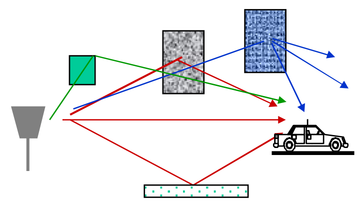
Statistical Multipath Model



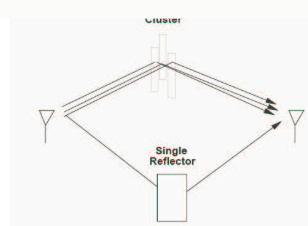
- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

Time Varying Impulse Response

- The received signal is the sum of the line-of-sight (LOS) path and all resolvable multipath components

$$r(t) = \Re\left\{\sum_{n=0}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j(2\pi f_c(t-\tau_n(t))+\phi_{D_n})}\right\}$$

- Two multipath components with delay τ_1 and τ_2 are resolvable if their delay difference significantly exceeds the inverse signal bandwidth
- if nonresolvable components are combined into a single multipath component with delay $\tau \approx \tau_1 \approx \tau_2$
- The parameters $\alpha_n(t)$, $\tau_n(t)$, and ϕ_{Dn} associated with each resolvable multipath component are characterized as random processes which we assume to be both stationary and ergodic.



Doppler shift

$$f_{D_n}(t) = \nu \cos(\theta_n(t))/\lambda$$

$$\phi_{D_n} = \int_t 2\pi f_{D_n}(t)dt$$

Time Varying Impulse Response

- The received signal can be rewritten as

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t-\tau_n(t))\right] e^{j2\pi f_c t}\right\}$$

  - where $\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}$ $\quad \phi_{D_n} = \int_t 2\pi f_{D_n}(t)dt$
- $\alpha_n(t)$ is a function of path loss and shadowing while $\phi_n(t)$ depends on delay and Doppler,
  - these two random processes are independent.
- Equivalent lowpass TV CIR $c(\tau,t)$ of the channel

$$r(t) = \Re\left\{\left(\int_{-\infty}^{\infty} c(\tau,t)u(t-\tau)d\tau\right) e^{j2\pi f_c t}\right\}$$

Time Varying Impulse Response

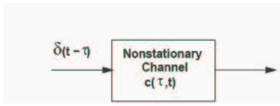
- Response of channel at t to impulse at t-τ:

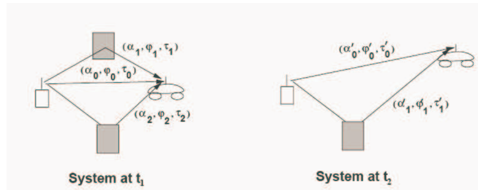
$$c(\tau,t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\varphi_n(t)} \delta(\tau - \tau_n(t))$$

  - t is time when impulse response is observed
  - t-τ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for MP component currently observed

Example of a Time-Varying Channel

- Each multipath component corresponds to a single reflector

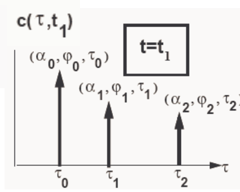


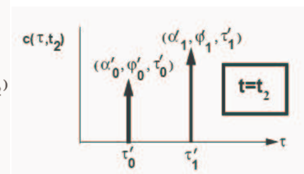


Time-varying case

$$c(\tau, t_1) = \sum_{n=0}^{2} \alpha_n e^{-j\phi_n} \delta(\tau - \tau_n)$$

$$c(\tau, t_2) = \sum_{n=0}^{1} \alpha'_n e^{-j\phi'_n} \delta(\tau - \tau'_n)$$





Time-invariant case

$$c(\tau) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n} \delta(\tau - \tau_n)$$

Received Signal Characteristics

- Received signal consists of many multipath components
- Assumptions:
  - Amplitudes change slowly
  - Phases change rapidly
    - Constructive and destructive addition of signal components
    - Amplitude fading of received signal (both wideband and narrowband signals)

