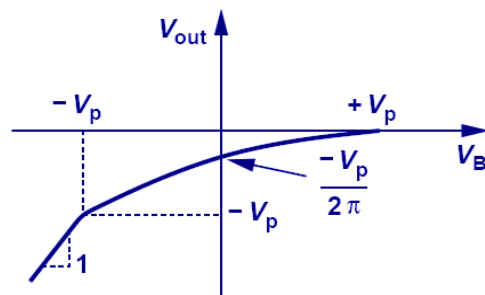


- The purpose of a limiter is to force the output to remain below certain value.
- In a), the addition of a 1 V battery forces the diode to turn on after V_1 has become greater than 1 V.

Limiter: When Battery Varies



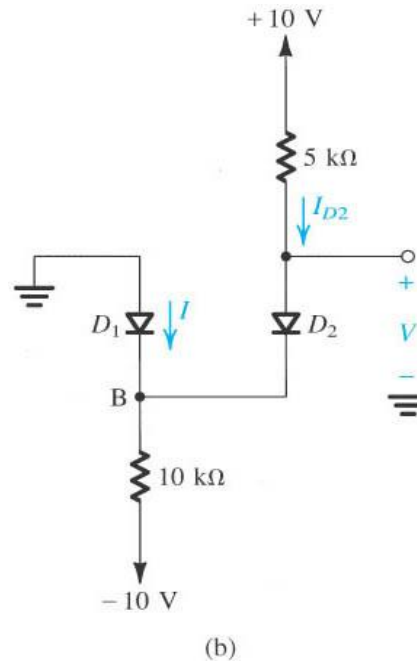
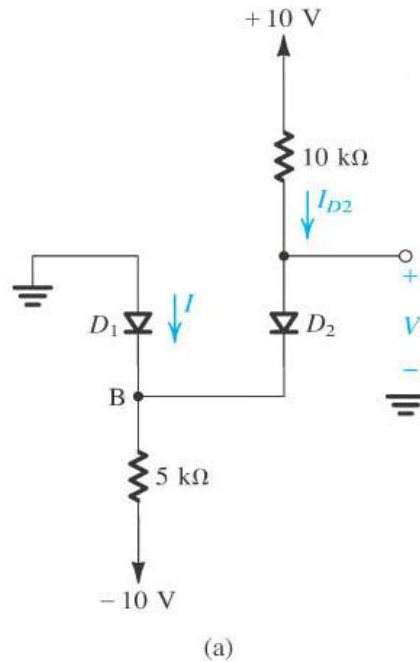
(a)



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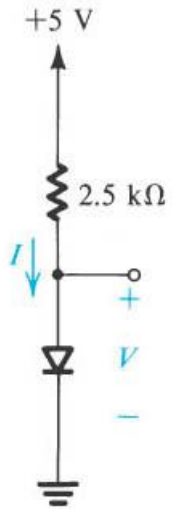
- An interesting case occurs when V_B (battery) varies.
- Rectification fails if V_B is greater than the input amplitude.

Complex Diode Circuit Examples

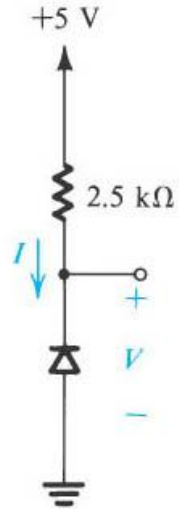


- Assuming the diodes ideal, find the values of I and V in the circuits above
- In these circuits, it may not be obvious at first sight whether none, one, or both diodes are conducting
- Make a plausible assumption → Proceed with the analysis → Check whether the solution is consistent

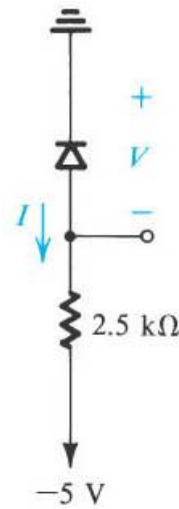
Diode Examples



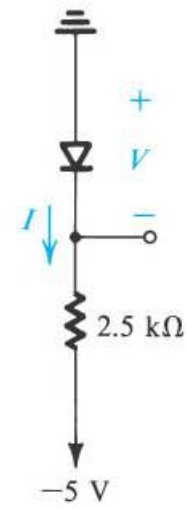
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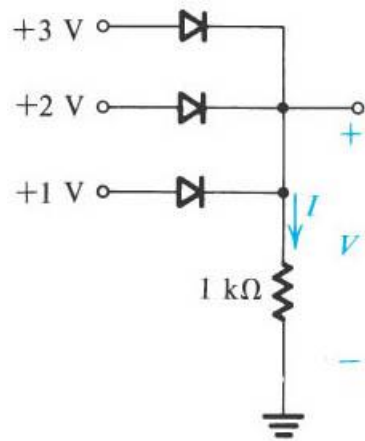
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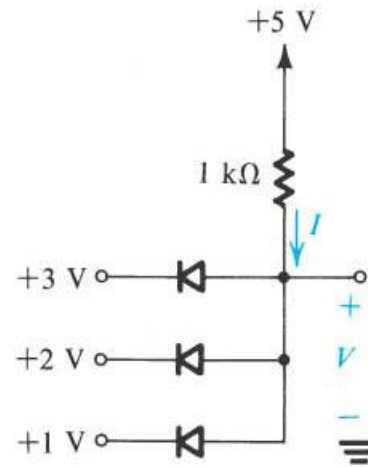
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(d)

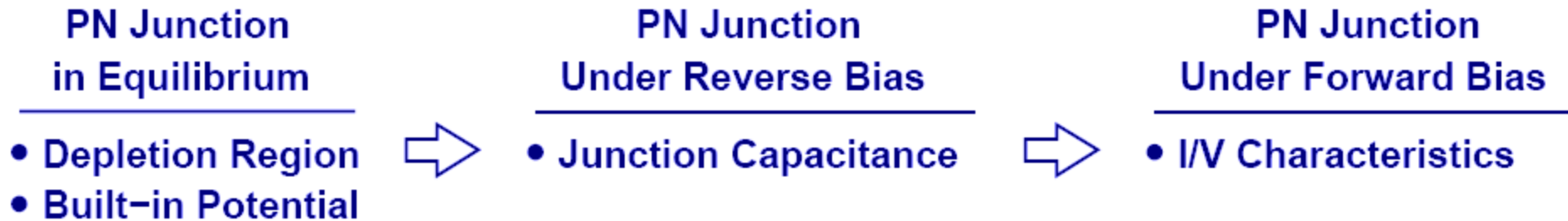


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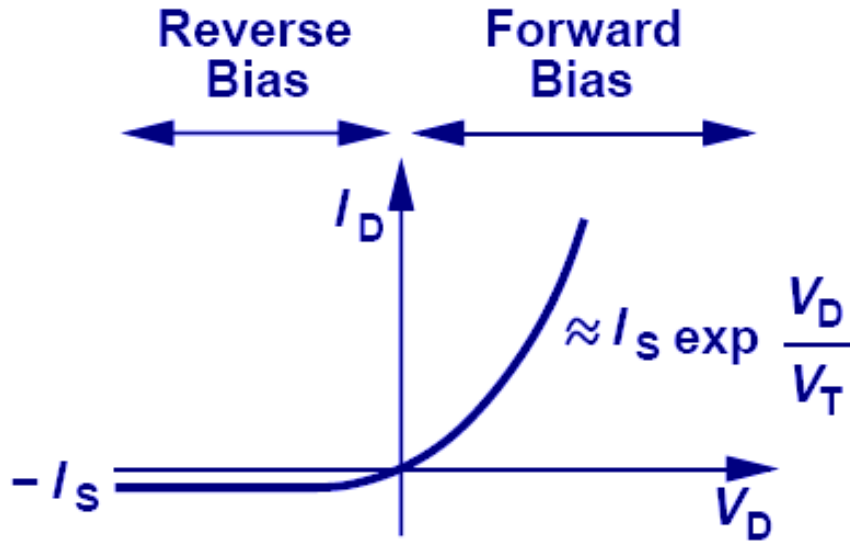
(f)

Diode's Three Operation Regions



- In order to understand the operation of a diode, it is necessary to study its three operation regions: equilibrium, reverse bias, and forward bias.

Terminal Characteristics of Junction Diodes



Two distinct regions

1. $V_D > 0$
2. $V_D < 0$

The current and voltage relationship of a PN junction is exponential in forward bias region, and relatively constant in reverse bias region.

Forward Bias

$$I_D = I_S (\exp(V_D/V_T) - 1) \approx I_S \exp(V_D/V_T)$$

I_S → Saturation current (proportional to surface area of pn junction)

$V_T = kT/q$ → Thermal voltage

k → Boltzmann's constant = 1.38×10^{-23} joules/Kelvin

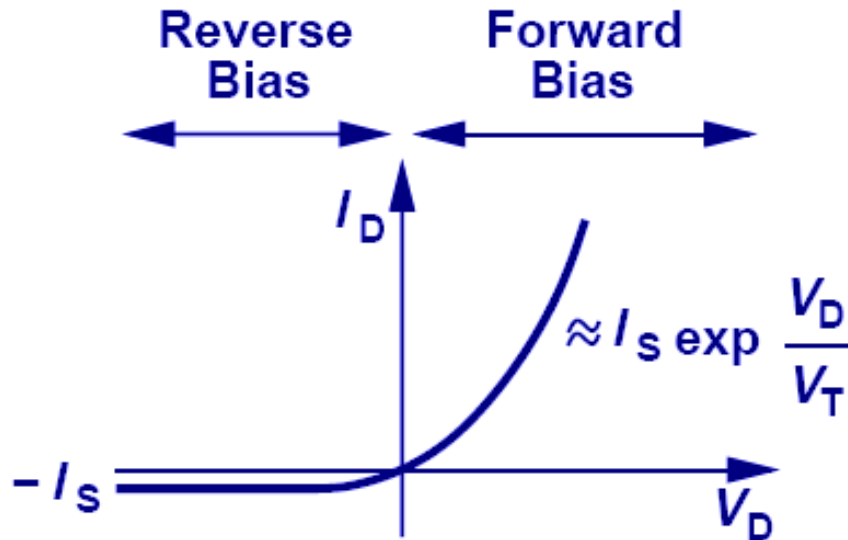
T → The absolute temperature in Kelvins = $273 + ^\circ C$

q → the magnitude of electronic charge = 1.60×10^{-19} coulomb

$$V_T \approx 26 \text{ mV at } T=300\text{K}$$

$$V_D = V_T \ln \frac{I_D}{I_S}$$

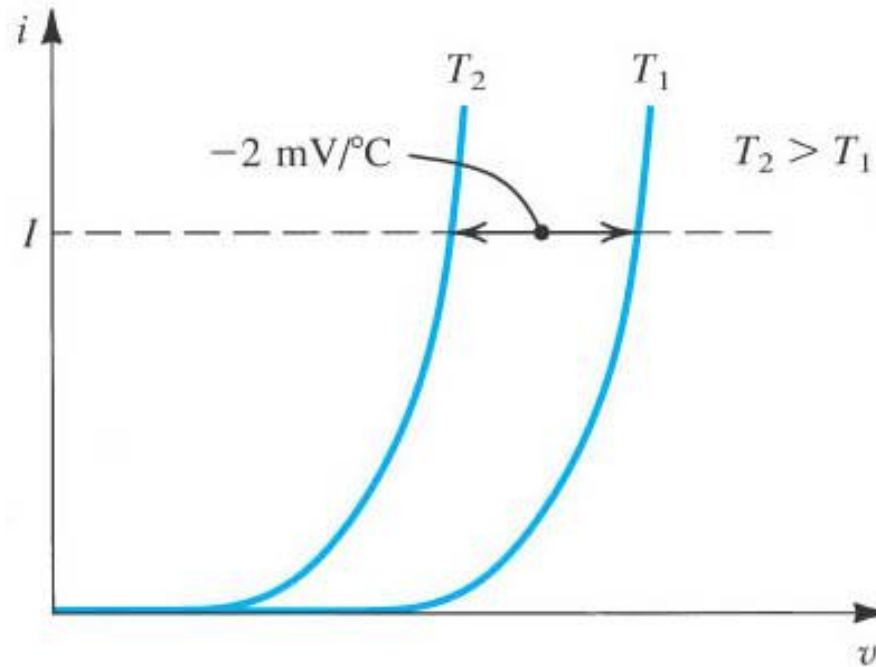
IV Characteristic of PN Junction



$$I_D = I_S \left(\exp \frac{V_D}{V_T} - 1 \right)$$

- The current and voltage relationship of a PN junction is exponential in forward bias region, and relatively constant in reverse bias region.

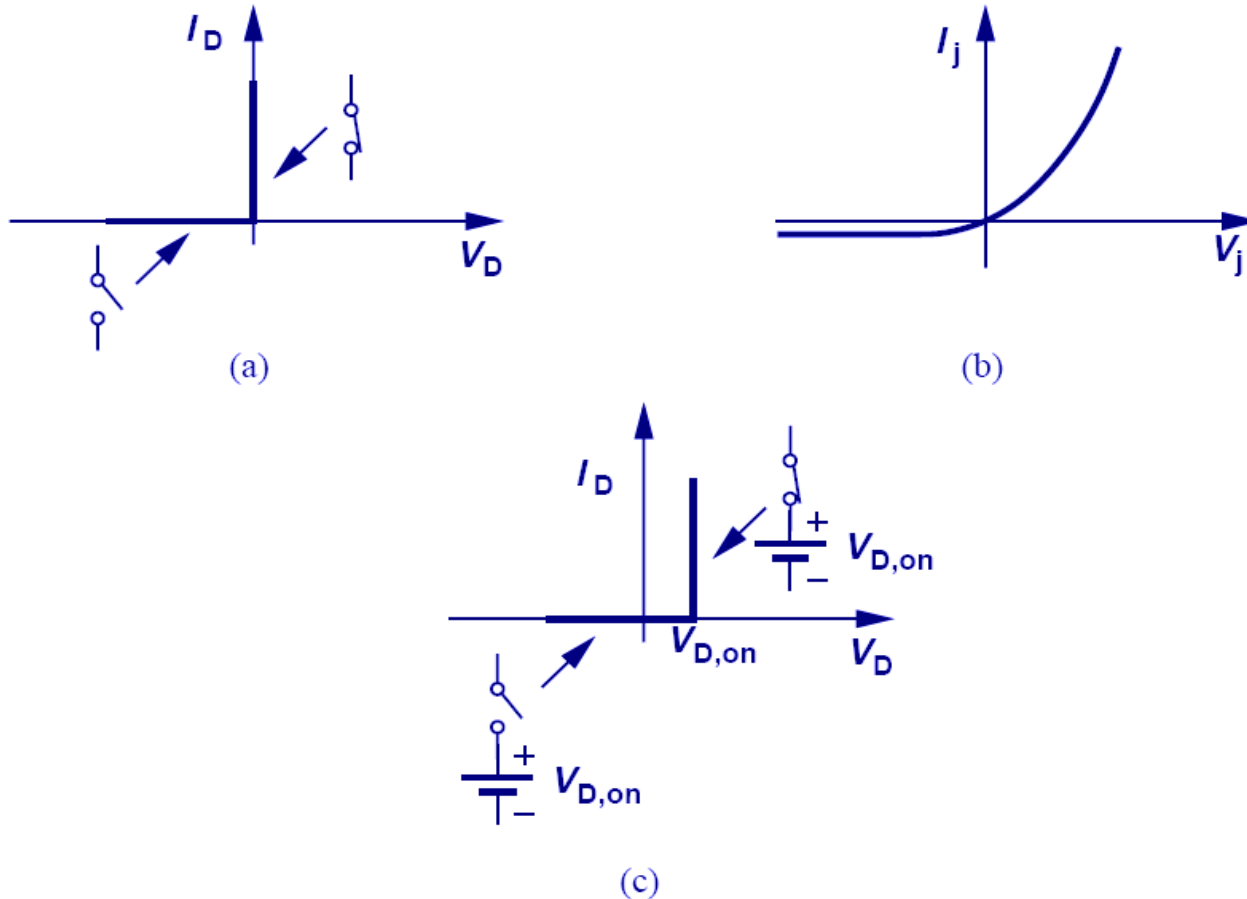
Temperature Dependence of the Diode Forward Characteristics



$$I = I_S \exp(q V / (kT))$$

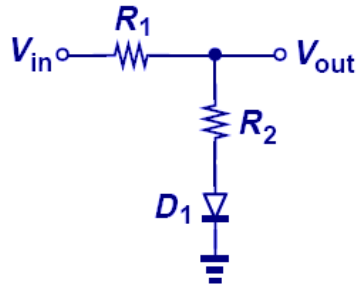
- I_S (saturation current) is also a function of temperature.
- At a constant current, there is a decrease of approximately 2mV in diode voltage for every 1°C degree increase

Different Models for Diode

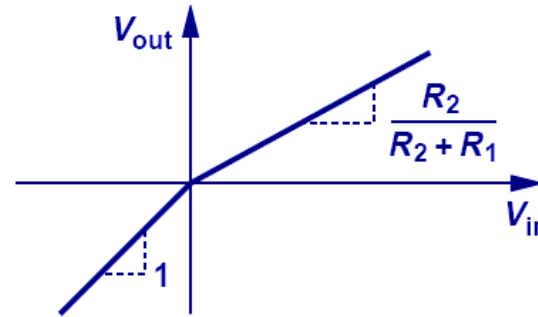


- Thus far, Diode models include the ideal model of diode, the exponential, and constant voltage models.

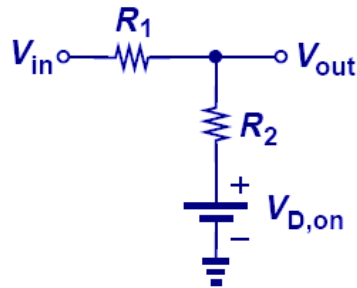
Input/Output Characteristics with Ideal and Constant-Voltage Models



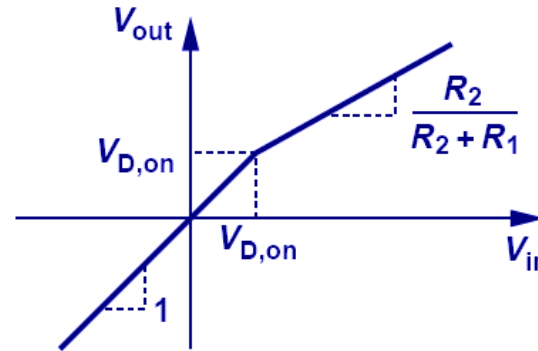
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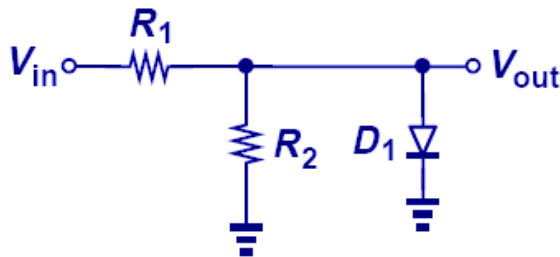
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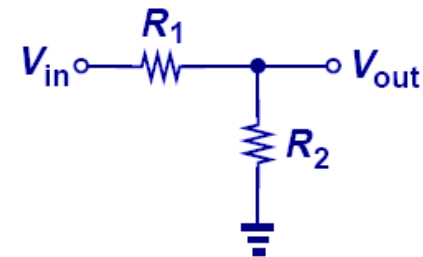
(d)

- The circuit above shows the difference between the ideal and constant-voltage model; the two models yield two different break points of slope.

