# Homework 1
# CS 6347: Statistical methods in AI/ML

Instructor: Vibhav Gogate
Vibhav.Gogate@utdallas.edu

Due date: Monday Feb. 12, 2021 via elearning

[ Key: AD: Book by Adnan Darwiche, KF: Koller Friedman]

- **WARNING:** Start early. Some problems are quite hard (e.g., Problem 10).
- **Each problem is worth 10 points**

**Problem 1: Propositional logic [Problems 2.6 to 2.9 from AD]**

- Prove the Refutation theorem, namely $\alpha \models \beta$ iff $\alpha \wedge \neg\beta$ is inconsistent.
- Prove the Deduction theorem, namely $\alpha \models \beta$ iff $\alpha \Rightarrow \beta$ is valid.
- Prove that if $\alpha \models \beta$ then $\alpha \wedge \beta$ is equivalent to $\alpha$.
- Prove that if $\alpha \models \beta$ then $\alpha \vee \beta$ is equivalent to $\beta$.

**Problem 2: Probability theory**

- Prove that the following two definitions of conditional independence are equivalent.
  1. $\Pr(\alpha|\beta \wedge \gamma) = \Pr(\alpha|\gamma)$
  2. $\Pr(\alpha \wedge \beta|\gamma) = \Pr(\alpha|\gamma)\Pr(\beta|\gamma)$

  [**Hint:** Derive one using the other using laws of probability.]

**Problem 3: Probability theory**
We solved the following problem in class.

- **Fact 1:** The probability that your partner fails a lie detector test given that he/she is cheating on you is 0.98 (or 98%). The probability that your partner fails the test given that he/she is not cheating on you is 0.02.

- **Fact 2:** You are a CS graduate student.
- **Fact 3:** You should break up with your partner if he/she is cheating on you with a probability greater than 0.05.

Today, you found that you failed the lie detector test. You are in panic mode and are sure that your partner will break up with you. Suddenly you realize that you had previously found the following, pretty reliable statistic on the internet: only 1 out of 10000 CS graduate students (since they are boring people) cheat on their partners. Given this new information and using sound probabilistic arguments, how do you convince your partner that you are not cheating on him/her?

We derived in class that with this new information the probability of cheating given that you have failed the test is much smaller than 0.05.

Here is the twist; your partner is not totally convinced. He/She tells you to take the test three more times on three different days. Unfortunately, you fail two out of the three tests. Can you still convince your partner that you are not cheating on him/her (assume that the three tests are independent of each other). Justify your answer using purely probabilistic arguments.

**Problem 4: Probability theory (Exercise 2.10 from Koller & Friedman)**

The question investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations. Let $\alpha$, $\beta$, and $\gamma$ be three propositional variables.

- Suppose we wish to calculate $\Pr(\alpha|\beta, \gamma)$ and we have no conditional independence information. Which of the following sets of numbers is sufficient for the calculation?

    1. $\Pr(\alpha, \beta)$, $\Pr(\alpha)$, $\Pr(\beta|\alpha)$ and $\Pr(\gamma|\alpha)$.
    2. $\Pr(\beta, \gamma)$, $\Pr(\alpha)$ and $\Pr(\beta, \gamma|\alpha)$
    3. $\Pr(\beta|\alpha)$, $\Pr(\gamma|\alpha)$ and $\Pr(\alpha)$.

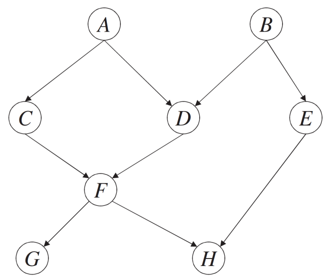
    For each case, justify your response either by showing how to calculate the desired answer or by explaining why this is not possible.

- Suppose we know that $\beta$ and $\gamma$ are conditionally independent given $\alpha$. Now which of the preceeding three sets is sufficient. Justify your response as before.

**Problem 5: Independence relations**

- Prove that Weak Union and Contraction hold for any probability distribution Pr.
- Provide a counter-example to the intersection property. (You cannot use the counter example given in AD, you have to make your own.)

**Problem 6: Bayesian networks (AD Exercise 4.1)**




| $A$ | $\Theta_A$ |
|---|---|
| 1 | .2 |
| 0 | .8 |

| $B$ | $\Theta_B$ |
|---|---|
| 1 | .7 |
| 0 | .3 |

| $B$ | $E$ | $\Theta_{E\|B}$ |
|---|---|---|
| 1 | 1 | .1 |
| 1 | 0 | .9 |
| 0 | 1 | .9 |
| 0 | 0 | .1 |

| $A$ | $B$ | $D$ | $\Theta_{D\|AB}$ |
|---|---|---|---|
| 1 | 1 | 1 | .5 |
| 1 | 1 | 0 | .5 |
| 1 | 0 | 1 | .6 |
| 1 | 0 | 0 | .4 |
| 0 | 1 | 1 | .1 |
| 0 | 1 | 0 | .9 |
| 0 | 0 | 1 | .8 |
| 0 | 0 | 0 | .2 |

Consider the Bayesian network given above.

1. List the Markovian assumptions asserted by the DAG.
2. Express $\Pr(a, b, c, d, e, f, g, h)$ in terms of network parameters.
3. Compute $\Pr(A = 0, B = 0)$ and $\Pr(E = 1|A = 1)$. Justify your answers.
4. True or false? Why?
   (a) $dsep(A, BH, E)$
   (b) $dsep(G, D, E)$
   (c) $dsep(AB, F, GH)$

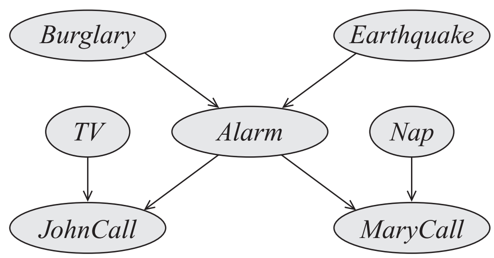
**Problem 7: Bayesian networks (AD Exercise 4.12)**
Construct two distinct DAGs over variables $A, B, C$, and $D$. Each DAG must have exactly four edges and the DAGs must agree on d-separation.

**Problem 8: Bayesian networks (AD Exercise 4.15)**
Identify a DAG that is a D-MAP for all distributions Pr over variables $\mathbf{X}$. Similarly, identify another DAG that is an I-MAP for all distributions Pr over variables $\mathbf{X}$.

**Problem 9: Bayesian networks (AD Exercise 4.17)**

Prove that for strictly positive distributions, if $\mathbf{B}_1$ and $\mathbf{B}_2$ are Markov blankets for some variable $X$, then $\mathbf{B}_1 \cap \mathbf{B}_2$ is also a Markov blanket for $X$. [**Hint:** Appeal to the intersection axiom.]

**Problem 10: Bayesian networks (Exercise 3.11 from KF)**




Consider the Burglary Alarm network given above. Construct a Bayesian network over all the node **except** the Alarm that is a minimal I-map for the marginal distribution over the remaining variables (namely, over $B, E, N, T, J, M$). Be sure to get all the dependencies from the original network.