Narrowband Model

- Assume delay spread T_m<<*1/B* $T_m = \max_n \tau_n - \tau_0$ or $T_m = \max_n |\tau_n - \bar{\tau}|$
- Then $u(t) \approx u(t-\tau)$.
- Received signal given by

$$r(t) = \Re\left\{ u(t) e^{j2\pi f_c t} \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} \right] \right\}$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=\delta(t)$

In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) \cos(\phi_n(t)) \qquad r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) \sin(\phi_n(t))$$

- Without dominant LOS path
- Assumptions: Amplitude, delays, Doppler are changing slowly

$$\alpha_n(t) \approx \alpha_n \quad \tau_n(t) \approx \tau_n \quad \text{and} \quad f_{D_n}(t) \approx f_{D_n}$$

- For *N(t)* large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian (sum of large # of random vars).
- Received signal completely characterized by its mean, autocorrelation, and cross correlation.
- If ϕ_0 and $\phi_n(t)$ uniform, the in-phase/quad components are mean zero, indep., and stationary

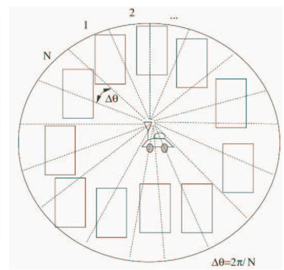
$$\phi_{D_n}(t) = \int_t 2\pi f_{D_n} dt = 2\pi f_{D_n} t \qquad \phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t - \phi_0$$

- Assuming uniform phase offsets, process is
  - Zero mean
  - Auto and cross correlation depend on AOAs of multipath

Auto and Cross Correlation

- Assume ϕ_n~U[0,2π] $\mathrm{E}[r_I(t)] = \mathrm{E}[\sum_n \alpha_n \cos\phi_n(t)] = \sum_n \mathrm{E}[\alpha_n]\mathrm{E}[\cos\phi_n(t)] = 0$
- Recall that θ_n is the multipath arrival angle
- Under uniform scattering, in phase and quad comps have no cross correlation

$$\begin{aligned} \mathrm{E}[r_I(t) r_Q(t)] &= \mathrm{E}\left[\sum_n \alpha_n \cos\phi_n(t) \sum_m \alpha_m \sin\phi_m(t)\right] \\ &= \sum_n \sum_m \mathrm{E}[\alpha_n\alpha_m]\mathrm{E}[\cos\phi_n(t)\sin\phi_m(t)] \\ &= \sum_n \mathrm{E}[\alpha_n^2]\mathrm{E}[\cos\phi_n(t)\sin\phi_n(t)] \\ &= 0. \end{aligned}$$



- The autocorrelation of $r_I(t)$ is given by

$$A_{r_I}(t,\tau) = \mathrm{E}[r_I(t) r_I(t+\tau)] = \sum_n \mathrm{E}[\alpha_n^2]\mathrm{E}[\cos\phi_n(t)\cos\phi_n(t+\tau)]$$

Auto and Cross Correlation

- Assume ϕ_n~U[0,2π]
- Recall that θ_n is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\Omega_p E_{\theta_n}[\cos 2\pi f_{D_n}\tau], \quad f_{D_n} = v\cos\theta_n/\lambda$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau) = .5\Omega_p E_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

$\Rightarrow A_{r_I,r_Q}(\tau) = .5\Omega_p E_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$ **=0**

$\Rightarrow A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$

Uniform AOAs

- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is

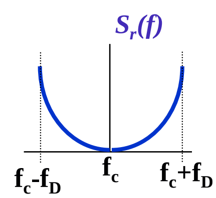
$$A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\Omega_p J_0(2\pi f_D \tau)$$