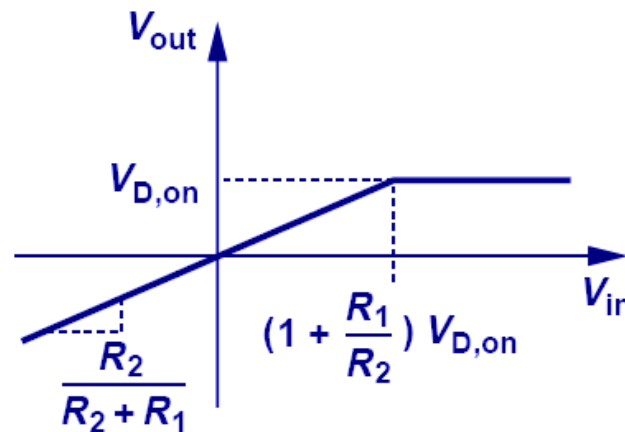
Input/Output Characteristics with a Constant-Voltage Model



(a)



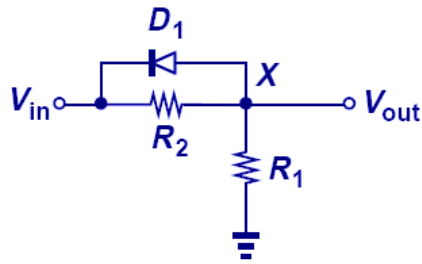
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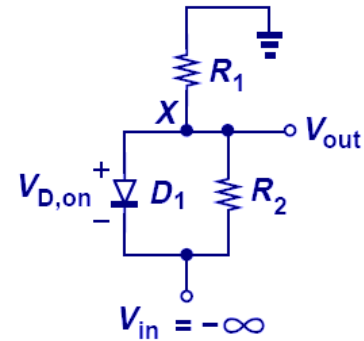
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- When using a constant-voltage model, the voltage drop across the diode is no longer zero but $V_{d,on}$ when it conducts.

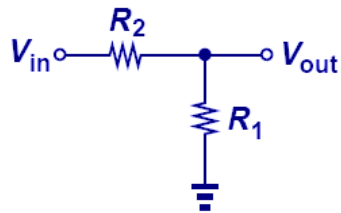
Another Constant-Voltage Model Example



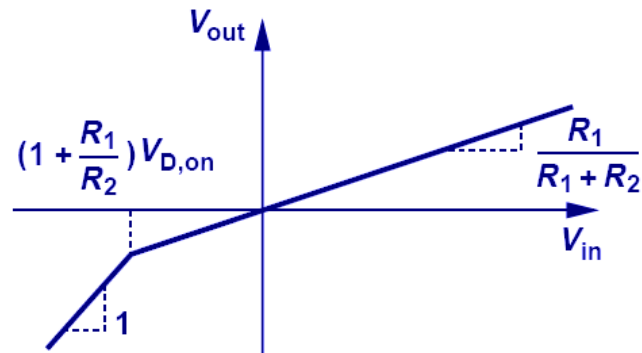
(a)



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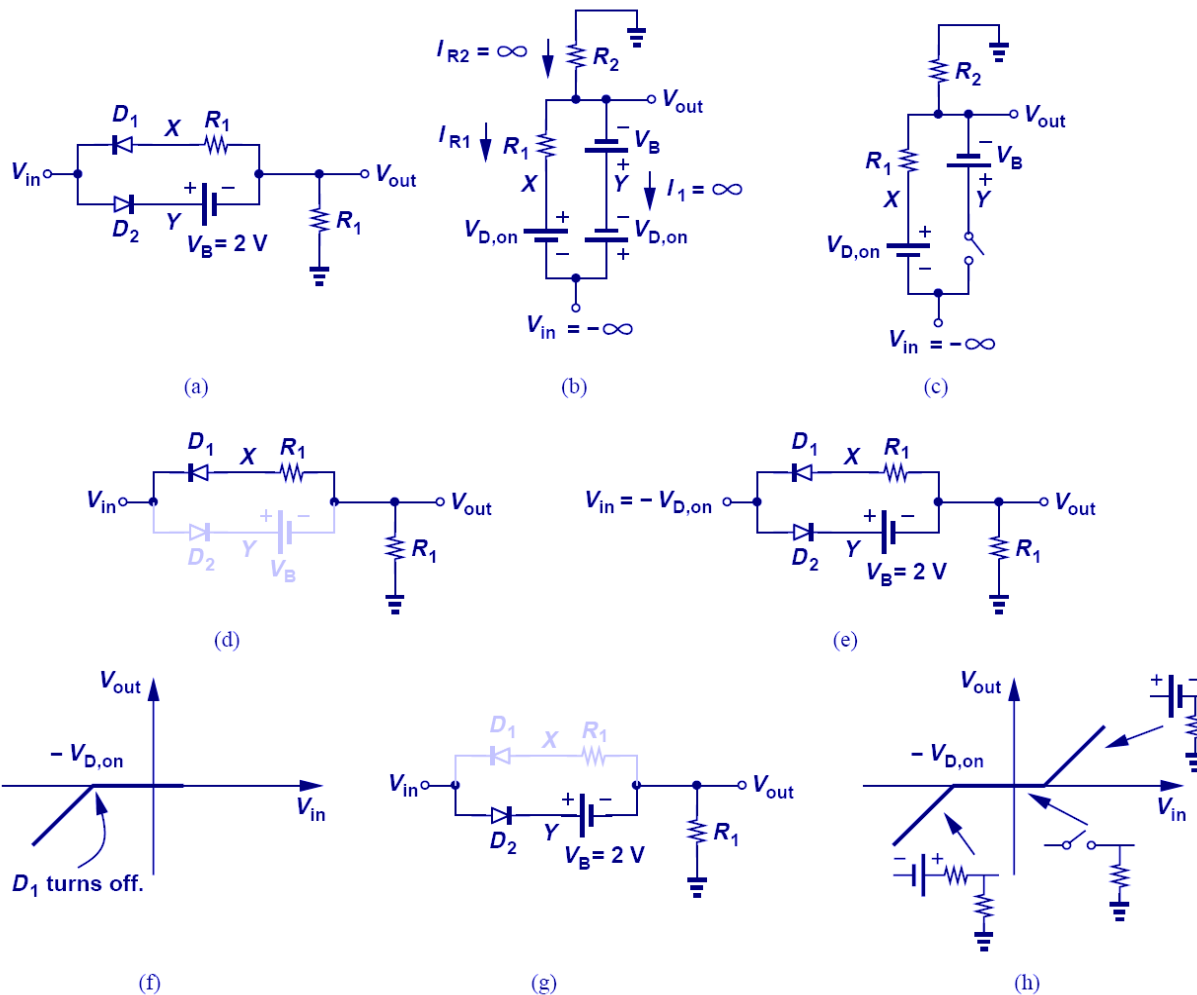
(c)



(d)

- In this example, since V_{in} is connected to the cathode, the diode conducts when V_{in} is very negative.
- The break point where the slope changes is when the current across R_1 is equal to the current across R_2 .

Another Constant-Voltage Model Example

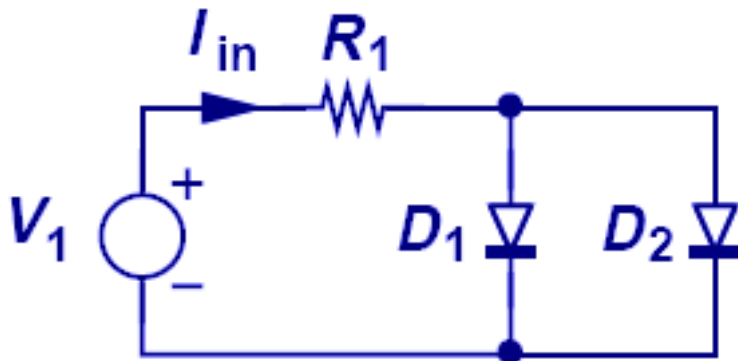


➤ This example shows the importance of good initial guess and careful confirmation.

Example: Constant Voltage Model

Example: Constant Voltage Model

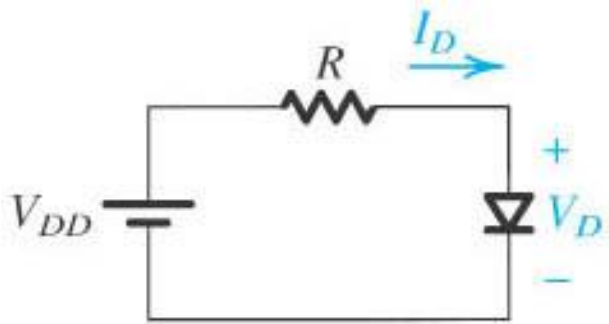
Exponential Model



$$I_{D1} = \frac{I_{in}}{1 + \frac{I_{s2}}{I_{s1}}}$$
$$I_{D2} = \frac{I_{in}}{1 + \frac{I_{s1}}{I_{s2}}}$$

- In this example, since the two diodes have different cross-section areas, only exponential model can be used.
- The two currents are solved by summing them with I_{in} , and equating their voltages.

Iterative Analysis Using the Exponential Model



$$I_1 = I_s e^{V_1/V_T} \quad I_2 = I_s e^{V_2/V_T}$$

$$V_2 - V_1 = V_T \ln \frac{I_2}{I_1}$$

- Determine the current and the diode voltage V_D for the circuit above with $V_{DD}=5V$ and $R=1K\Omega$. Assume that the diode has a current of 1 mA at a voltage of 0.7V.
- Apply KVL or KCL as usual
- Drive two independent equations between I_D and V_D
- Start with good initial guess for either I_D or V_D
- Iterate between two equations of I_D and V_D until values of I_D or V_D is very close.

How to choose a diode model to utilize?

- Utilize the ideal model so as to develop a quick understanding of the circuit
- If the ideal model is insufficient, employ the constant-voltage model
- For more accurate analysis with smaller signal levels, we need to resort to the exponential model.
 - Exponential model is often complicated.
 - Thus, we do first approximation to exponential model → Small-signal model

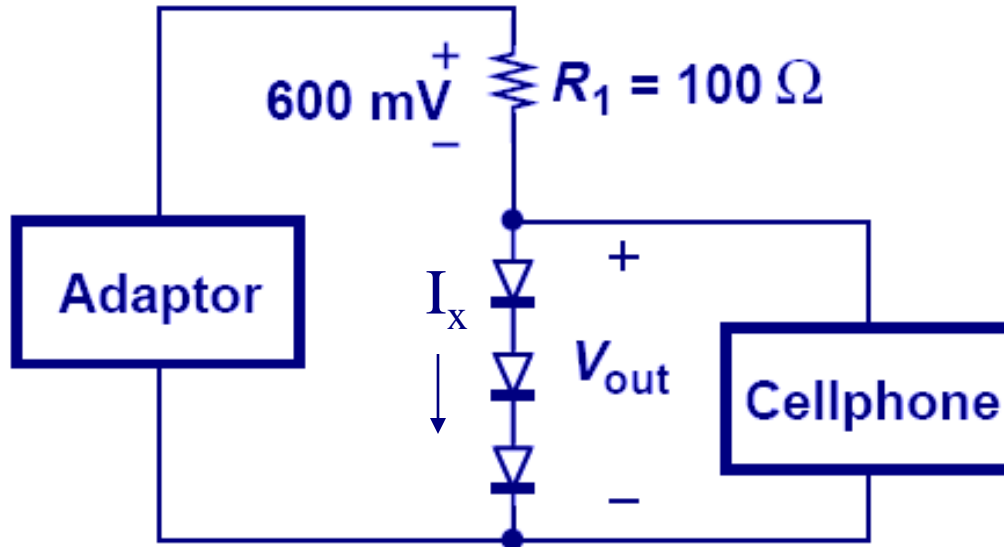
$$\text{Exp}[x] \approx 1 + x + \frac{x^2}{2} + \dots$$

HOT for $\text{abs}(x) \ll 1$

Cell Phone Adapter

$$V_{ad} = 3V$$

$$V_{out} = 2.4V, I_X = 6mA$$



$$I_s = 2.602 \times 10^{-16} \text{ A and } V_T = 26 \text{ mV}$$

➤ $V_{out} = 3V_{D,on}$ is used to charge cell phones.

➤ However, if I_X changes, iterative method is often needed to obtain a solution, thus motivating a simpler technique.

$$V_{ad} \rightarrow 3.1V$$

$$V_{out} = 3V_D$$

$$= 3V_T \ln \frac{I_X}{I_s}$$

$$I_X = \frac{V_{ad} - V_{out}}{R_1}$$

$$V_{out} = 2.411V$$

Large-Signal and Small-Signal Operations

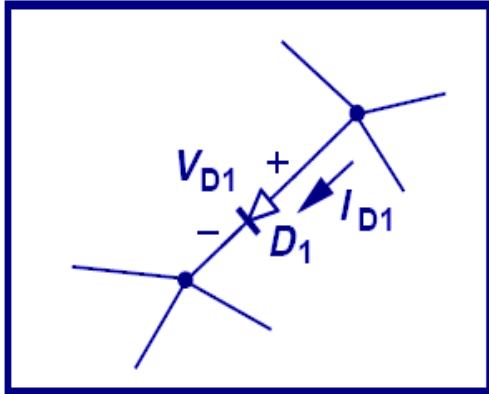
➤ Large-Signal Operation

- “General Model” such as exponential I/V characteristics
- Arbitrarily large voltage and current changes
- Complicates the analysis

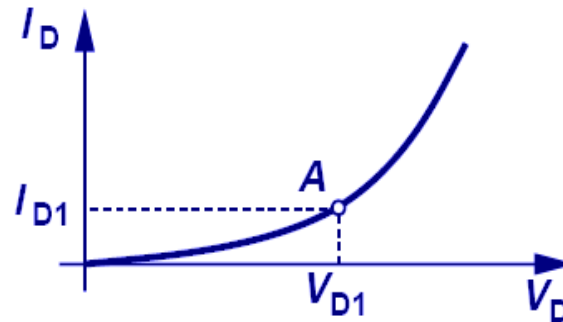
➤ Small-Signal Operation

$$\text{Exp}[x] \approx 1 + x + \underbrace{x^2/2 + \dots}_{\text{HOT for } \text{abs}(x) \ll 1}$$

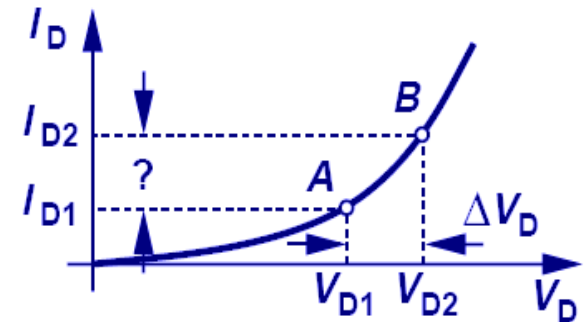
Small-Signal Analysis



(a)



(b)

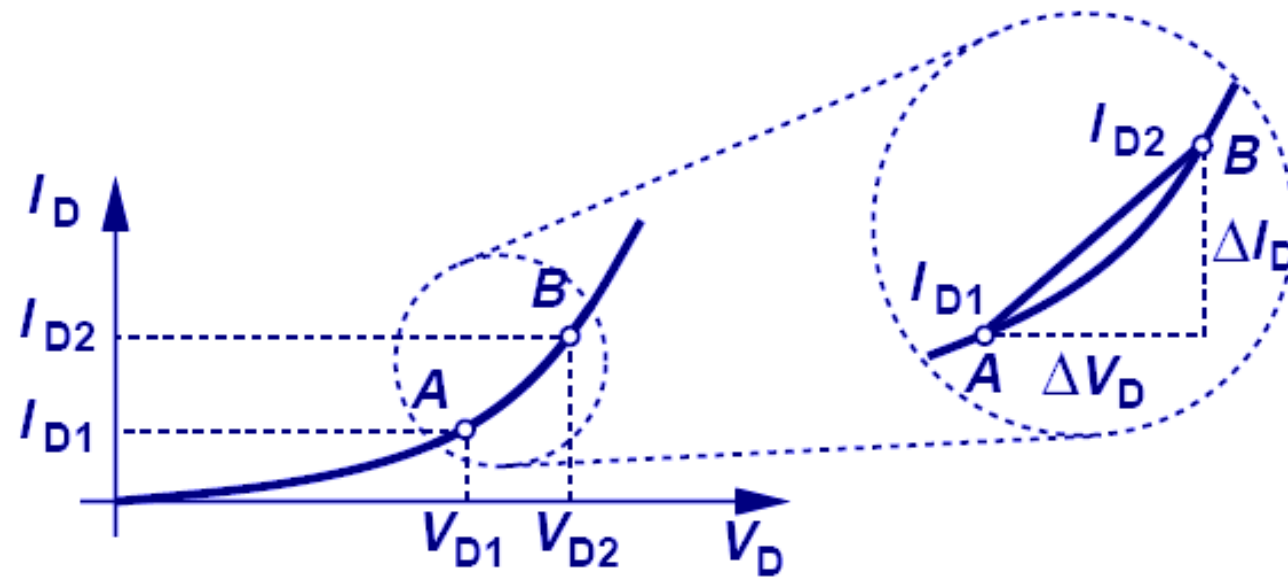


(c)

$$\Delta I_D = \frac{\Delta V}{V_T} I_{D1}$$

- **Small-signal analysis is performed around a bias point by perturbing the voltage by a small amount and observing the resulting linear current perturbation.**

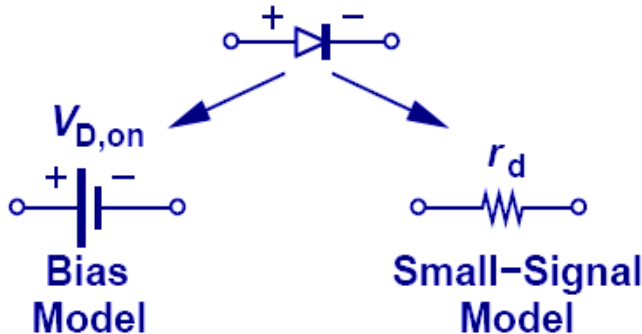
Small-Signal Analysis in Detail



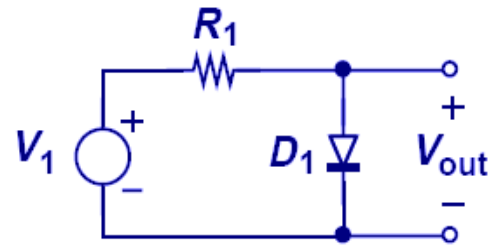
$$\begin{aligned}\frac{\Delta I_D}{\Delta V_D} &= \left. \frac{dI_D}{dV_D} \right|_{V_D=V_{D1}} \\ &= \frac{I_s}{V_T} \exp \frac{I_{D1}}{V_T} \\ &= \frac{I_{D1}}{V_T}\end{aligned}$$

- If two points on the IV curve of a diode are close enough, the trajectory connecting the first to the second point is like a line, with the slope being the proportionality factor between change in voltage and change in current.

