UNIVERSITY OF TEXAS AT DALLAS
Department of Electrical Engineering

EE 6391 - Signaling and Coding for Wireless Communication Systems
Solutions to Problem Set #5: Digital Modulation

Date assigned:
Date due:

Reading: "The potential and limitations of adaptive modulation over slow Rayleigh fading channels," J. Torrance, D. Didascalou, and L. Hanzo, IEE Colloquium on the Future of Mobile Multimedia Communications, 1996.

Solution 5.1

P9.2 $\bar{\gamma} = 20$ dB (100 in linear scale).

$$P_{out} = p(\gamma < \gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}} = 0.1, \quad \Rightarrow \quad \gamma_0 = 10.53$$

SNR after the truncated channel inversion

$$\sigma = \frac{1}{E_{\gamma_0}[1/\gamma]} = \int_{10.53}^{\infty} \frac{p(\gamma)}{\gamma} d\gamma = 56.29$$

$$M(\sigma) \leq 1 + \frac{1.5\sigma}{-\ln(5 \times 10^{-3})} = 16.93$$

P9.3 The power adaptation is truncated channel inversion, so we need only find σ and γ_0. For QPSK modulation, with a constant SNR of $\sigma = 4.77$, we get $P_b = Q(\sqrt{(2\sigma)}) = 10^{-3}$. Setting

$$\sigma = \frac{1}{E_{\gamma_0}[1/\gamma]} = \frac{\Gamma(0, \gamma_0/100)}{100} = 4.77$$

and solving for γ_0 numerically, we obtain $\gamma_0 \simeq 10^{-7}$. Thus,

$$P_{out} = p(\gamma < \gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}} \simeq 10^{-9}$$

Very small outage!

Solution 5.2

(a) Using $P_b \leq 0.2e^{-1.5\gamma/(M-1)}$, express

$$\bar{P}_b = \int_0^{\infty} 0.2e^{-1.5\gamma/(M-1)} \frac{e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}} d\gamma = \frac{0.2}{1 + \frac{1.5\bar{\gamma}}{M-1}}$$

(b) $\bar{\gamma} = 20$ dB $\Rightarrow$ 100

$$M \leq 1 + \frac{1.5\bar{\gamma}}{-\ln(5P_b)} = 1.7538$$

(c) Adaptive methods from figure 9.3 give about $C/B \simeq 4$ bits/symbol. This is clearly much better than part (b) answer.

Solution 5.3 $\gamma_1 = 3.16$, $\gamma_2 = 10$, $\gamma_3 = 31.62$, and $\gamma_4 = 100$.

(a)

$$\sum_{i=1}^{4} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i} \right) p(\gamma_i) = 1, \qquad \Rightarrow \qquad \gamma_0 = 0.866$$

The optimal power control policy is given by

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{0.866} - \frac{1}{\gamma_i} \; \forall \gamma_i \end{cases}$$

(b)

$$\frac{C}{B} = \sum_{i=1}^{4} \log_2 \left(\frac{\gamma_i}{\gamma_0} \right) p(\gamma_i) = 4.526$$

(c)

$$E_{\gamma_0}[\frac{1}{\gamma}] = \frac{1}{\sigma} = \sum_{i=1}^{4} \frac{p(\gamma_i)}{\gamma_i} = 0.1549, \quad \Rightarrow \quad \sigma = 6.45$$

For $\gamma_0 \leq \gamma_1$

$$\frac{R}{B} = \log_2 \left(1 + \frac{1}{\sum_{i=1}^{4} \frac{p(\gamma_i)}{\gamma_i}} \right) p(\gamma_0 \geq \gamma_i) = 2.8983$$

For $\gamma_1 < \gamma_0 \leq \gamma_2$

$$\frac{R}{B} = \log_2 \left(1 + \frac{1}{\sum_{i=2}^{4} \frac{p(\gamma_i)}{\gamma_i}} \right) p(\gamma_0 \geq \gamma_i) = 3.1 \quad (*)$$

For $\gamma_2 < \gamma_0 \leq \gamma_3$

$$\frac{R}{B} = \log_2 \left(1 + \frac{1}{\sum_{i=3}^{4} \frac{p(\gamma_i)}{\gamma_i}} \right) p(\gamma_0 \geq \gamma_i) = 2.76$$

So, since (*) gives the highest spectral efficiency, 10 (dB) $< \gamma_0 \leq 15$ (dB) should be selected.

(d) Constellations are restricted to 2^{x_i} size for $x_i = 0, 1, 2, ...$

$$M(\gamma_i) \leq 1 + \frac{1.5\gamma}{-\ln(5BER)}$$

For $\gamma_1 = 5$ dB, $M_1 = 1.89$, $2^{x_1} \leq M_1$, $x_1 = 0$.
For $\gamma_2 = 10$ dB, $M_2 = 3.83$, $2^{x_2} \leq M_2$, $x_2 = 1$.
For $\gamma_2 = 10$ dB, $M_3 = 9.95$, $2^{x_3} \leq M_3$, $x_3 = 3$.
For $\gamma_2 = 10$ dB, $M_4 = 29.31$, $2^{x_4} \leq M_4$, $x_2 = 4$.
Spectral efficiency is given by

$$E[\log_2 M(\gamma_i)] = \sum_{i=2}^{4} \log_2 M(\gamma_i) p(\gamma_i) = 0.2 \sum_{i=2}^{4} x_i = 1.6\text{bits/symbol}$$