- The PSD of received signal is

Decorrelates over roughly half a wavelength

$$S_r(f) = .5[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)]$$

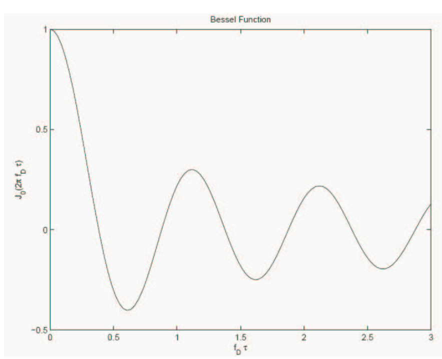
$$S_{r_I}(f) = \mathcal{F}[.5\Omega_p J_0(2\pi f_D \tau)]$$

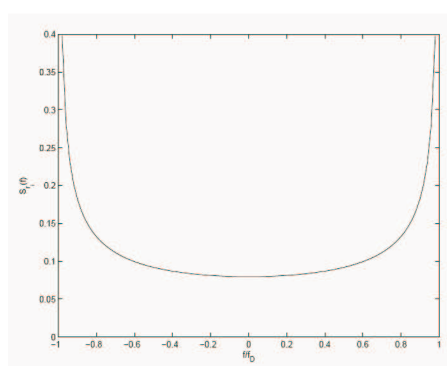


Used to generate simulation values

Bessel Function

- Bessel function





Signal Envelope and Power Distributions

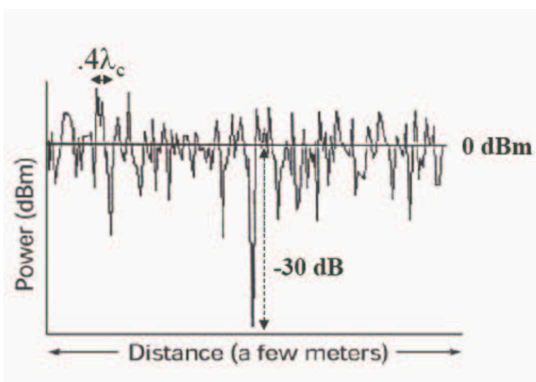
- CLT approx. leads to Rayleigh distribution (power is exponential)
- For any two Gaussian random variables X and Y , both with mean zero and equal variance σ^2

$Z = \sqrt{X^2 + Y^2}$ is Rayleigh-distributed and Z^2 is exponentially distributed

- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

Narrowband Fading

- Narrowband fading



Ricean Fading

- LOS component with power $s^2 = \alpha_0^2$

$$p_Z(z) = \frac{z}{\sigma^2}\exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0,$$

- The Rician distribution is often described in terms of a fading parameter *K*

$$P_r = \int_0^\infty z^2 p_Z(z)dx = s^2 + 2\sigma^2 \qquad K = \frac{s^2}{2\sigma^2}$$

Nakagami Fading

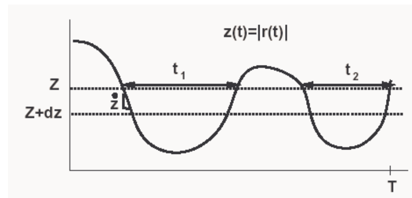
- More general fading distribution was developed whose parameters can be adjusted to fit a variety of empirical measurements.
- This distribution is called the Nakagami fading distribution

$$p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m}\exp\left[\frac{-mz^2}{P_r}\right], \quad m \geq .5$$

  - where P_r is the average received power and $\Gamma(\cdot)$ is the Gamma function.