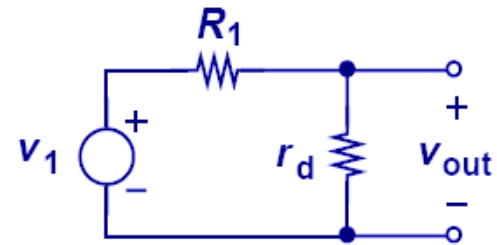
Small-Signal Incremental Resistance



(a)



(b)



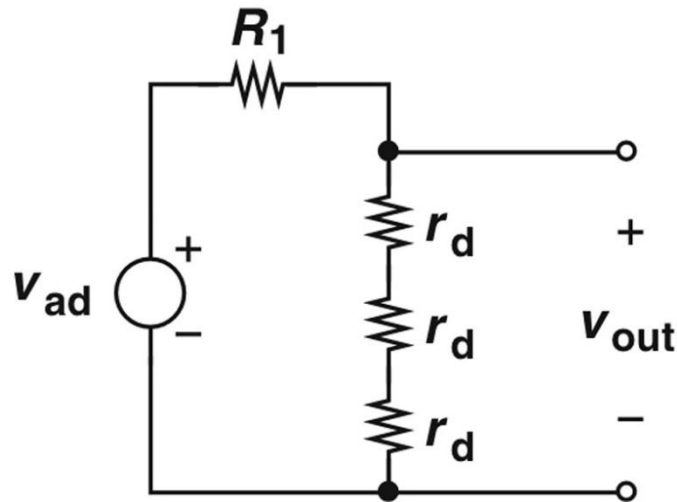
(c)

Small-signal resistance

$$r_d = \frac{V_T}{I_D}$$

- Since there's a linear relationship between the small signal current and voltage of a diode, the diode can be viewed as a linear resistor when only small changes are of interest.

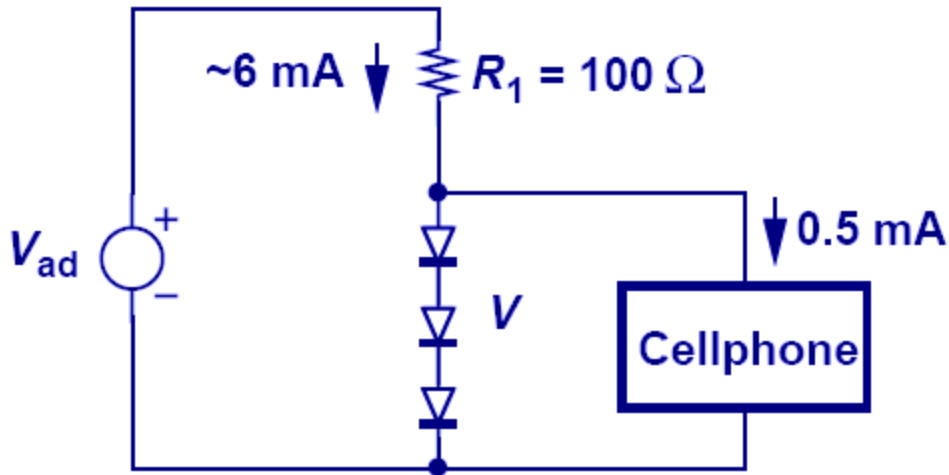
Adapter Example Revisited



$$v_{out} = \frac{3r_d}{R_1 + 3r_d} v_{ad}$$
$$= 11.5mV$$

- With our understanding of small-signal analysis, we can revisit our cell phone charger example and easily solve it with just algebra instead of iterations.

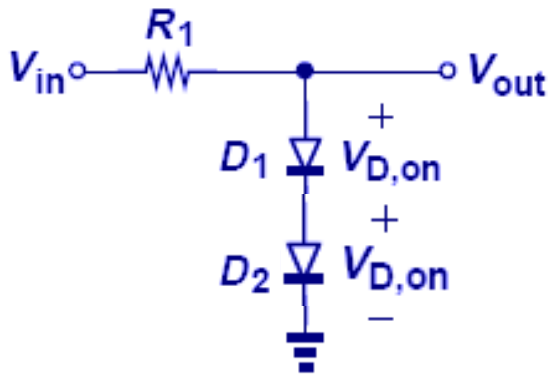
Simple is Beautiful



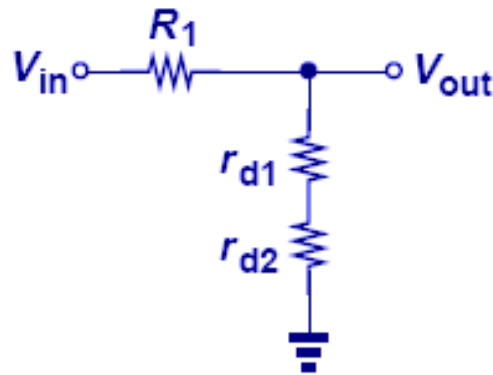
$$\begin{aligned}\Delta V_{out} &= \Delta I_D \cdot (3r_d) \\ &= 0.5 \text{ mA} (3 \times 4.33 \Omega) \\ &= 6.5 \text{ mV}\end{aligned}$$

- In this example we study the effect of cell phone pulling some current from the diodes. Using small signal analysis, this is easily done. However, imagine the nightmare, if we were to solve it using non-linear equations.

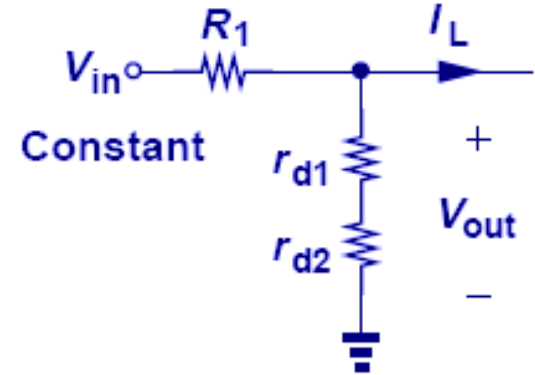
Line Regulation VS. Load Regulation



(a)



(b)



(c)

$$\frac{V_{out}}{V_{in}} = \frac{r_{D1} + r_{D2}}{r_{D1} + r_{D2} + R_1}$$

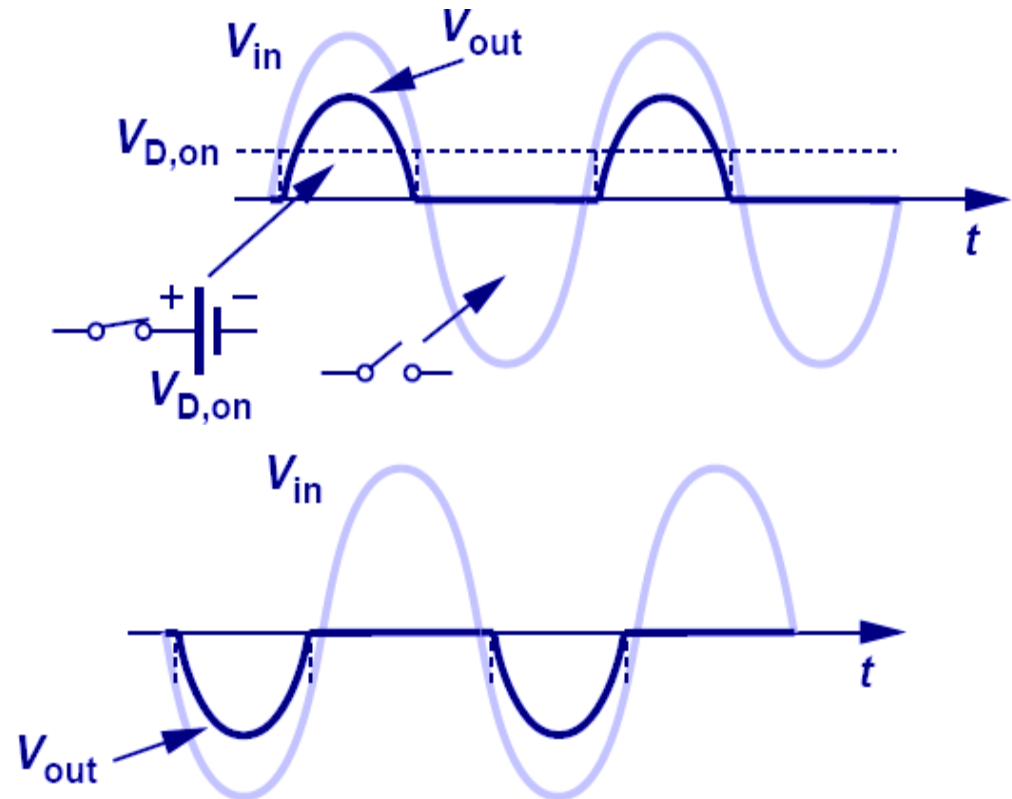
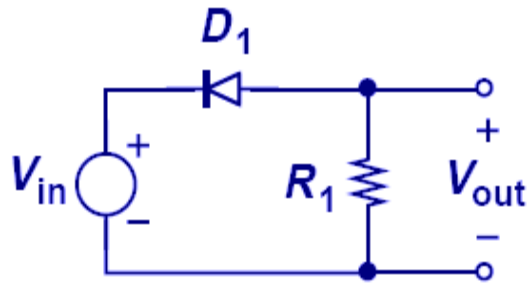
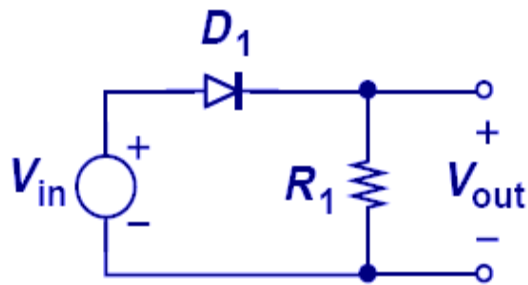
$$\left| \frac{V_{out}}{i_L} \right| = (r_{D1} + r_{D2}) \parallel R_1$$

- **Line regulation is the suppression of change in V_{out} due to change in V_{in} (b).**
- **Load regulation is the suppression of change in V_{out} due to change in load current (c).**

Applications of Diode

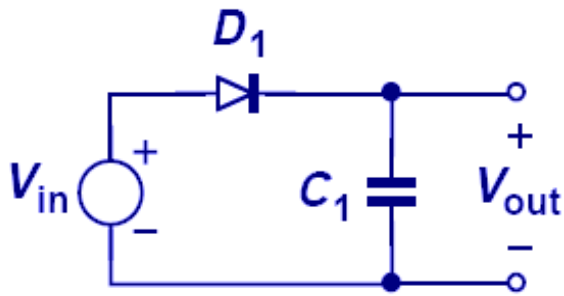


Half-Wave Rectifier

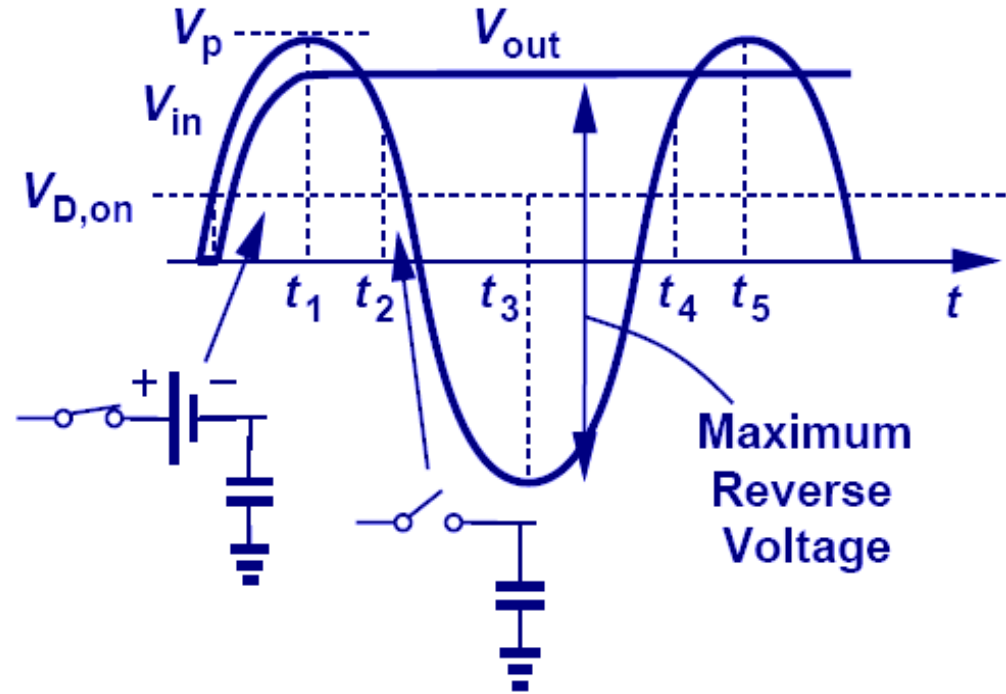


- A very common application of diodes is half-wave rectification, where either the positive or negative half of the input is blocked.
- But, how do we generate a constant output?

Diode-Capacitor Circuit: Constant Voltage Model



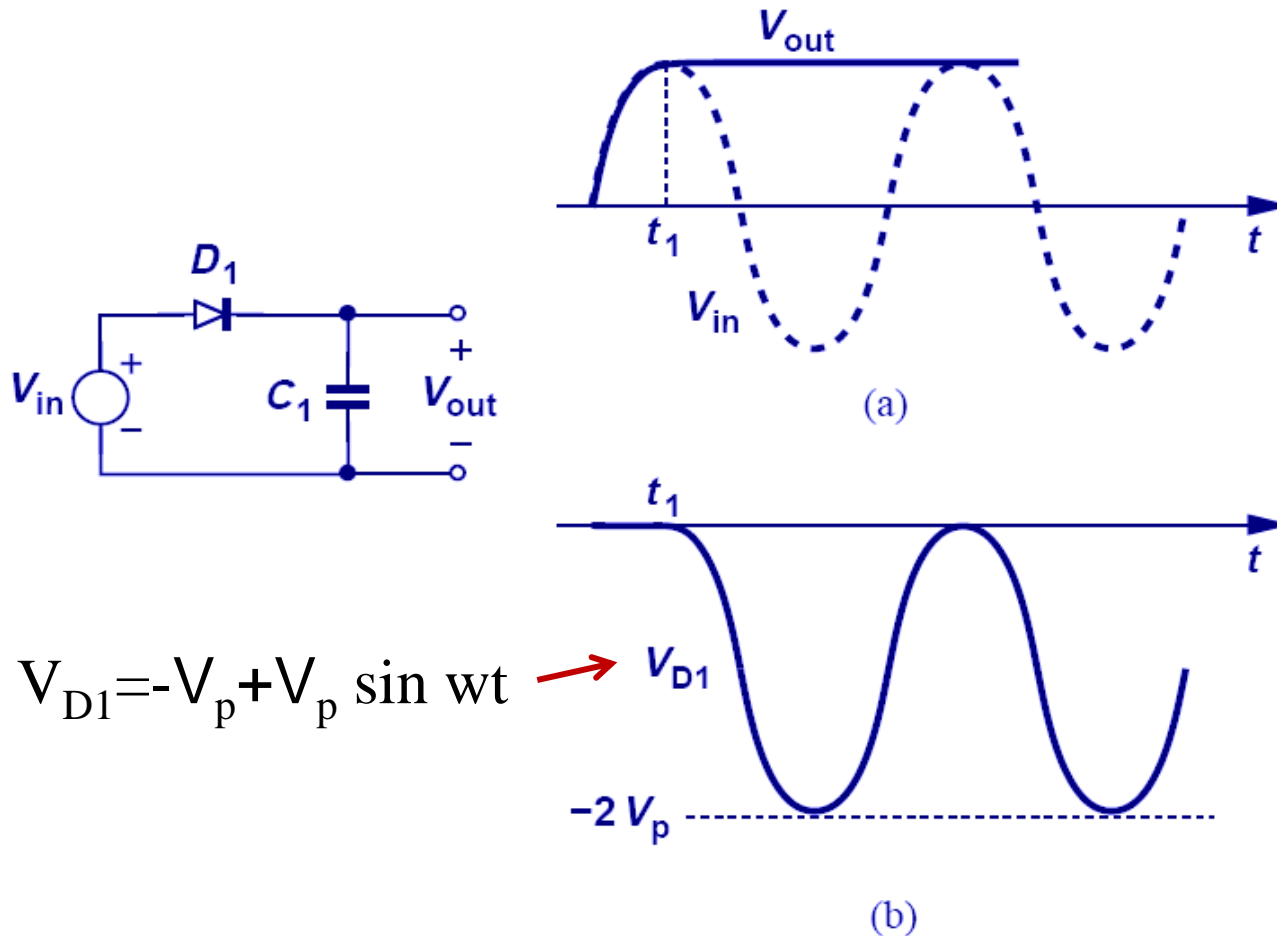
(a)



(b)

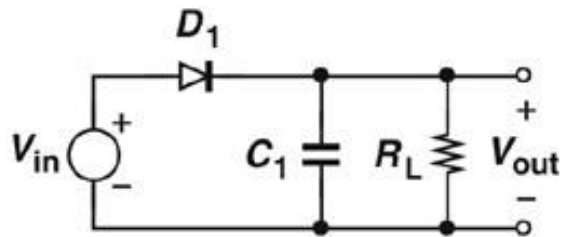
- If the resistor in half-wave rectifier is replaced by a capacitor, a fixed voltage output is obtained since the capacitor (assumed ideal) has no path to discharge.

