Approximate Inference in Probabilistic Graphical Models with Determinism

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Introduction

In the proposed thesis, we study a special class of belief networks which contain both probabilistic and *deterministic information*. Deterministic information occurs as *zero probabilities* in the belief network. A majority of the work in the belief network community (see for example papers in conferences like UAI, AAAI, IJCAI and NIPS) addresses probabilistic inference tasks under the assumption that the underlying joint distribution represented by the belief network is strictly positive i.e. devoid of any determinism. The positivity assumption is problematic because (a) modeling many real-world problems such as genetic linkage analysis (Fishelson & Geiger 2003) requires that the inference method reason with both probabilistic and deterministic information and (b) inference is harder in presence of determinism or extreme probabilities (Dagum & Luby 1993).

The purpose of the proposed thesis is to study both the representational and algorithmic issues involved in modeling deterministic information along with the usual probabilistic information in a belief network.

Progress to Date

Hybrid Mixed Networks and Hybrid Dynamic Mixed Networks

Focusing on representational issues, (Dechter & Larkin 2001) introduced the framework of mixed networks. The mixed network framework allows one to reason about deterministic information in the belief network by explicitly modeling it as constraint relations. The mixed network framework (Dechter & Mateescu 2004) is restricted in its applicability to real-world domains because it assumes that all random variables are discrete. Therefore in our work published at UAI (Gogate & Dechter 2005; Gogate *et al.* 2005), we introduced two new modeling frameworks of Hybrid Mixed Networks (HMN) and Hybrid Dynamic Mixed Networks (HDMN) which extend the mixed networks framework to (a) handle discrete-time stochastic processes and (b) handle variables having both continuous and discrete domains.

The main motivation for developing HMN and HDMN as modeling frameworks is a real-world problem of inferring car travel activity of individuals. Given historical GPS

data of a person and his/her current location, some sample queries that can be answered by our probabilistic HDMN model are: (a) Infer the locations of driver's common destinations, such as home and church and (b) Predict the future movements, both in the short term (will he make the next right?) and in terms of distant goals (is he going to work?) (i.e. his route to destination).

Expectation Propagation and Rao-Blackwellised Importance Sampling for mixed networks

Focusing on algorithmic issues, the most popular approximate query processing algorithms for belief networks are Generalized Belief Propagation and Rao-Blackwellised Importance sampling. We therefore extend these algorithms to HMNs and HDMNs so that they accommodate and exploit discrete constraints in the presence of continuous probabilistic functions. Extending Generalized Belief Propagation to handle constraints is easy, extension to continuous variables is a little more intricate but still straightforward. The presence of constraints introduces a principles challenge for Importance Sampling algorithms and Particle Filtering, however and we explain why below.

It is well known (Cheng & Druzdzel 2000; Yuan & Druzdzel 2006) that the quality of approximation of importance sampling techniques deteriorate in the presence of zero probabilities. The primary reason for this poor performance is the generation of a large number of samples having zero weights. These zero weight samples do not contribute to the inference task and are effectively rejected; *the rejection problem*. We can circumvent the rejection problem by enforcing adaptive consistency (Dechter 2003). However, adaptive consistency is time and space exponential in a graph parameter called treewidth and therefore is not practical when the treewidth is large. In (Gogate & Dechter 2005; Gogate *et al.* 2005), we suggest an approximate method called IJGP-sampling to reduce the amount of rejection. IJGP-sampling samples from the output of a Generalized Belief Propagation algorithm called *iterative join graph propagation algorithm* (IJGP). IJGP achieves constraint propagation and therefore most inconsistencies are deduced before sampling which in turn substantially reduces the rejection rate. IJGP proves to be an ideal candidate for constructing the sampling distribution because IJGP also computes a very good approximation to the true posterior

(which is the quantity of interest) and it is known that the performance of importance sampling is highly dependent on how close the sampling distribution is to the exact posterior distribution (Cheng & Druzdzel 2000). We also show that IJGP-sampling can be integrated into the w-cutset Rao-blackwellised sampling scheme (Bidyuk & Dechter 2003) resulting in a new scheme of IJGP-RB-sampling. Our empirical results which are reported in (Gogate & Dechter 2005; Gogate *et al.* 2005) demonstrate both the practical effectiveness and the superior performance of IJGP-RB-sampling to competing approaches.

SampleSearch: A scheme that searches for consistent samples

Although IJGP-RB-Sampling works quite well when very few zeros are present in the belief network (i.e. small amount of determinism), we found in (Gogate & Dechter 2006) that when the belief network has a substantial number of zero probabilities, IJGP-RB-Sampling may fail to generate even a single sample having non-zero weight. The problem is that constraint propagation achieved by IJGP is not enough to eliminate the rejection problem.

Therefore, in a recent paper (Gogate & Dechter 2007), we present an alternative stronger approach referred to as *SampleSearch* which guarantees that all samples generated will have non-zero weight. In this scheme, when a sample is supposed to be rejected, the algorithm continues instead with systematic search until a non-zero weight sample is generated. The use of search with sampling is problematic because search introduces bias into the sampling process.

To remove the bias we present two weighting schemes. The first weighting scheme guarantees unbiasedness but requires a substantial amount of computation. The second scheme is a linear time procedure that approximates the first scheme such that the resulting estimates are asymptotically unbiased. We found that (Gogate & Dechter 2007) as the number of deterministic information in the belief network is increased our SampleSearch scheme outperforms IJGP-RB-sampling and other competing schemes available in belief network literature.

Proposed Plan for Future

Efficient Storage strategies for SampleSearch

The asymptotically unbiased estimator of SampleSearch requires that we store all partial assignments that were generated during sampling. This requires an additional $O(N * d * n)$ space where N is the number of samples, d is the maximum domain size and n is the number of variables. Although the storage required is linear in N, the cost might be prohibitive for large values of N, n and d. Therefore, we propose to use the recently developed AND/OR multi-valued decision diagrams (AOMDDD) framework (Mateescu & Dechter 2006) for efficiently storing the generated samples. The stored AOMDD structure might also be useful for making our SampleSearch scheme an adaptive importance sampling scheme (Cheng & Druzdzel 2000) in which the generated (cached) samples are used to update the sampling distribution.

Bounding algorithms

Although, we have extended algorithms like generalized belief propagation and Importance sampling (Gogate & Dechter 2005; Gogate *et al.* 2005; Gogate & Dechter 2007; 2006) to handle determinism in belief networks, these algorithms do not have any guarantees in terms of upper/lower bounds. Therefore, we propose to develop bounding schemes that output high confidence lower/upper bounds on various inference tasks such as belief updating and probability of evidence. Our preliminary results on lower bounding probability of evidence which use the Markov inequality are very promising and we plan to build upon this approach.

Sampling in AND/OR space

Sampling algorithms appearing in the literature do not take advantage of the decomposition of graph admitted by the belief network. For example, even if the graph admitted by the belief network is a tree; making inference tractable for algorithms like variable elimination, the bounds on the sampling error are dependent solely on the mean/variance of the posterior distribution (Dagum & Luby 1997). We propose to develop novel sampling techniques that decompose the mean/variance along an AND/OR graph (Dechter & Mateescu 2004). Our hope is that better bounds on sampling error can be derived by taking advantage of the AND/OR structure.

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