UNIVERSITY OF TEXAS AT DALLAS
Department of Electrical Engineering

EE 6391 - Signaling and Coding for Wireless Communication Systems
Solutions to Problem Set #4: Digital Modulation

Date assigned:
Date due:

Reading: "The potential and limitations of adaptive modulation over slow Rayleigh fading channels," J. Torrance, D. Didascalou, and L. Hanzo, IEE Colloquium on the Future of Mobile Multimedia Communications, 1996.

Solution 4.1

For DPSK modulation in Rayleigh fading, $\bar{P}_b = 1/\bar{\gamma}_b \quad \Rightarrow \quad \bar{\gamma}_b = 500$.
$N_0 B = 3 \times 106{-}12\text{mW}$. $Rightarrow$ $P_{\text{target}} = \bar{\gamma}_b N_0 B = 1.5 \times 10^{-6}\text{mW} = -88.24\text{dBm}$.
¡p¿ Now consider shadowing:

$$P_out = P[P_r < P_{\text{target}}] = P[\Psi < P_{\text{target}} - \bar{P}_r] = \Phi\left(\frac{P_{\text{target}} - \bar{P}_r}{\sigma}\right)$$

$$\Rightarrow \Phi^{-1}(0.01) = 2.327 = \frac{P_{\text{target}} - \bar{P}_r}{\sigma}$$

$$\bar{P}_r = -74.28 \ \text{ dBm} = 3.73 \times 10^{-8}\text{mW} = P_t\left(\frac{\lambda}{4\pi d}\right) \quad \Rightarrow \quad d = 1372.4 \ \text{ m}$$

Solution 4.2 In the initial part of the problem, average probability of bit error should have been given as 10^{-4} instead of 10^{-3}.

(a) $\bar{P}_r = P_t K \left(\frac{d}{d_0}\right)^{-3} = 10^{-4}\text{mW} = -40$ dBm.

(b) $\bar{P}_b \simeq \frac{1}{4\bar{\gamma}_b} = 10^{-4}$, $\Rightarrow \bar{\gamma}_b = 0.25 \times 10^4$.
$P_{\text{target}} = \bar{\gamma}_b N_0 B = 0.25 \times 10^4 \times 10^{-14} \times 30 \times 10^3 = 0.75 \times 10^{-6} = -61.2494\text{dBm}$.

(c) Find percentage of cell coverage with Rayleigh Fading and log-normal shadowing using

$$C = \Phi(a) + \exp\left(\frac{2 - 2ab}{b^2}\right)\Phi\left(\frac{2 - ab}{b}\right)$$

where

$$a = \frac{P_{\text{target}} - \bar{P}_r}{\sigma_{\psi_{\text{dB}}}} = (-61.2494 + 40)/4 = -5.3124$$

and

$$b = \frac{10\gamma \log_{10}(e)}{\sigma_{\psi_{\text{dB}}}} = \frac{30 \times 0.434}{4} = 3.2550$$

Solution 4.3 Let us denote by $p_{\gamma_1}(\gamma_1)$ and $p_{\gamma_2}(\gamma_2)$ the pdfs of the two branches, by $P_{\gamma_1}(\gamma_1)$ and $P_{\gamma_2}(\gamma_2)$ their respective cdfs, and by $\bar{\gamma}_1$ and $\bar{\gamma}_2$ their respective average SNRs. The paper entitled as "Analysis and Optimization of Switched Diversity Systems" by Young-Chai Ko, Mohamed-Slim Alouini, and Marvin K. Simon discuss in detail the derivation of general expression of CDF of SSC with unequal average SNR branches. The CDF for general case (equation (62)) is given by

$$P_{\gamma_{SSC}}(\gamma) = \begin{cases} \frac{P_{\gamma_1}(\gamma_T)P_{\gamma_2}(\gamma_T)}{P_{\gamma_1}(\gamma_T)+P_{\gamma_2}(\gamma_T)}(P_{\gamma_1}(\gamma) + P_{\gamma_2}(\gamma)) & \gamma \leq \gamma_T \\ \frac{P_{\gamma_1}(\gamma_T)P_{\gamma_2}(\gamma_T)}{P_{\gamma_1}(\gamma_T)+P_{\gamma_2}(\gamma_T)}(P_{\gamma_1}(\gamma) + P_{\gamma_2}(\gamma) - 2) + \frac{P_{\gamma_1}(\gamma_T)P_{\gamma_1}(\gamma)+P_{\gamma_2}(\gamma_T)P_{\gamma_2}(\gamma)}{P_{\gamma_1}(\gamma_T)+P_{\gamma_2}(\gamma_T)} & \gamma > \gamma_T \end{cases}$$

If the branches are i.i.d., then, by setting $P_{\gamma_1}(\gamma_T) = P_{\gamma_2}(\gamma_T)$ and $P_{\gamma_1}(\gamma) = P_{\gamma_2}(\gamma)$, the above equation will reduce to i.i.d. case in the text.

Solution 4.4

$$\bar{P}_b = c_1 \int_0^{\pi/2} \int_0^{\infty} e^{-c_2\gamma_1} p_{\gamma_1}(\gamma_1) d\gamma_1 \int_0^{\infty} e^{-c_2\gamma_2} p_{\gamma_2}(\gamma_2) d\gamma_2 d\phi$$

where $c_1 = 1/\pi$ and $c_2 = 1/\sin^2(\phi)$ for BPSK modulation. Using the relation given in the question for pdf, we obtain

$$\bar{P}_b = \frac{1}{\pi}(0.01\bar{\gamma})^2 \int_0^{\pi/2} \sin^2(\phi) d\phi = 0.25 \times 10^{-2}$$

Note that $\int \sin^2(\phi) d\phi = \phi/2 - \sin(2\phi)/4$.