Diode-Capacitor Circuit: Ideal Model

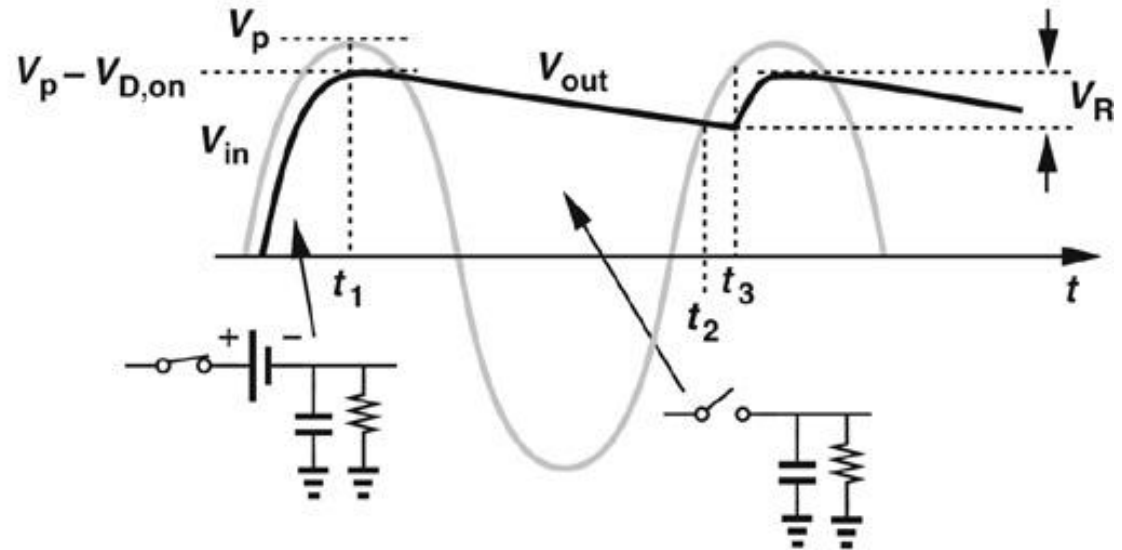


➤ Note that (b) is just like V_{in} , only shifted down.

Diode-Capacitor With Load Resistor



(a)



(b)

- A path is available for capacitor to discharge. Therefore, V_{out} will not be constant and a ripple exists.

Peak to Peak amplitude of Ripple

$$e^{-x} \approx 1 - x \quad \text{for } x \ll 1$$

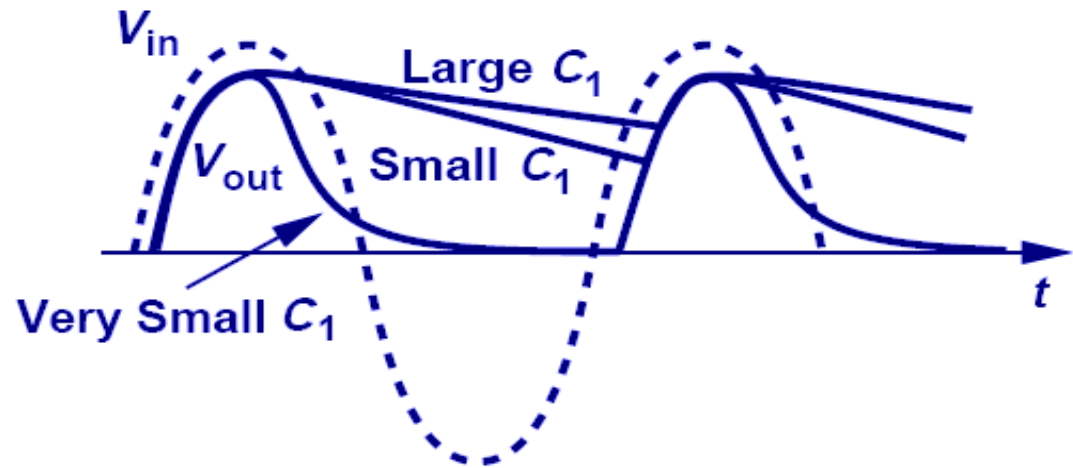
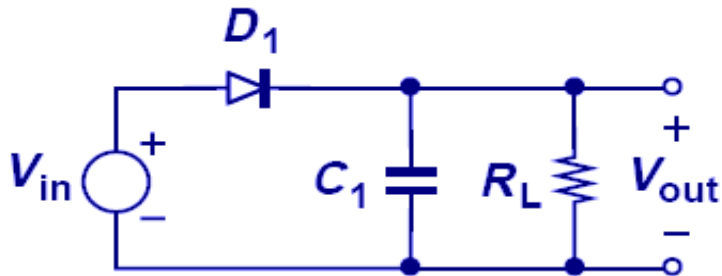
$$V_{out}(t) = (V_p - V_{D,on}) \exp \frac{-t}{R_L C_1} \quad 0 \leq t \leq T_{in}$$

$$V_{out}(t) \approx (V_p - V_{D,on}) \left(1 - \frac{t}{R_L C_1}\right) \approx (V_p - V_{D,on}) - \underbrace{\frac{V_p - V_{D,on}}{R_L} \frac{t}{C_1}}_{V_R}$$

$$V_R \approx \frac{V_p - V_{D,on}}{R_L} \cdot \frac{T_{in}}{C_1} \approx \frac{V_p - V_{D,on}}{R_L C_1 f_{in}} \quad t \approx T_{in}$$

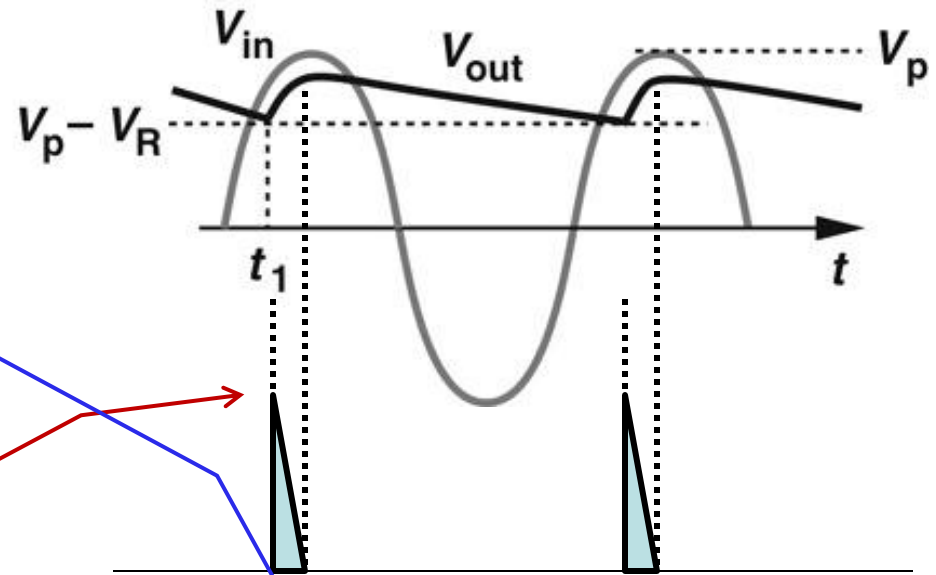
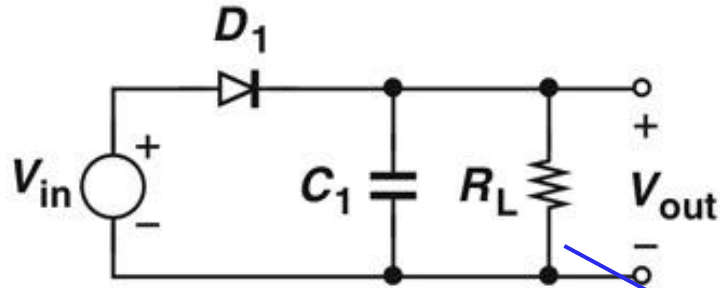
- The ripple amplitude is the decaying part of the exponential.
- Ripple voltage becomes a problem if it goes above 5 to 10% of the output voltage.

Behavior for Different Capacitor Values



➤ For large C_1 , V_{out} has small ripple.

Maximum Diode Current



$$V_p - V_R = V_p \sin \omega_{in} t_1$$

$$I_{D1}(t) = C_1 \frac{dV_{out}(t)}{dt} + \frac{V_p}{R_L}$$

$$= C_1 \omega_{in} V_p \cos \omega_{in} t_1 + \frac{V_p}{R_L}$$

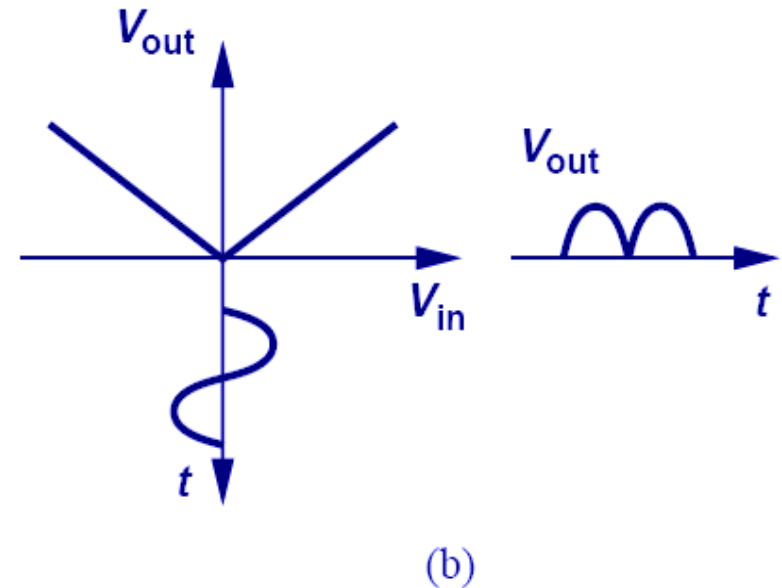
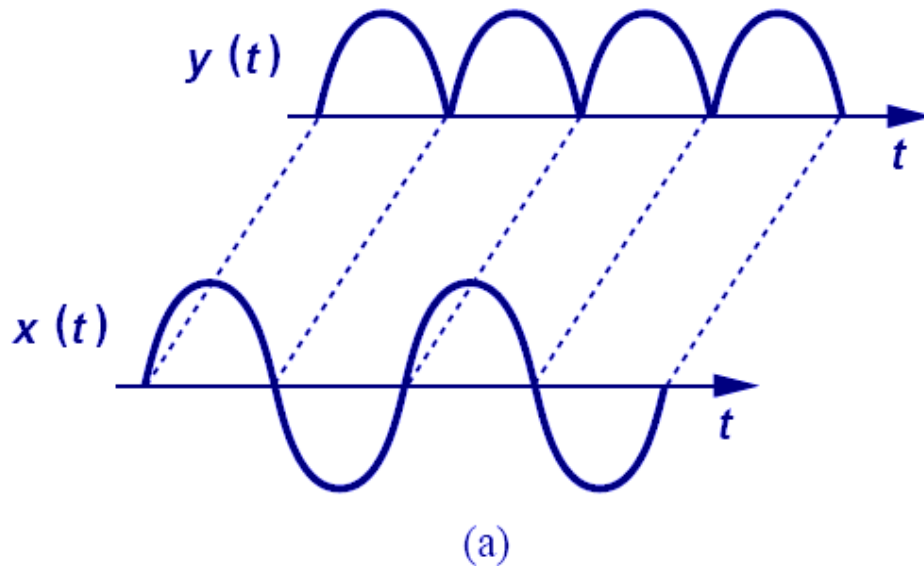
This current reaches a peak at $t=t_1$

$$I_p = C_1 \omega_{in} V_p \sqrt{1 - \left(1 - \frac{V_R}{V_p}\right)^2} + \frac{V_p}{R_L}$$

$$I_p \approx C_1 \omega_{in} V_p \sqrt{\frac{2V_R}{V_p} + \frac{V_p}{R_L}} \approx \frac{V_p}{R_L} \left(R_L C_1 \omega_{in} \sqrt{\frac{2V_R}{V_p} + 1} \right)$$

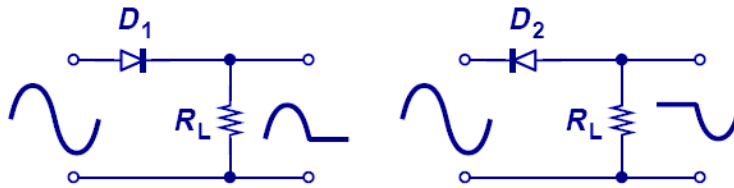
- The diode has its maximum current at t_1 , since that's when the slope of V_{out} is the greatest.
- This current has to be carefully controlled so it does not damage the device.

Full-Wave Rectifier

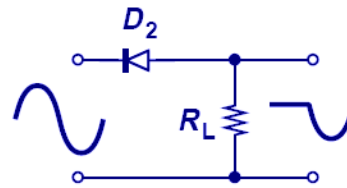


- A full-wave rectifier passes both the negative and positive half cycles of the input, while inverting the negative half of the input.
- As proved later, a full-wave rectifier reduces the ripple by a factor of two.

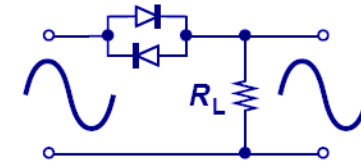
The Evolution of Full-Wave Rectifier



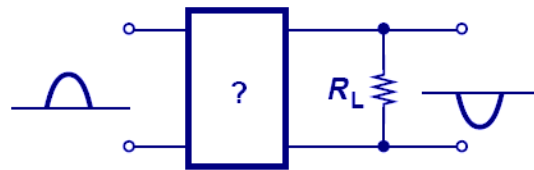
(a)



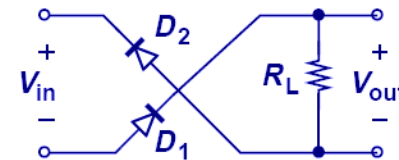
(b)



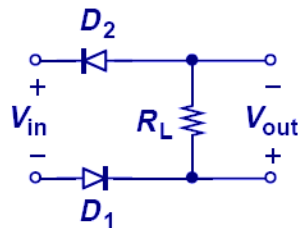
(c)



(d)



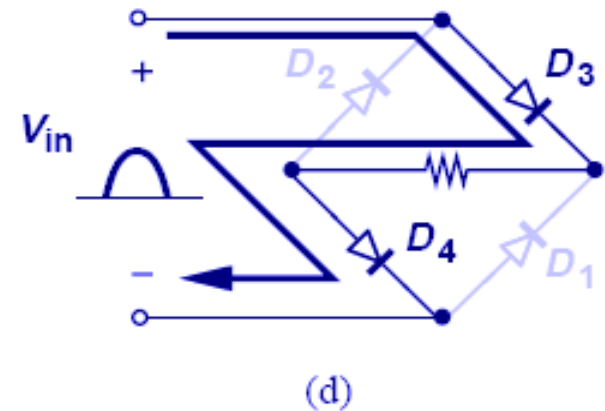
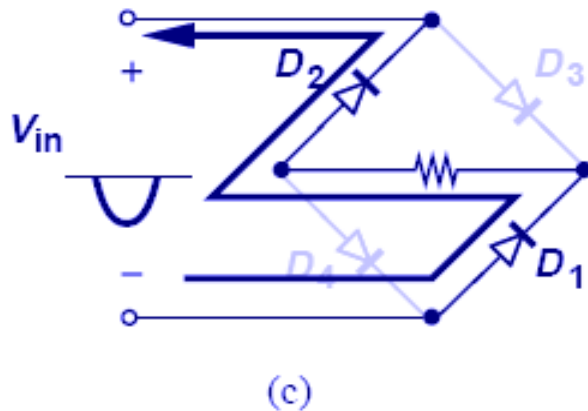
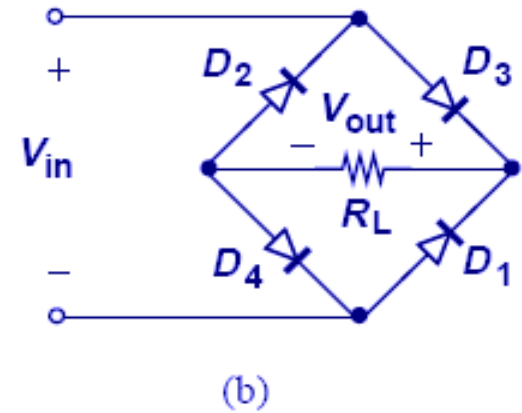
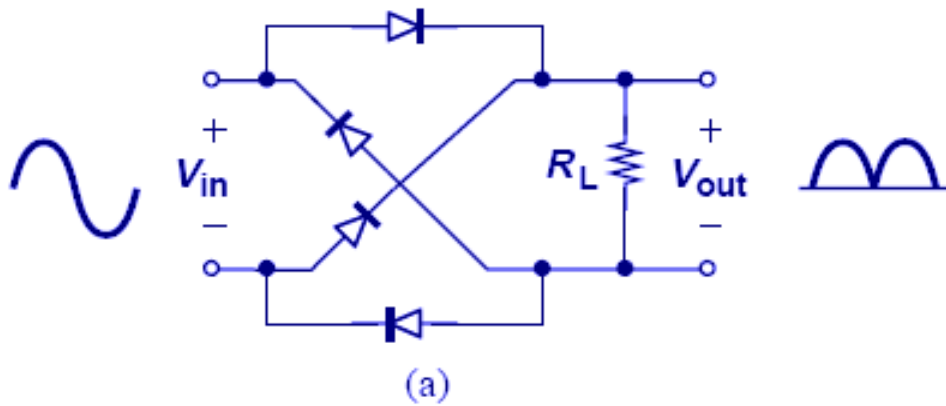
(e)



(f)