Level Crossing Rate

- The envelope level crossing rate L_Z is defined as the expected rate (in crossings per second) at which the signal envelope crosses the level Z in the downward direction.
- Obtaining L_Z requires the joint distribution of the signal envelope $z = |r|$ and its derivative with respect to time $\dot{z}$, $p(z, \dot{z})$.
- For Rayleigh channels with threshold γ_0
- For Ricean channels



$$L_Z = \sqrt{2\pi} f_D \rho e^{-\rho^2} \qquad \rho = \sqrt{\gamma_0 / P_r}$$

$$L_Z = \sqrt{2\pi(K+1)} f_D \rho e^{-K-(K+1)\rho^2} I_0(2\rho\sqrt{K(K+1)}),$$

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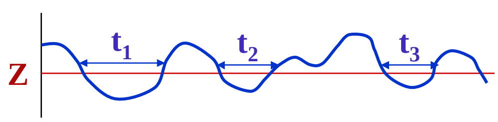
Average Fade Duration

- How long a signal stays below target R (SNR γ)



$$p(z(t) < Z) = \frac{1}{T}\sum_i t_i.$$

- Derived from level crossing rate of fading process
- For Rayleigh fading

$$\bar{t}_Z = (e^{\rho^2} - 1)/(\rho f_D \sqrt{2\pi})$$

  - Depends on ratio of target to average level (ρ)
  - Inversely proportional to Doppler frequency

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Average Fade Duration

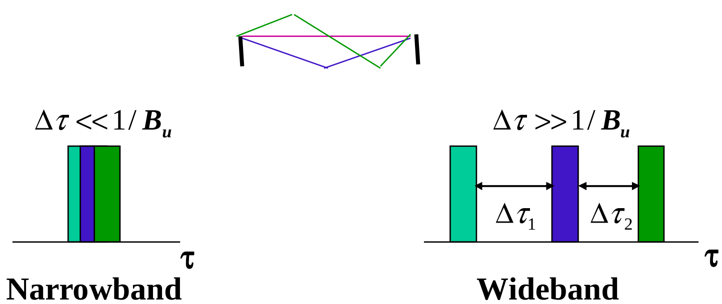
- The average fade duration indicates the number of bits or symbols affected by a deep fade.
- Specifically, consider an uncoded system with bit time *Tb*.
- Suppose the probability of bit error is high when $z < Z$
  - Then, if $T_b \approx t_Z$, the system will likely experience single error events, where bits that are received in error have the previous and subsequent bits received correctly (since $z > Z$ for these bits).
  - On the other hand, if $T_b << t_Z$ then many subsequent bits are received with $z < Z$ so large bursts of errors are likely.

Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath
- Uniform scattering makes autocorrelation of inphase and quad follow Bessel function
  - Signal components decorrelated over half wavelength
  - Cross correlation is zero (in-phase/quadrature indep.)
- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Average fade duration important for system issues

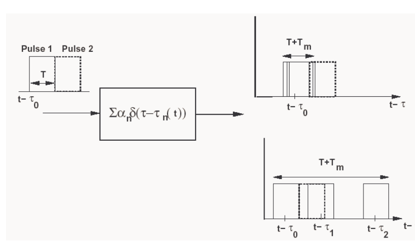
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth



Wideband Fading Models

- Multipath resolution

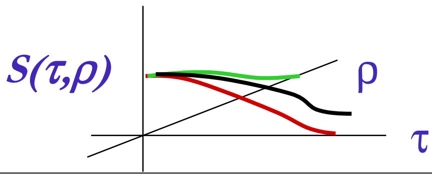


Scattering Function

- $S(\tau,\rho)$=Fourier transform of $c(\tau,t)$ relative to t, the deterministic scattering function

$$S(\tau,\rho) = \int_{-\infty}^{\infty} c(\tau,t)e^{-j2\pi\rho t}dt$$

- Typically characterize its statistics, since $c(\tau,t)$ is different in different environments
- Since underlying process Gaussian, need only characterize mean (0) and correlation
- Autocorrelation: $E[c(\tau_1,t)c(\tau_2,t+\Delta t)]=A_c(\tau,\Delta t)$



