Exploiting Logical Structure in Probabilistic Inference

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RECAP

- Graph-based Exact methods.
- Bucket elimination and Junction tree elimination
  - Convert the primal graph to a tree-decomposition
  - Perform message passing over the tree-decomposition
- w-cutset conditioning
  - Remove variables until treewidth is bounded by w
  - Conditioning on the removed variables and Bucket elimination on each assignment.
  - Space: $O(\exp(w))$. Time: $O(\exp(w+k))$. What is k?
- AND/OR search space (Treewidth: w^*)
  - Space: $O(n)$ vs. $O(\exp(w^*))$
  - Time: $O(\exp(w^*))$ vs. $O(\exp(w^* \log(n)))$

Today: Logic-based AND/OR search

- Can yield substantial reduction in complexity
  - Use logical propagation and pruning techniques
- Exploit context-specific independence (CSI) and determinism
  - CSI: Identical values in a factor (a CPT or a potential)
  - Determinism: Zeros in a factor

Graphical models as Weighted Logic

A	B	Value	Formula	Weight
0	0	0	$\neg A \wedge \neg B$	0
0	1	0.27	$\neg A \wedge B$	0.27
1	0	0.56	$A \wedge \neg B$	0.56
1	1	0.1	$A \wedge B$	0.1

- A graphical model is a set of mutually exclusive and exhaustive weighted formulas (F_i, w_i)
- The distribution it represents is given by

$$\Pr(\mathbf{x}) = \frac{1}{Z} \prod_i \phi_i(\mathbf{x})$$

where $\phi_i(\mathbf{x}) = w_i$ if $\mathbf{x}$ satisfies F_i and 1 otherwise.

Logic-based dynamic AND/OR Search: Example

- $(A \vee B \vee C \vee D \vee E, w_1)$
- $(A \vee B \vee C \vee F \vee G, w_2)$
- $(D \vee E \vee H, w_3)$
- $(F \vee G \vee J, w_4)$

What If I condition on A?

Logic-based dynamic AND/OR Search: Example

- $(A \vee B \vee C \vee D \vee E, w_1)$
- $(A \vee B \vee C \vee F \vee G, w_2)$
- $(D \vee E \vee H, w_3)$
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For A = True

- $2^2 \times (w_1 \times w_2)$
- $(D \vee E \vee H, w_3)$
- $(F \vee G \vee J, w_4)$

The two formulas are independent

For A = False

- $(B \vee C \vee D \vee E, w_1)$
- $(B \vee C \vee F \vee G, w_2)$
- $(D \vee E \vee H, w_3)$
- $(F \vee G \vee J, w_4)$

Can further condition on B

Logical Conditioning and Decomposition Algorithm

Algorithm LCD($\mathcal{F} = \{F_i, w_i\}$)

- **If** $\mathcal{F}$ is empty **Return** 1
- **If** $\mathcal{F}$ can be decomposed into k subsets such that no two subsets share a variable **then Return** $\prod_{i=1}^{k} LCD(\mathcal{F}_i)$
- Select a variable X_i to condition on.
- $v = 0$
- For $x_i \in \{True, False\}$ do
  - $w =$ Product of weights of all clauses in $\mathcal{F}$ that evaluate to true given $X_i = x_i$.
  - $\mathcal{F}' =$ Remove all clauses in $\mathcal{F}$ that evaluate to true or false given $X_i = x_i$.
  - Let p be the number of variables that appear in $\mathcal{F}$ but not in $\mathcal{F}'$. Multiply w with 2^p
  - $v = v + w \times$ LCD($\mathcal{F}'$)
- **Return** v

Logical Conditioning and Decomposition Algorithm

- Improvements
  - Heuristics for Conditioning
  - Condition on Formulas instead of variables!
    - (Gogate and Domingos, UAI 2010)
  - Caching
- Complexity
  - Same as AND/OR search (worst case)
  - Much smaller than AND/OR search (average case) if the problem has local structure (i.e., CSI and determinism)
- Logical Elimination?
- Logical Elimination and Conditioning?