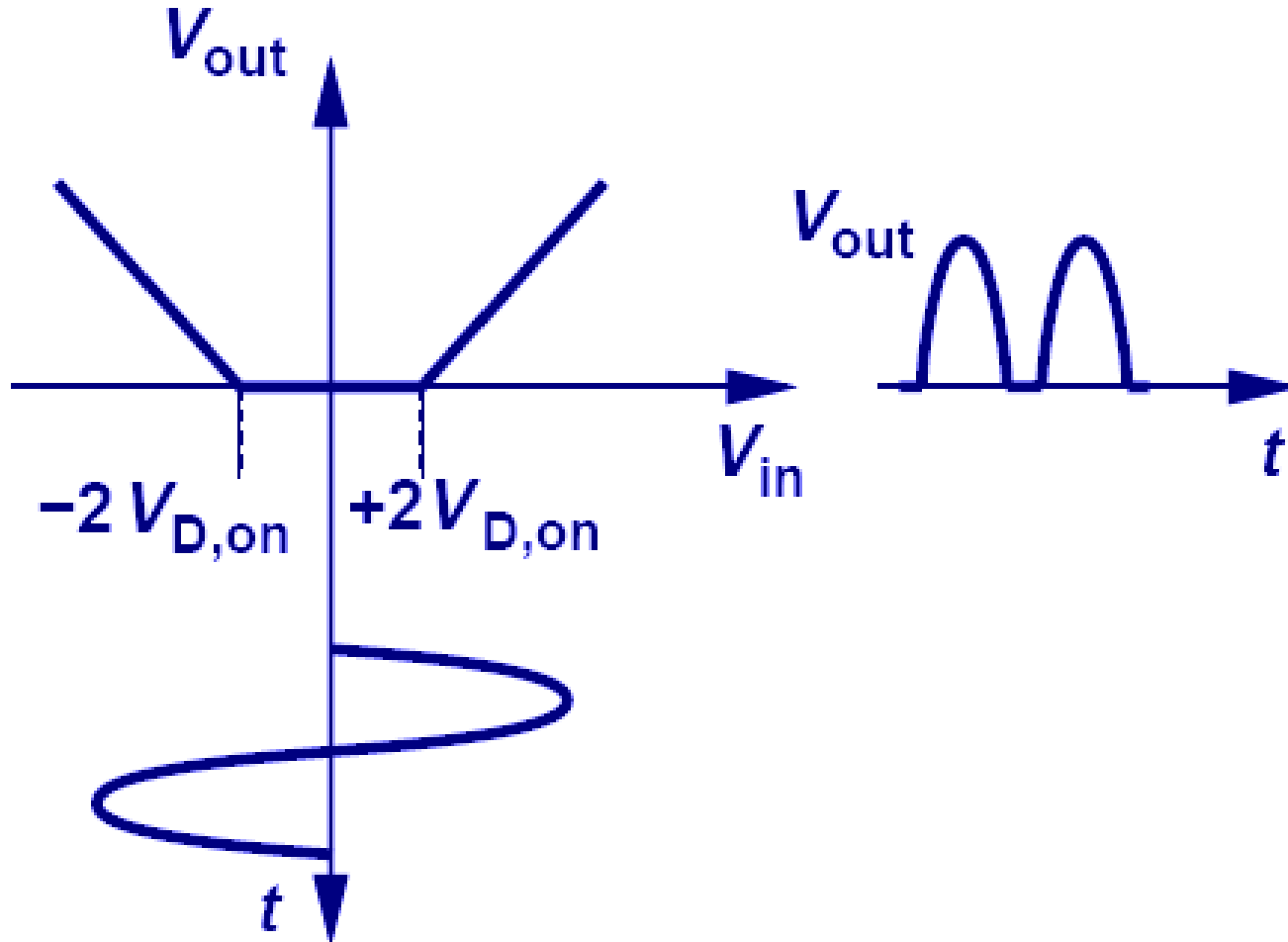
➤ Figures (d) and (e) show the topology that inverts the negative half cycle of the input.

Full-Wave Rectifier: Bridge Rectifier



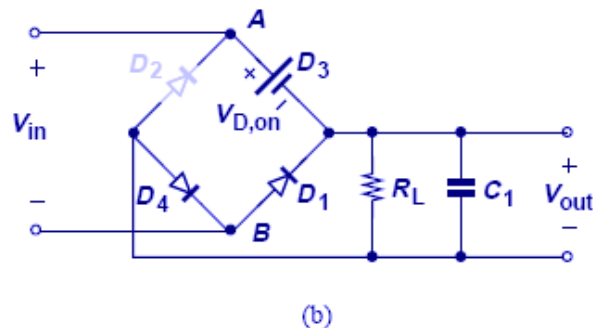
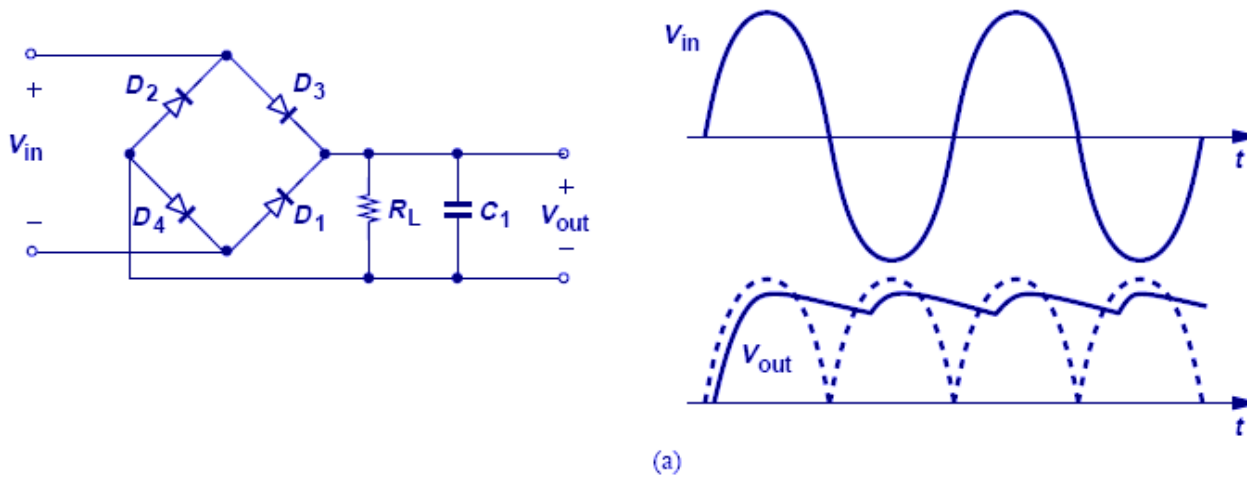
➤ The figure above shows a full-wave rectifier, where D_1 and D_2 pass/invert the negative half cycle of input and D_3 and D_4 pass the positive half cycle.

Input/Output Characteristics of a Full-Wave Rectifier (Constant-Voltage Model)



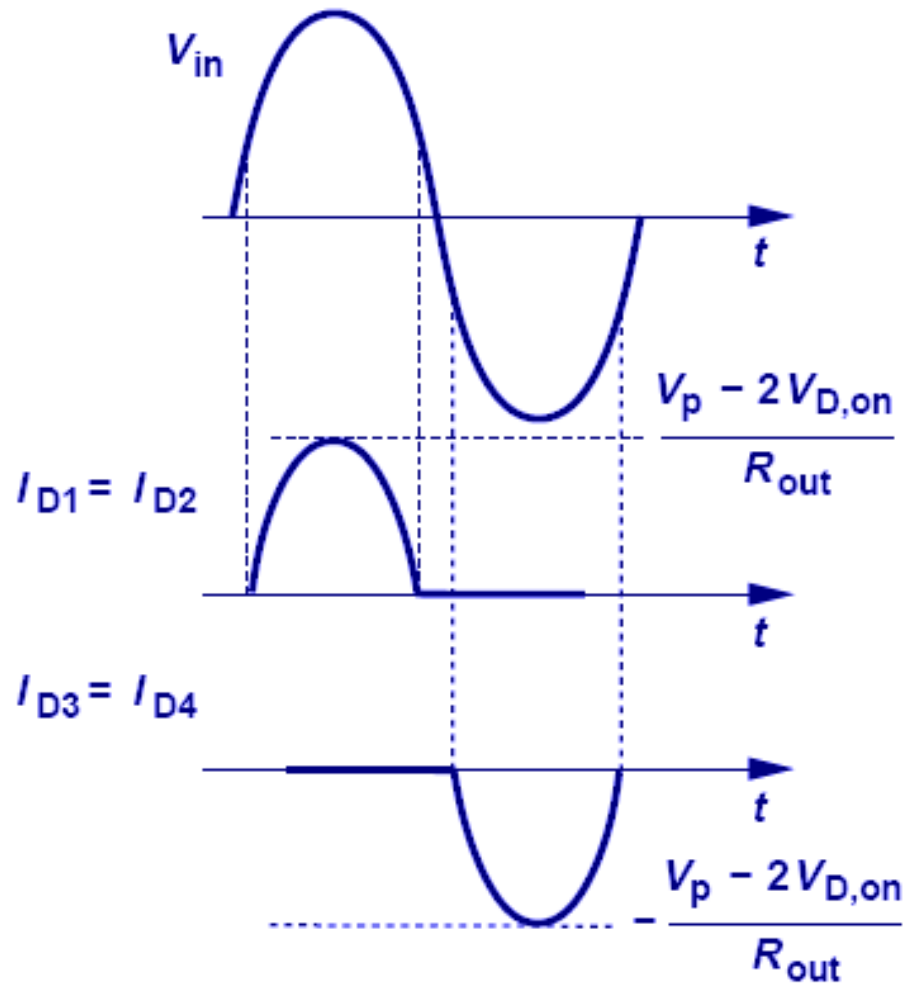
- The dead-zone around V_{in} arises because V_{in} must exceed $2V_{D,ON}$ to turn on the bridge.

Complete Full-Wave Rectifier

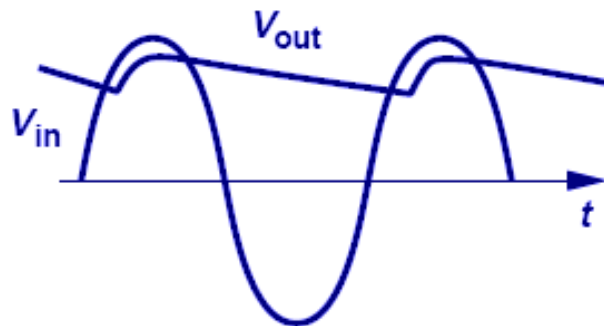
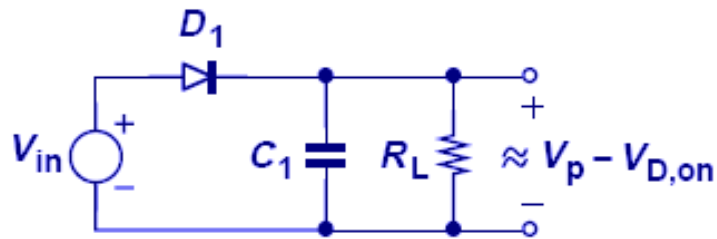


- Since C_1 only gets $\frac{1}{2}$ of period to discharge, ripple voltage is decreased by a factor of 2. Also (b) shows that each diode is subjected to approximately one V_p reverse bias drop (versus $2V_p$ in half-wave rectifier).

Current Carried by Each Diode in the Full-Wave Rectifier

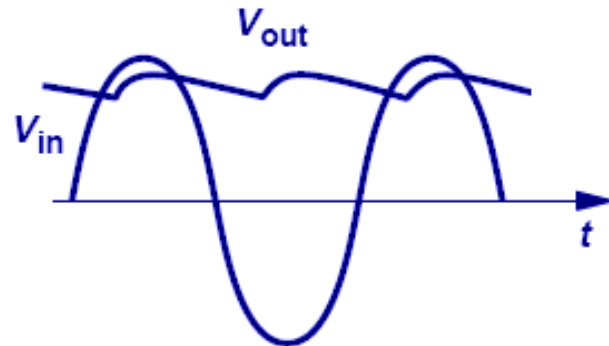
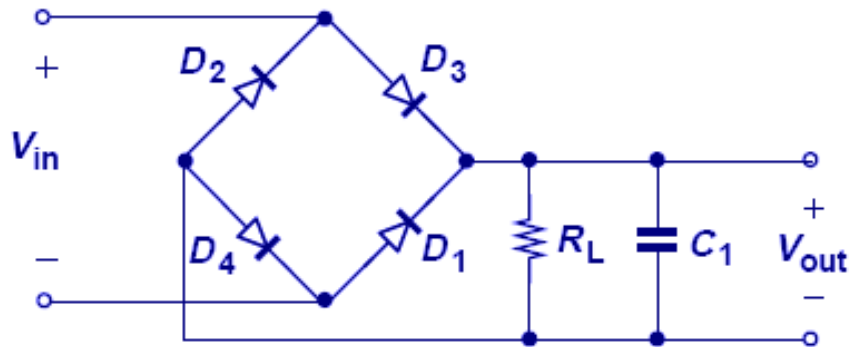


Summary of Half and Full-Wave Rectifiers



Reverse Bias $\approx 2V_p$

(a)

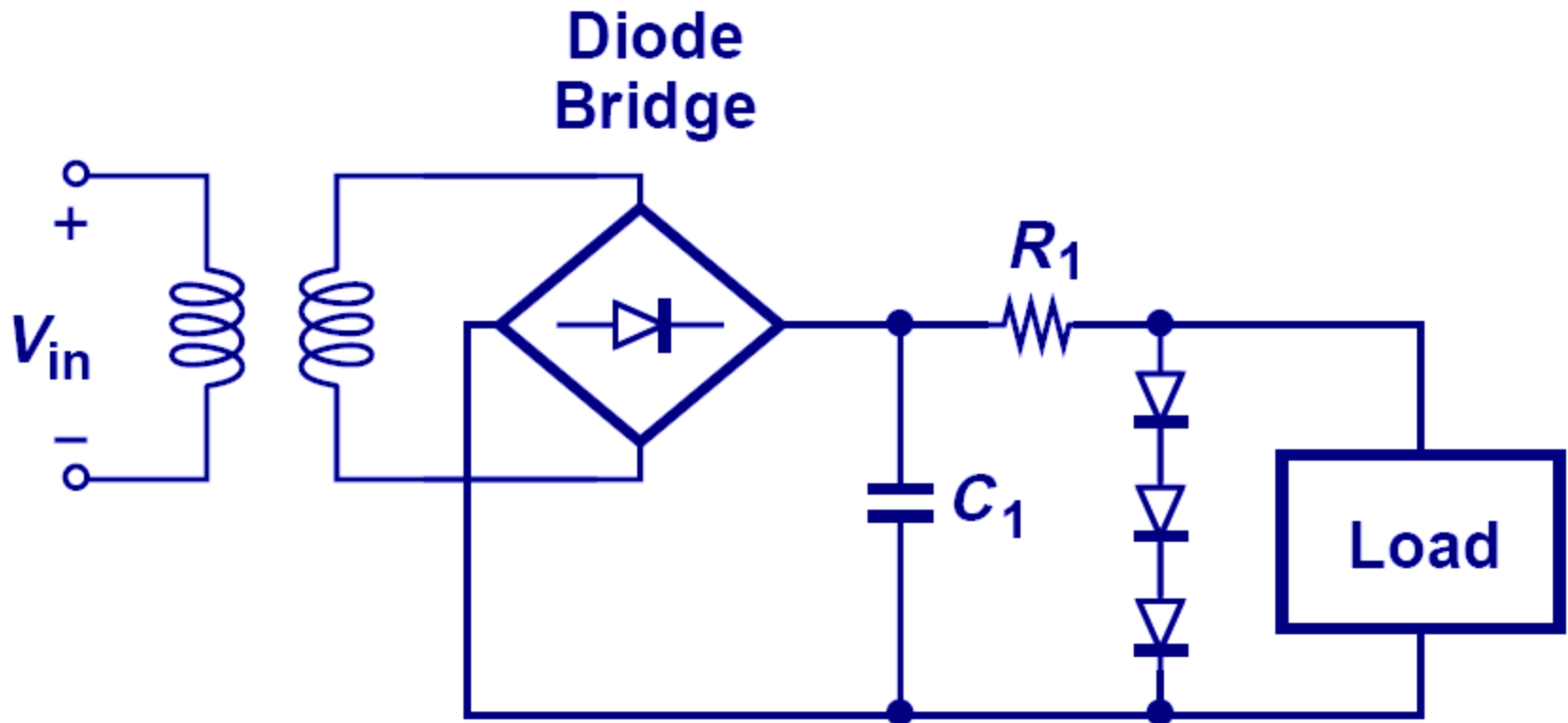


Reverse Bias $\approx V_p$

(b)

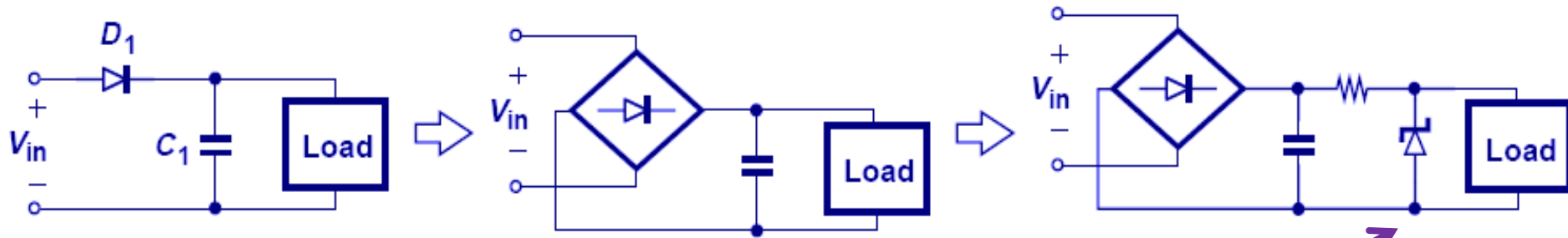
- Full-wave rectifier is more suited to adapter and charger applications.