WSSUS Channel Model

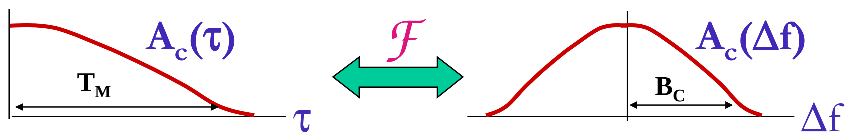
- Widesense stationary (WSS)
- Uncorrelated scattering (US)
- The scattering function for random channels is defined as the Fourier transform of $A_c(\tau;\Delta t)$

$$S_c(\tau,\rho) = \int_{-\infty}^{\infty} A_c(\tau,\Delta t)e^{-j2\pi\rho\Delta t}d\Delta t$$

- The scattering function characterizes the average output power associated with the channel as a function of the multipath delay τ and Doppler ρ.

Multipath Intensity Profile

- Defined as $A_c(\tau,\Delta t=0)= A_c(\tau)$
- Average delay spread T_M and rms delay spread σ_τ defined relative to $A_c(\tau)$
  - Approximate max delay of significant m.p.
- Coherence bandwidth $B_c=1/T_M$
  - Maximum frequency over which $A_c(\Delta f)=F[A_c(\tau)]>0$
  - $A_c(\Delta f)=0$ implies signals separated in frequency by Δf will be uncorrelated after passing through channel



Multipath Intensity Profile

- Average and rms delay spread are typically defined in terms of the power delay profile $A_c(\tau)$ as

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}$$

And

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau-\mu_{T_m})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}}$$

- Note that if we define the pdf p_{Tm} of the random delay spread T_m

$$p_{T_m}(\tau) = \frac{A_c(\tau)}{\int_0^\infty A_c(\tau)d\tau}$$

ISI Channels and Coherence BW

- When $T_s >> \sigma_{Tm}$ the system experiences negligible ISI.
- For calculations one can assume that $T_s << \sigma_{Tm}$ implies $T_s < \sigma_{Tm}/10$ and $T_s >> \sigma_{Tm}$ implies $T_s > 10\sigma_{Tm}$
- Time-Frequency Correlation Function

$$\begin{aligned} A_C(f_1, f_2; \Delta t) &= E\left[\int_{-\infty}^{\infty} c^*(\tau_1; t) e^{j2\pi f_1 \tau_1} d\tau_1 \int_{-\infty}^{\infty} c(\tau_2; t+\Delta t) e^{-j2\pi f_2 \tau_2} d\tau_2\right] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[c^*(\tau_1; t) c(\tau_2; t+\Delta t)] e^{j2\pi f_1 \tau_1} e^{-j2\pi f_2 \tau_2} d\tau_1 d\tau_2 \\ &= \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi (f_2 - f_1)\tau} d\tau. \\ &= A_C(\Delta f; \Delta t) \end{aligned}$$

Frequency correlation function

If we define $A_C(\Delta f) \triangleq A_C(\Delta f; 0)$

$$A_C(\Delta f) = \int_{-\infty}^{\infty} A_c(\tau) e^{-j2\pi \Delta f \tau} d\tau$$

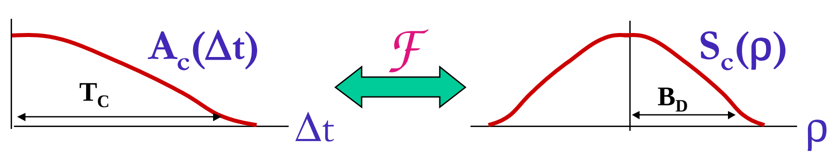
- The frequency B_c where $A_C(\Delta f) \approx 0$ for all $\Delta f > B_c$ is called the **coherence bandwidth** of the channel.
- Flat fading vs frequency-selective fading

Doppler Power Spectrum

- $S_c(\rho) = F[A_c(\tau=0, \Delta t)] = F[A_c(\Delta t)]$

$$S_C(\Delta f; \rho) = \int_{-\infty}^{\infty} A_C(\Delta f; \Delta t) e^{-j2\pi\rho\Delta t} d\Delta t.$$