Voltage Regulator

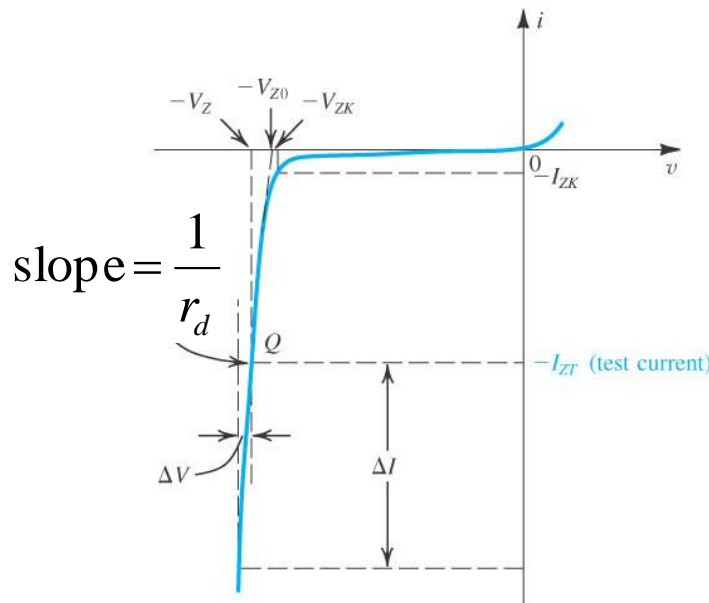
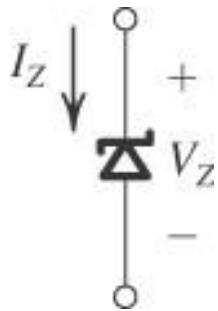


- The ripple created by the rectifier can be unacceptable to sensitive load; therefore, a regulator is required to obtain a very stable output.
- Three diodes operate as a primitive regulator.

Evolution of AC-DC Converter

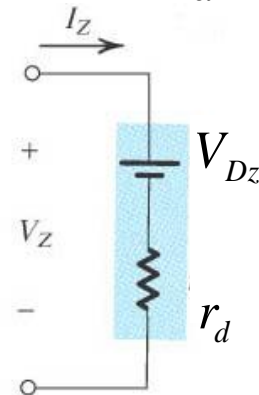


Operation in The Reverse Breakdown Region – Zener Diodes

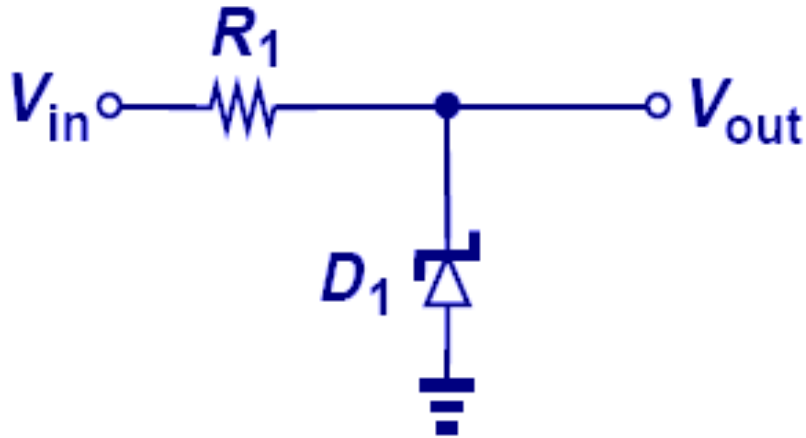


$$V_z = V_{Dz} + r_d I_z$$

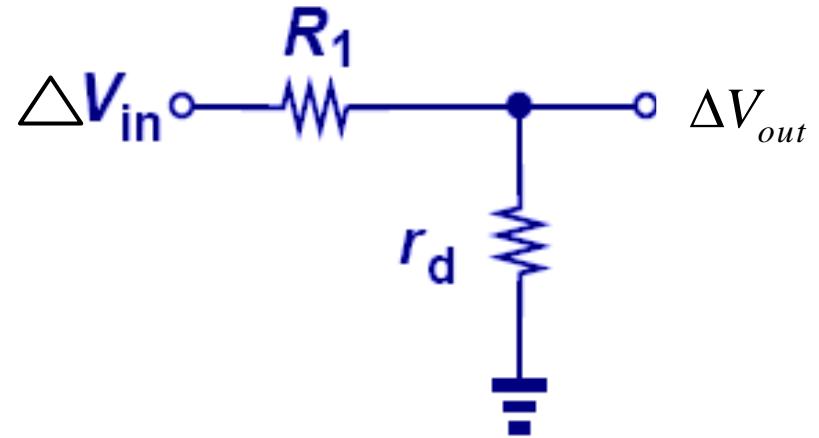
$$\Delta V = r_d \Delta I$$



Voltage Regulation With Zener Diode



(a)

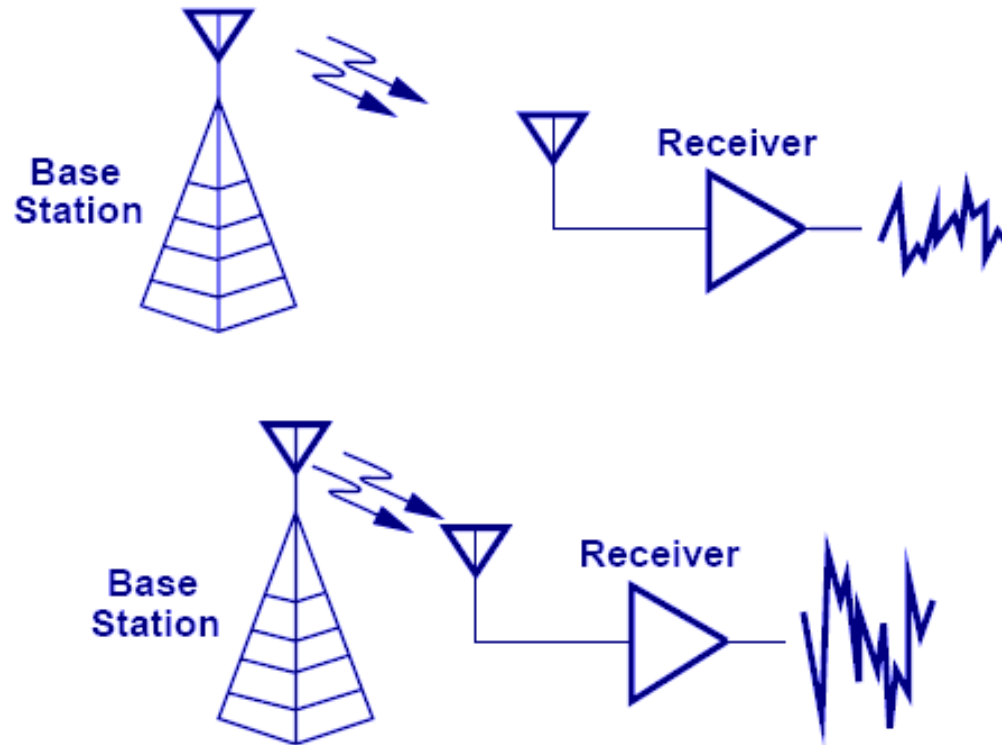


(b)

$$\Delta V_{out} = \frac{r_D}{r_D + R_1} \Delta V_{in}$$

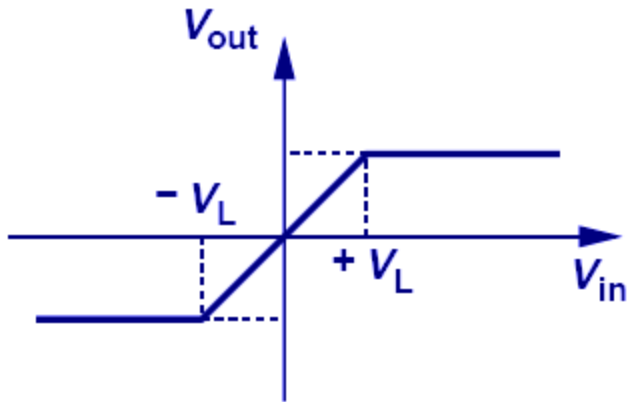
- Voltage regulation can be accomplished with Zener diode. Since r_d is small, large change in the input will not be reflected at the output.

Limiting Circuits

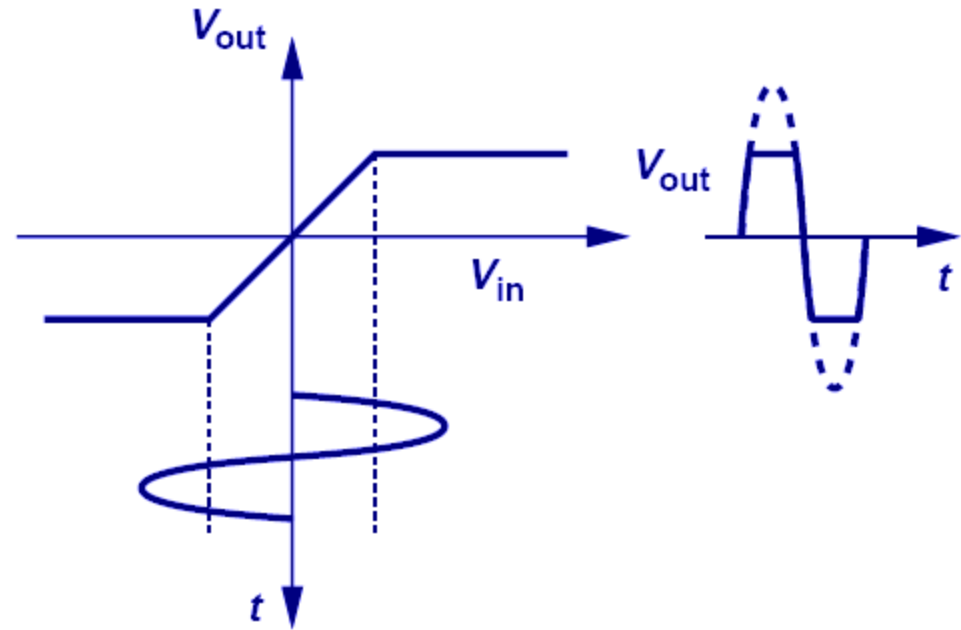


- The motivation of having limiting circuits is to keep the signal below a threshold so it will not saturate the entire circuitry.
- When a receiver is close to a base station, signals are large and limiting circuits may be required.

Input/Output Characteristics



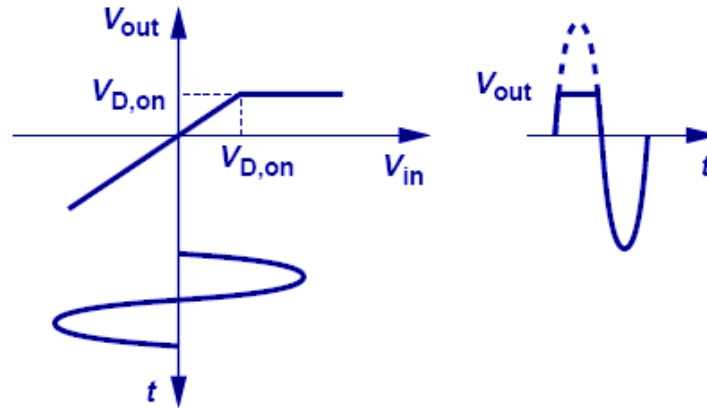
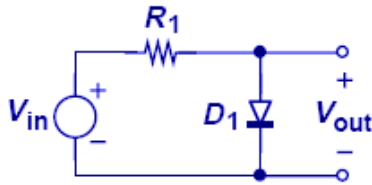
(a)



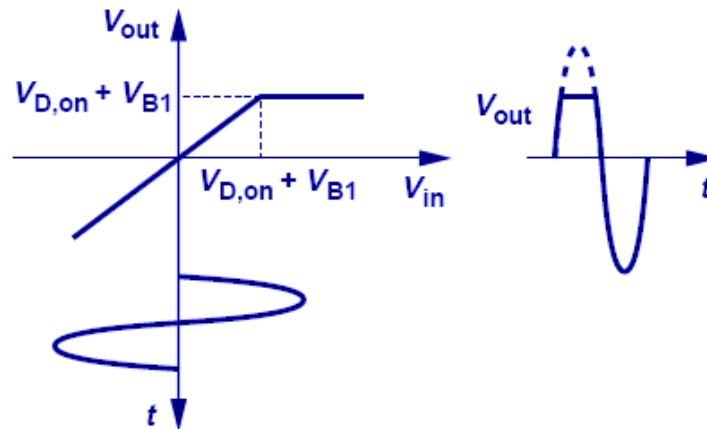
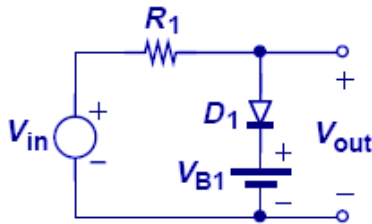
(b)

➤ **Note the clipping of the output voltage.**

Limiting Circuit Using a Diode: Positive Cycle Clipping



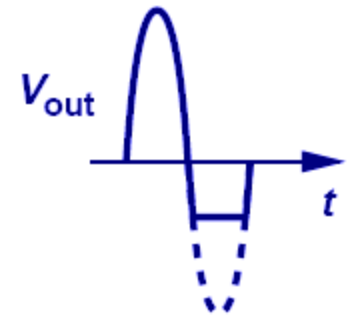
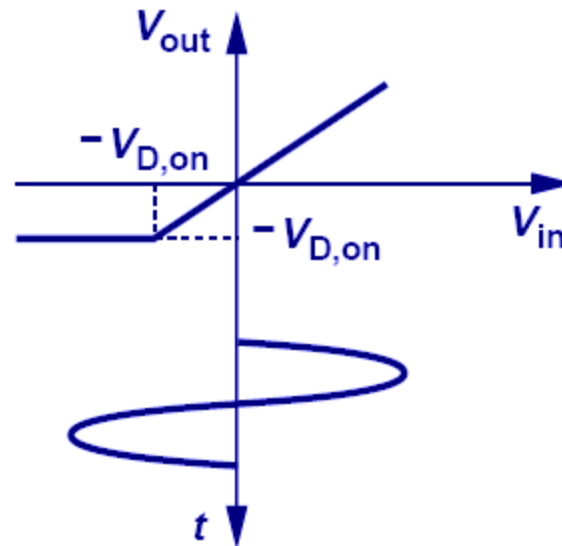
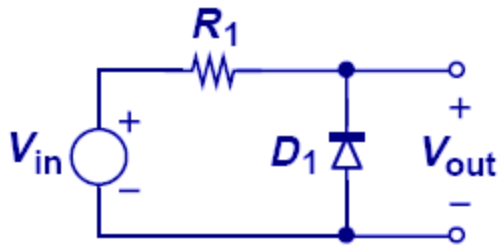
(a)



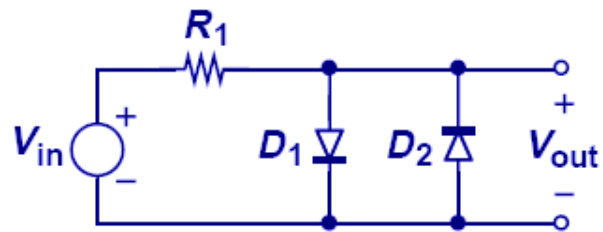
(b)

- As was studied in the past, the combination of resistor-diode creates limiting effect.

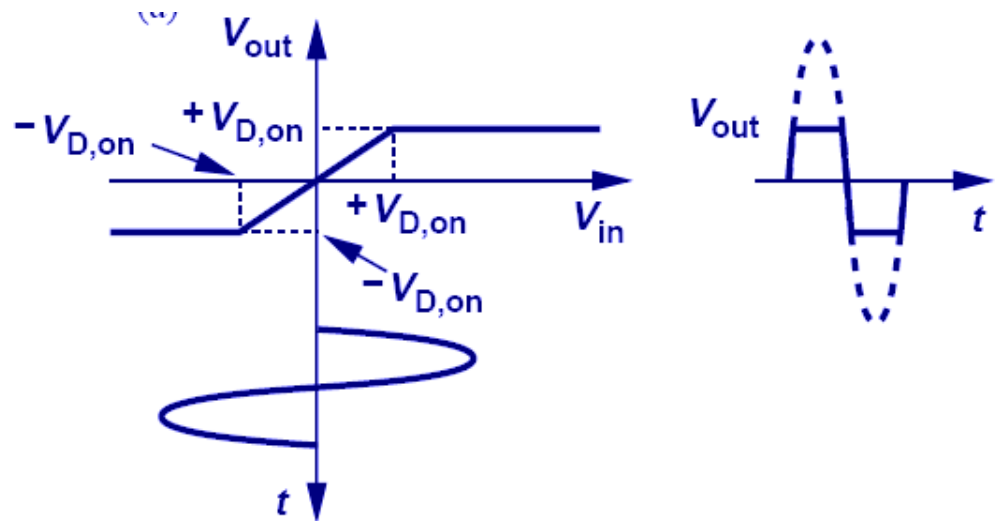
Limiting Circuit Using a Diode: Negative Cycle Clipping



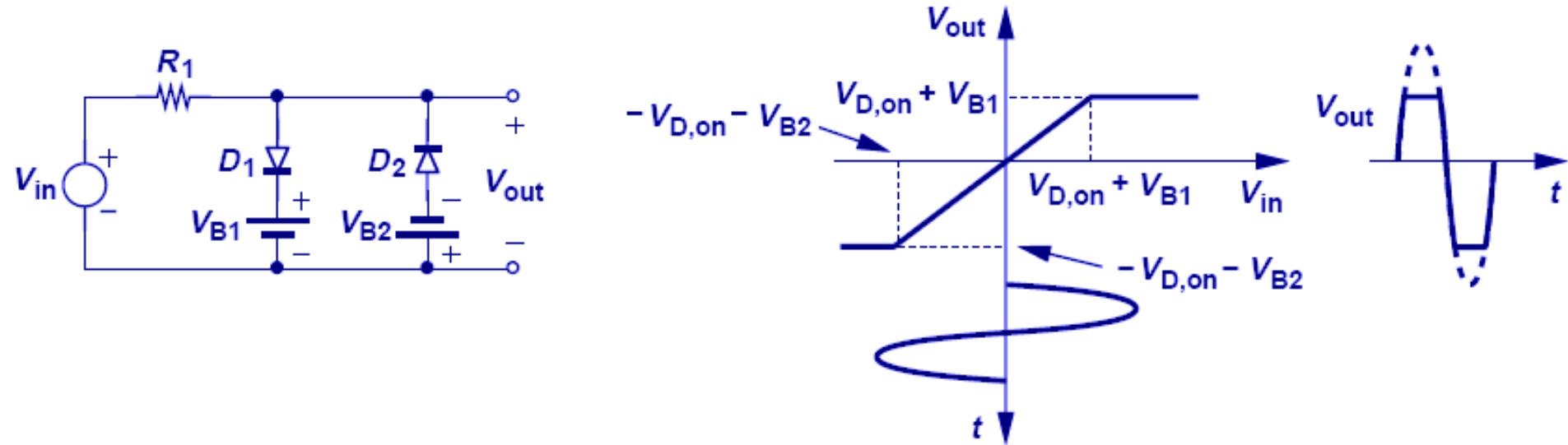
Limiting Circuit Using a Diode: Positive and Negative Cycle Clipping



Anti-parallel
configuration

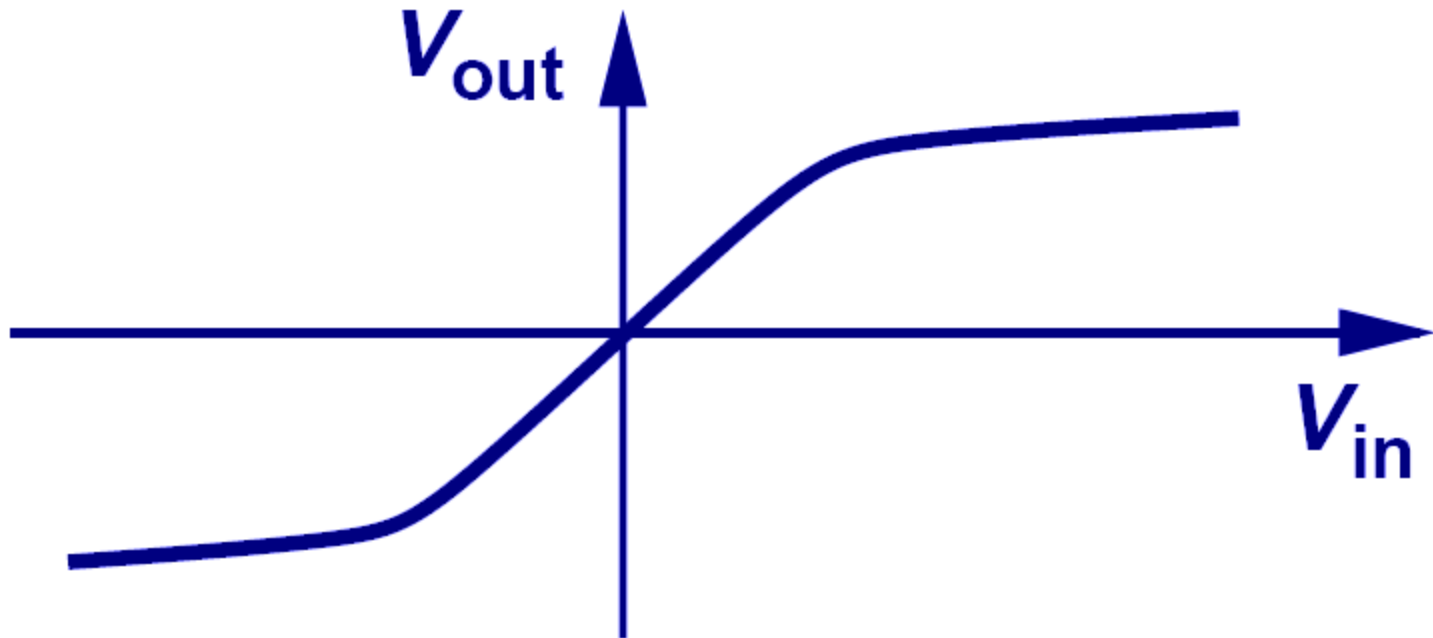


General Voltage Limiting Circuit



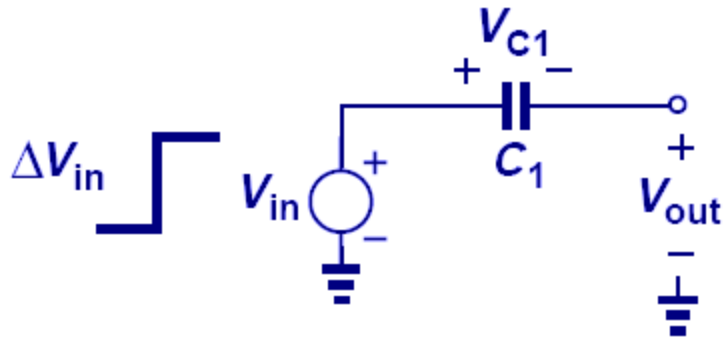
- **Two batteries in series with the antiparalle diodes control the limiting voltages.**

Non-idealities in Limiting Circuits

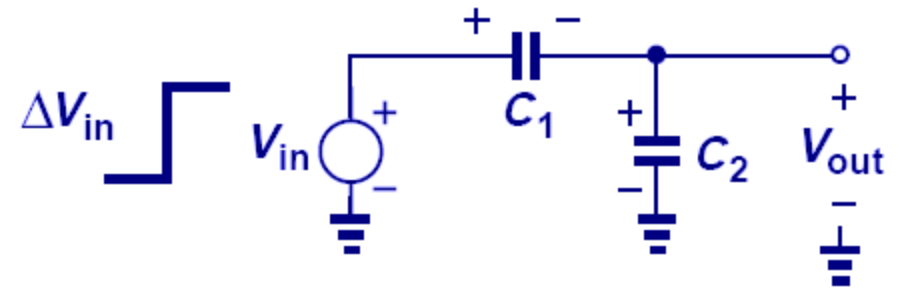


- The clipping region is not exactly flat since as V_{in} increases, the currents through diodes change, and so does the voltage drop.

Capacitive Divider



(a)

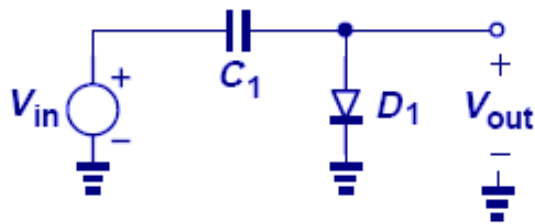


(b)

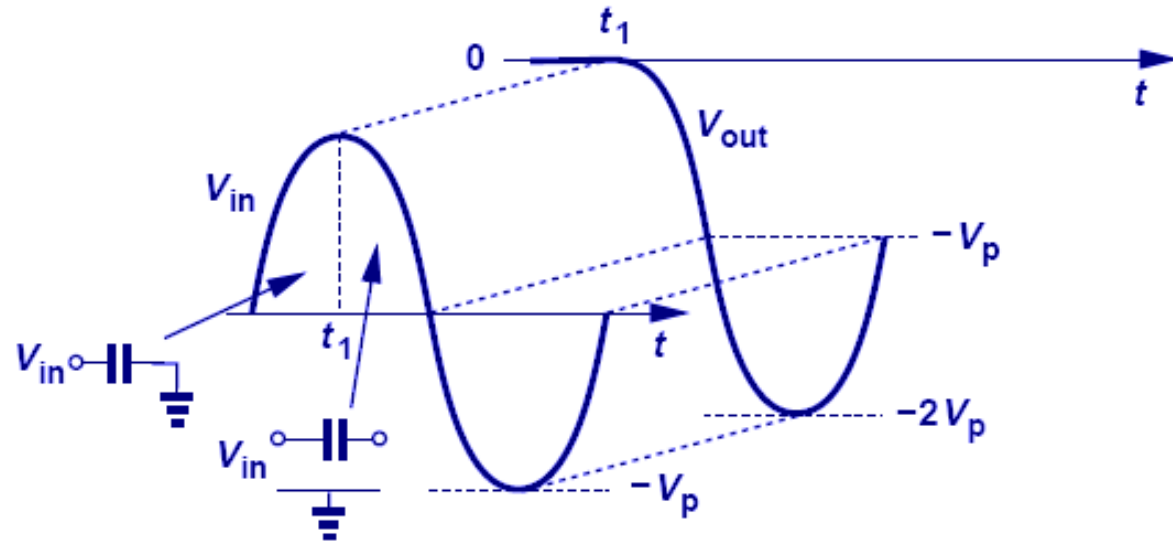
$$\Delta V_{out} = \Delta V_{in}$$

$$\Delta V_{out} = \frac{C_1}{C_1 + C_2} \Delta V_{in}$$

Waveform Shifter: Peak at $-2V_p$



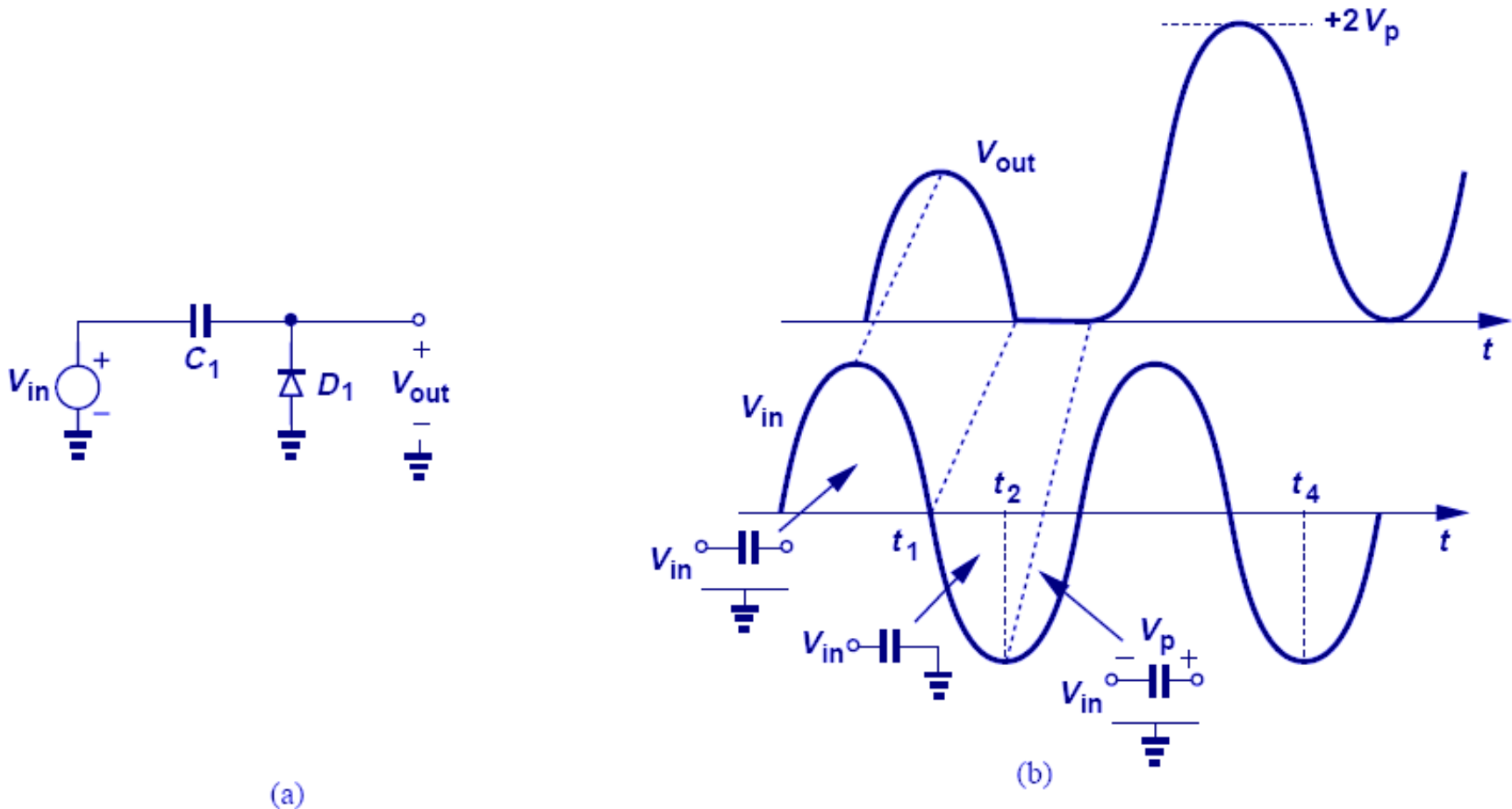
(a)



(b)

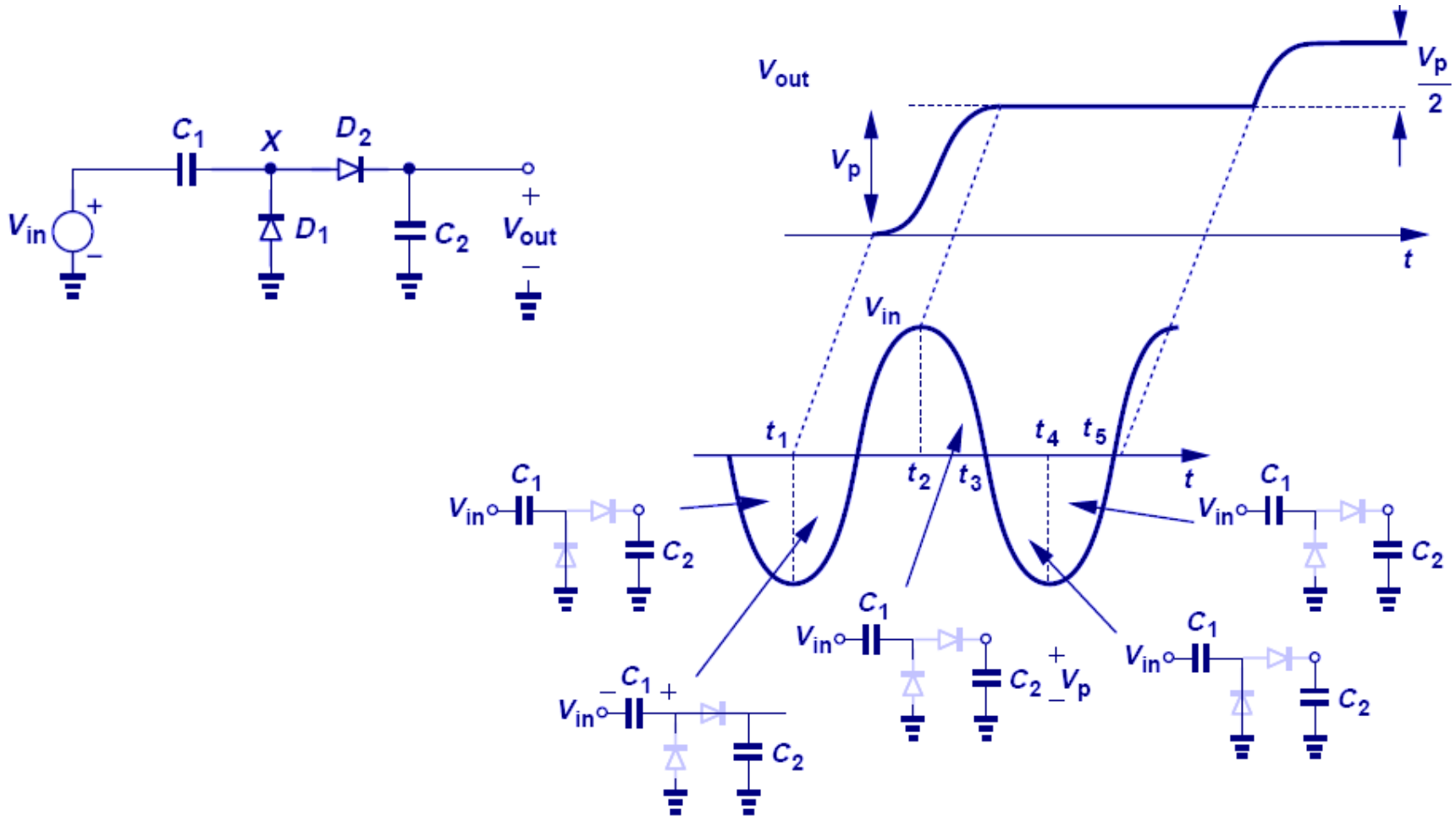
- As V_{in} increases, D_1 turns on and V_{out} is zero.
- As V_{in} decreases, D_1 turns off, and V_{out} drops with V_{in} from zero. The lowest V_{out} can go is $-2V_p$, doubling the voltage.

Waveform Shifter: Peak at $2V_p$



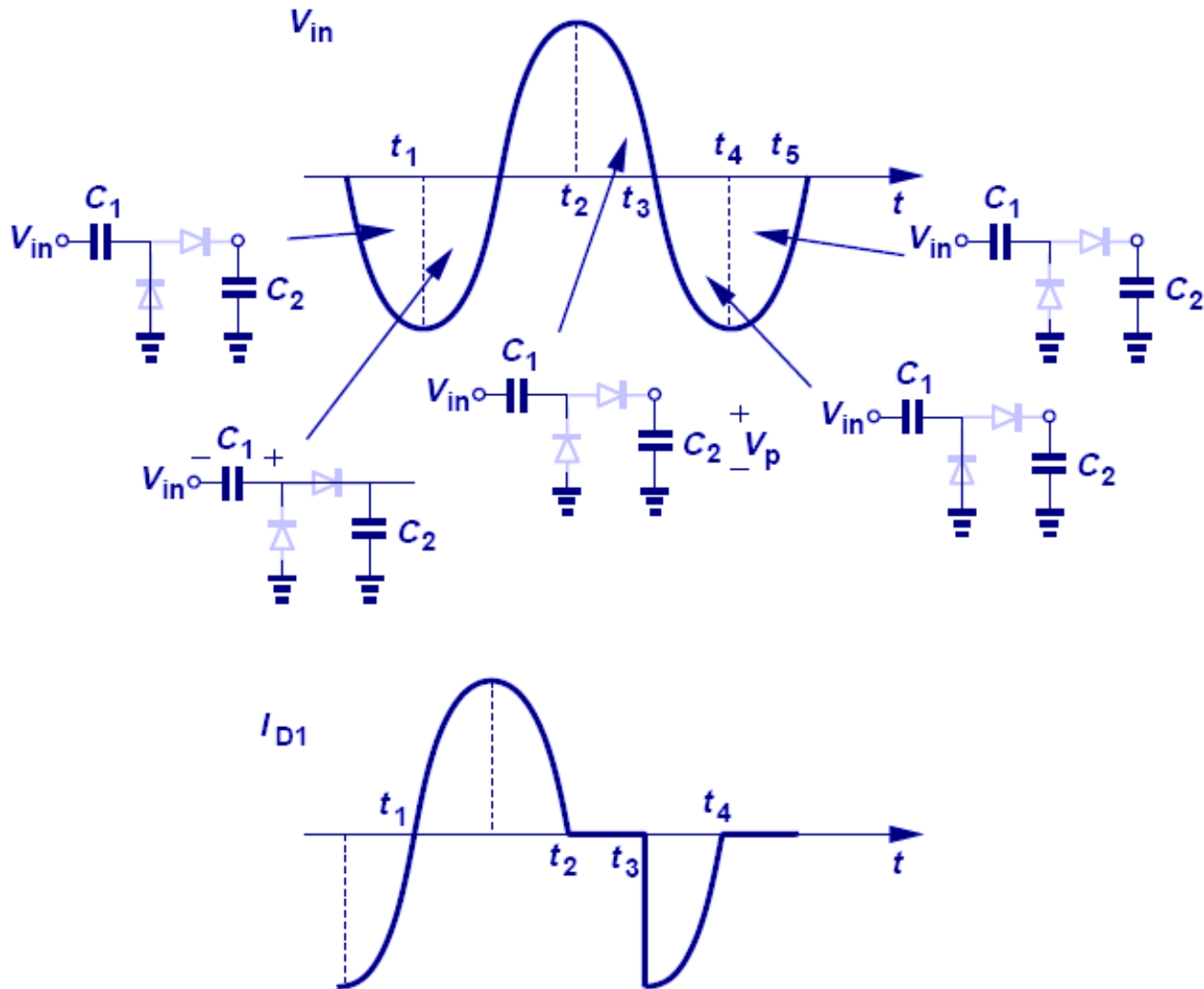
- Similarly, when the terminals of the diode are switched, a voltage doubler with peak value at $2V_p$ can be conceived.