- Doppler spread B_d
  - Maximum doppler for which $S_c(\rho) \geq 0$.
- Coherence time $T_c = 1/B_d$
  - Maximum time over which $A_c(\Delta t) > 0$
  - $A_c(\Delta t) = 0$ implies signals separated by Δt will be uncorrelated after passing through channel

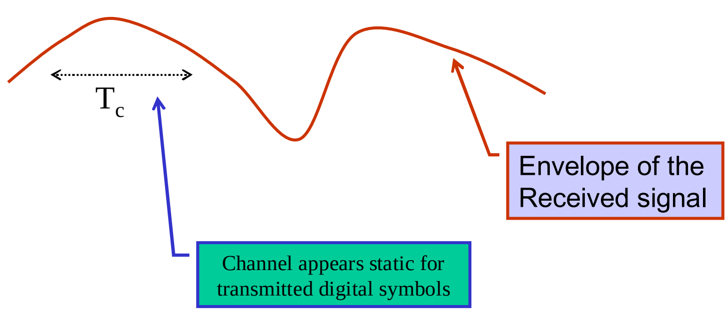


Channel Coherence Time

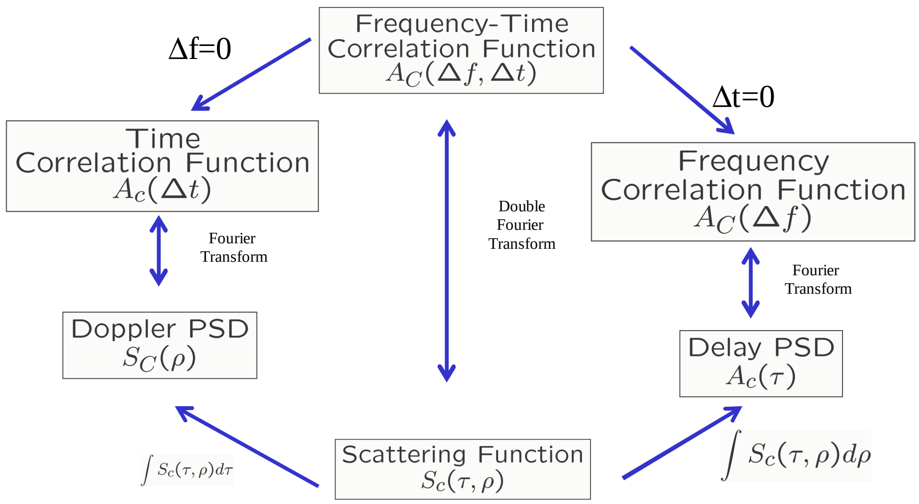
- We define the **channel coherence time** *T_c* to be the range of values over which Time Correlation Function $A_C(\Delta t)$ is approximately nonzero.
- Thus, the time-varying channel decorrelates after approximately T_c seconds.
- The function $S_C(\rho)$ is called the **Doppler power spectrum** of the channel:
- The maximum ρ value for which $|S_C(\rho)|$ is greater than zero is called the **Doppler spread** of the channel, and is denoted by B_D.
- By the Fourier transform relationship between $A_C(\Delta t)$ and $S_C(\rho)$, $B_D \approx 1/T_c$. If the transmitter and reflectors are all stationary and the receiver is moving with velocity v, then $B_D \leq v/\lambda = f_D$.

Coherence Time

- Doppler spread and coherence time describe the time varying nature of the channel
- It is a statistical measure of the time duration over which the channel impulse response is essentially invariant



Relationships among Channel Correlation Functions



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Main Points

- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Average fade duration important for system issues
- Wideband models characterized by scattering function: measures power vs delay and doppler
- M.P. delay spread defines maximum delay of significant multipath components, inverse is coherence bandwidth of channel
- Doppler spread defines maximum nonzero doppler, its inverse is coherence time

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