Voltage Doubler

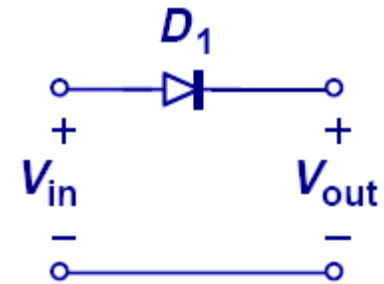


➤ The output increases by V_p , $V_p/2$, $V_p/4$, etc in each input cycle, eventually settling to $2V_p$.

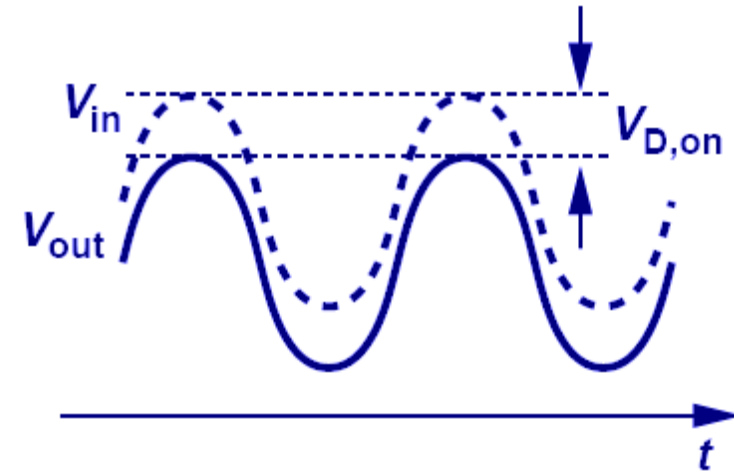
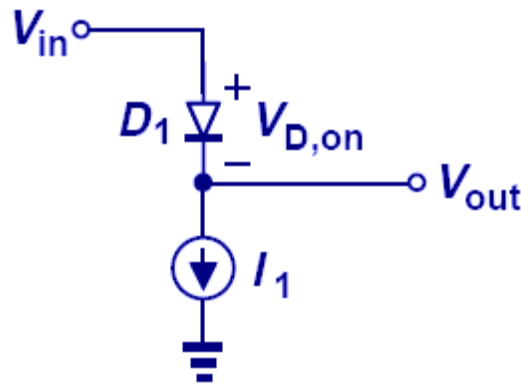
Current thru D_1 in Voltage Doubler



Another Application: Voltage Shifter

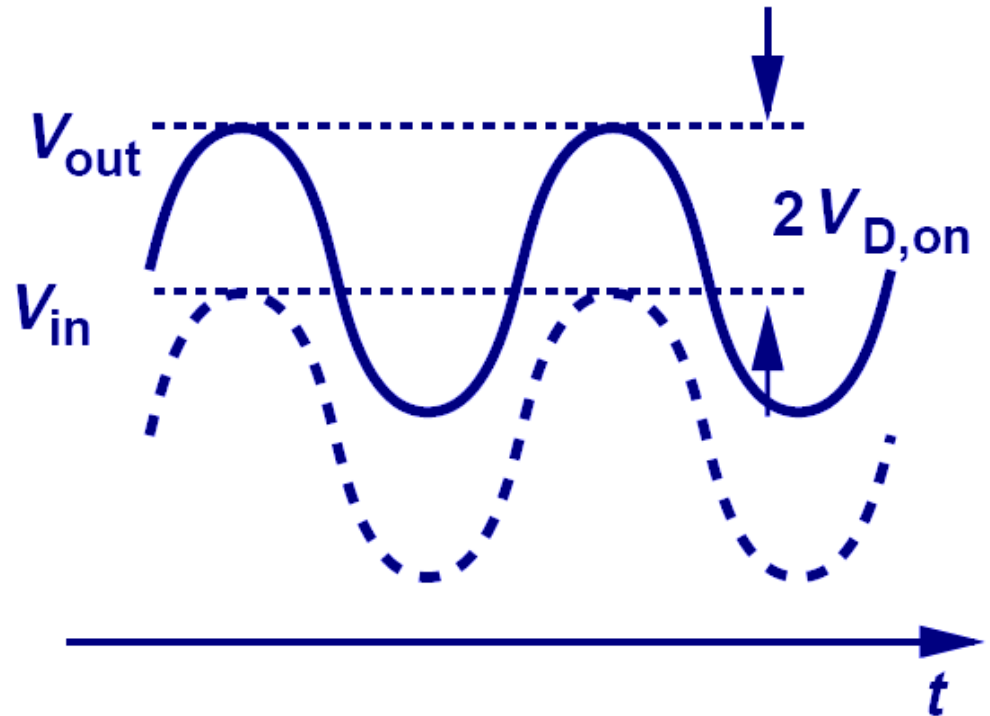
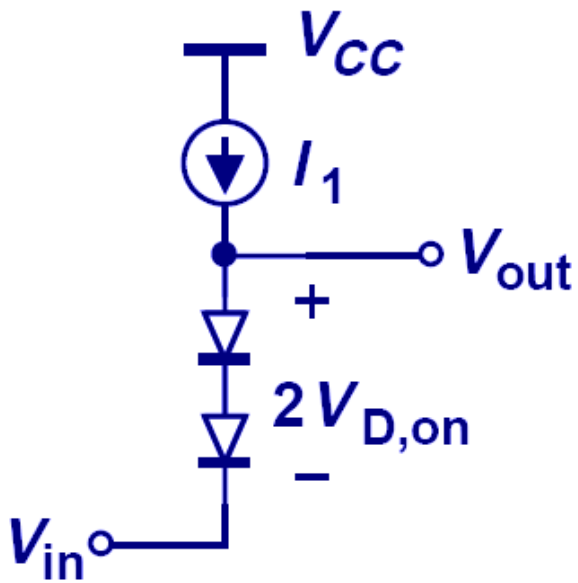


(a)

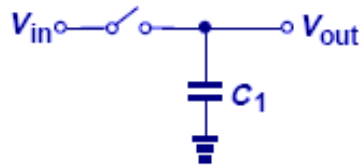


(b)

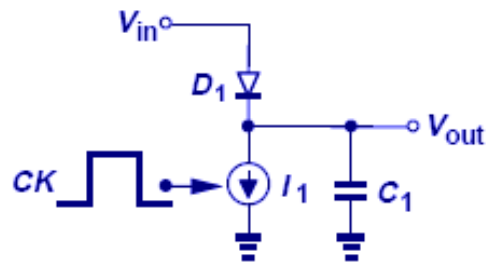
Voltage Shifter ($2V_{D,ON}$)



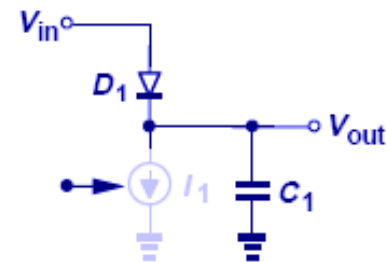
Diode as Electronic Switch



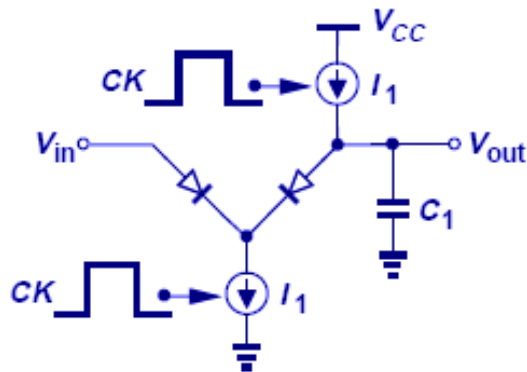
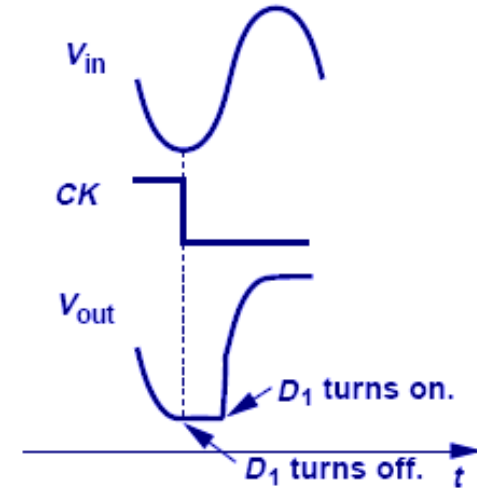
(a)



(b)



(c)



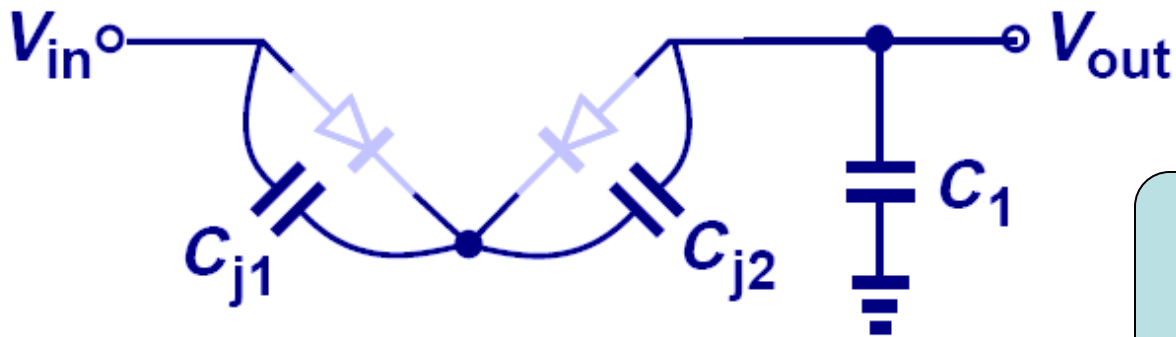
(d)



(e)

➤ Diode as a switch finds application in logic circuits and data converters.

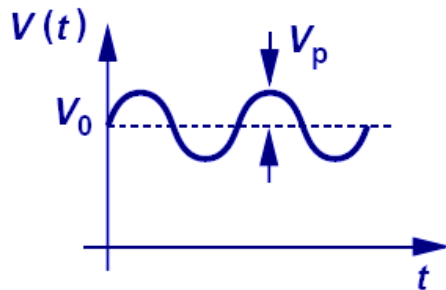
Junction Feedthrough



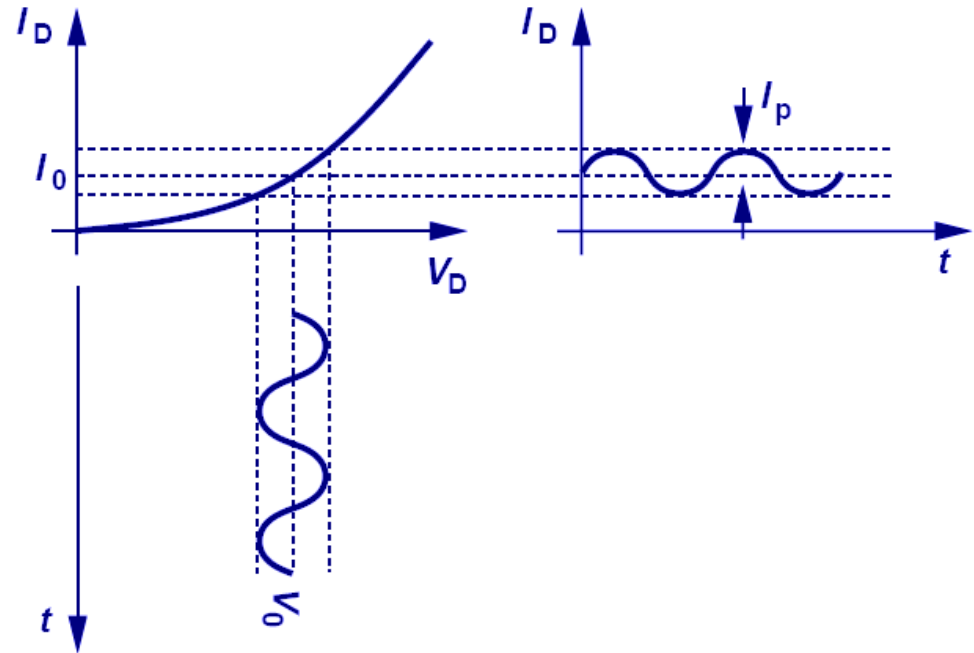
$$\Delta V_{out} = \frac{C_j/2}{C_j/2 + C_1} \Delta V_{in}$$

- For the circuit shown in part e) of the previous slide, a small feedthrough from input to output via the junction capacitors exists even if the diodes are reverse biased
- Therefore, C_1 has to be large enough to minimize this feedthrough.

Small Sinusoidal Analysis



(a)



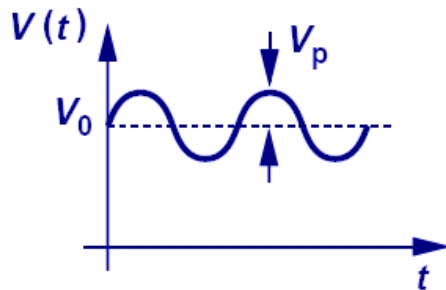
(b)

$$V(t) = V_0 + V_p \cos \omega t$$

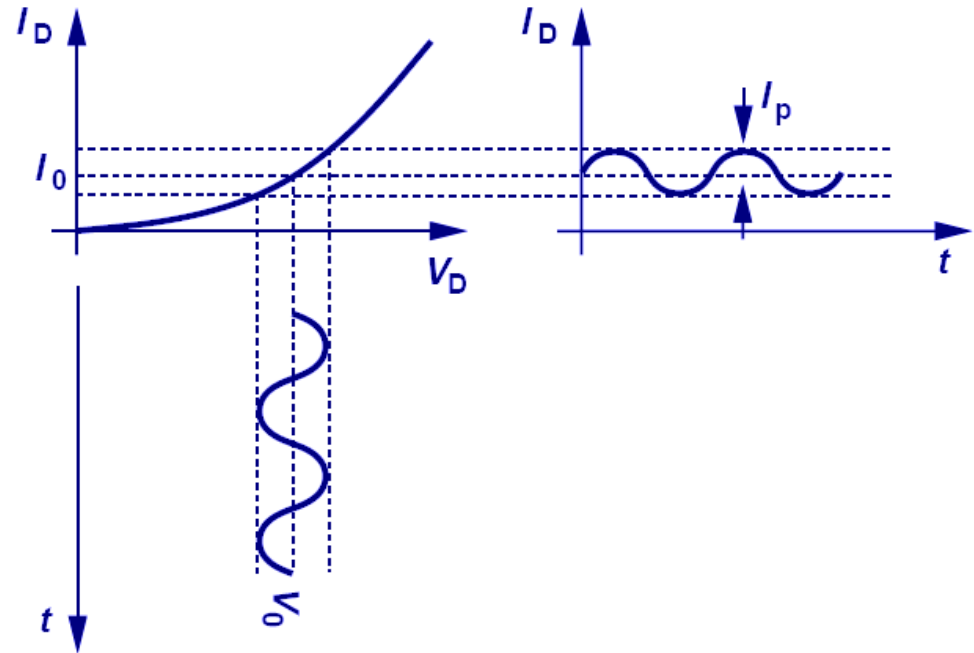
$$I_D(t) = I_0 + I_p \cos \omega t = I_s \exp \frac{V_0}{V_T} + \frac{V_T}{I_0} V_p \cos \omega t$$

- **If a sinusoidal voltage with small amplitude is applied, the resulting current is also a small sinusoid around a DC value.**

Small Sinusoidal Analysis



(a)



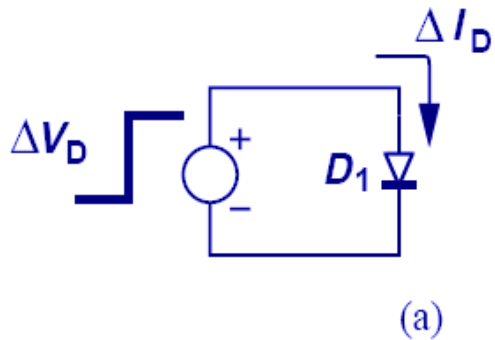
(b)

$$V(t) = V_0 + V_p \cos \omega t$$

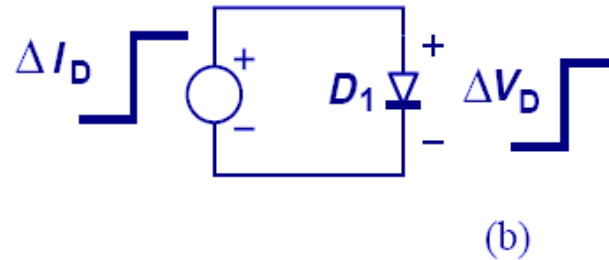
$$I_D(t) = I_0 + I_p \cos \omega t = I_s \exp \left(\frac{V_0}{V_T} + \frac{V_T}{I_0} V_p \cos \omega t \right)$$

- **If a sinusoidal voltage with small amplitude is applied, the resulting current is also a small sinusoid around a DC value.**

Cause and Effect



$$\begin{aligned}\Delta I_D &= \frac{\Delta V_D}{r_d} \\ &= \Delta V_D \frac{I_{D1}}{V_T}\end{aligned}$$



$$\begin{aligned}\Delta V_D &= \Delta I_D r_d \\ &= \Delta I_D \frac{V_T}{I_{D1}}\end{aligned}$$

- In (a), voltage is the cause and current is the effect. In (b), the other way around.