Understanding Closed-End Fund Puzzles
A Stochastic Turnover Perspective

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This version: September 2000

Abstract

This paper presents a dynamic equilibrium model to study the pricing of most closed-end funds and their tendency to sell at discounts from the value of their assets. Based on the institutional structure of closed-end funds, our model demonstrates the importance of a stochastic turnover factor in determining both the levels and the dynamics of closed-end fund discounts. For example, discounts are likely to increase when (1) the turnover becomes less persistent, (2) a fund’s portfolio composition is less transparent, (3) volatilities of the underlying securities increase, (4) the interest rate rises, and (5) investors become more risk averse. Our model also implies weak return autocorrelations, the excess return volatility, and positive cross correlations between current discounts and future returns, which are consistent with empirical findings. In addition, we document that the level of discounts decreases with both the persistence and the volatility of discounts, while it increases with the volatility of returns from the net asset value. At the same time, cross correlations decrease with return volatilities of underlying assets and the persistence of discounts. We also show that the closed-end fund returns are both more predictable and more positively skewed than that of the returns from the corresponding net asset value. This evidence supports our model implications. The structure of our model sheds light on other issues, such as corporate spin-offs and “excess volatility” of stock market prices.

*This paper is developed from part of my dissertation. I would like to thank my advising committee, John Y. Campbell, Burton G. Malkiel, and Timothy Van Zandt, for their encouragement and advice. I am also indebted to Ben Bernanke, James Cardon, Ted Day, Charles Jones, Chris Leach, Larry Merville, Dale Osborne, Ram Rao, Mark Watson, and seminar participants at Federal Reserve Bank of Boston, Princeton University, Rutgers University, University of Colorado, and The University of Texas at Dallas for helpful comments. The address of the author is: School of Management; The University of Texas at Dallas; Richardson, TX 75083. Email: yexiaoxu@apache.utdallas.edu
Introduction

There are many empirical observations that have puzzled researchers in finance. One which has not been fully understood is the so called “closed-end fund puzzle.” Quite often the shares of closed-end funds in the secondary market sell at prices below the market value of the securities they hold. This may be a startling counterexample to the generally accepted efficient market hypothesis. Dozens of explanations, both economic and psychological, have been offered in an attempt to resolve the puzzle, and are best summarized in a book by Anderson and Born (1992). As shown by Malkiel (1977, 1995), “rational” explanations can at best account for only half of the cross-sectional variation of fund discounts. Some of the important factors include the percentage of a closed-end fund portfolio held in restricted (not marketable and, therefore, hard to value) securities, distribution policy, investor sentiment, and the amount of a fund’s unrealized capital gains. The dynamic behaviors of discounts are more challenging. Various empirical studies (for example, Thompson, 1978; Hardouvis, La Porta, and Wizman, 1993; and Pontiff, 1994) have shown that funds with positive premia provide negative abnormal future returns while funds with discounts earn positive abnormal future returns. The economically motivated explanations such as bid-ask spreads, differences in dividend taxation, investor sentiment, and so on, do not fully account for such cross-correlations.

Therefore, it is useful to take another look at the institutional characteristics of closed-end funds. Just like an open-end mutual fund, a closed-end fund holds a diversified portfolio. There are, however, some institutional reasons for the structure of closed-end funds. We recognize that the portfolio holdings of a typical closed-end fund can be divided into three groups. The first group consists of illiquid assets. For an open-end mutual fund, liquidity could be an issue when shareholders demand to redeem their shares. Such an issue does not apply to a closed-end fund since redemption is not allowed. In fact, many closed-end funds hold restricted securities (such as “letter” stocks, real estate assets, private placements, etc.) and are established mainly for long-term investment purposes. The rest of the portfolio consists of liquid assets that can be bought and sold at the prevailing market prices. The fact that these assets are liquid, however, does not necessarily mean that shareholders know exactly what assets are held in the closed-end fund portfolio. In other words, these liquid assets are not completely “observable” by shareholders at all points in time. For this reason, the liquid assets should be further classified as the observable security group and the unobservable security group. As a result, since investors are unable to fully explore all potential arbitrage opportunities by

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1 Essentially the bid-ask spreads represent transactions costs. If investors dislike high costs, the price of a closed-end fund will be low (the discount will be large) when the spread is large. Other things being equal, the future fund returns should be high.

2 As a stylized fact, stock prices are relatively high in their ex-dividend days. Therefore, the net asset value will be high (the discount will be large) when its underlying securities pay out dividends. When the closed-end fund pays out its dividend, however, its future returns should be high.
mimicking a closed-end fund’s portfolio, discounts or premia on closed-end funds can persist and will not invalidate the efficient market hypothesis. At the same time, however, “limited investment opportunities” with respect to the underlying securities will affect investors’ abilities to realize their optimal portfolio allocations which will influence closed-end fund prices.

Turnover is another important characteristic of any actively managed portfolios. It is a popular practice to rebalance portfolios or to exploit changes in relative prices. In reality, however, the level of turnover for a typical fund is far beyond the need to balance a portfolio. Since turnover also creates potential tax liabilities, it may not be in the best interest of shareholders. From an empirical perspective (see for example Malkiel, 1995; Day, Wang, and Xu, 2000), excess turnover does not enhance the portfolio value in general, even for mutual funds. Moreover, the efficient market hypothesis implies the irrelevance of turnover if fund managers do not possess true insider information. Therefore, a rational investor could perceive turnover activities simply as stochastic weights shifting among the underlying securities of a closed-end fund. In other words, turnover is stochastic from the investors’ perspectives and is a potential risk bearable by investors. In practice, one cannot diversify away such “stochastic turnover” risk by simply holding many closed-end funds. This is shown empirically later in the paper. Such a systematic risk in closed-end funds will help to create discounts on fund prices.

Based on the above institutional features of closed-end funds, we propose a simple dynamic rational expectations equilibrium model to explore these intuitions in pricing closed-end funds. Our model offers an alternative explanation of the discount phenomena in addition to many other important factors that influence the dynamics of a closed-end fund pricing. In particular, our model suggests that discounts or premia can exist because of investors’ inability to optimally allocate their portfolios as well as face stochastic turnover risk. Furthermore, discounts of a closed-end fund are related to the volatility and persistence in the turnover, the risk-free interest rates, the investment opportunities, the behaviors of underlying securities, and the degree of investors’ risk aversions. Some of these predictions have been demonstrated empirically, for example Pontiff (1997). Finally, a number of testable implications of the model are discussed throughout the paper. Empirical evidences are provided on most of the testable implications. For example, the level of discounts decreases with both the persistence and the volatility of discounts, while it increases with the volatility of returns from the net asset value. At the same time, cross correlations decrease with return volatilities of underlying assets and the persistence of discounts. We also show that the fund returns are both more predictable and more positively skewed than that of the returns from the corresponding net asset value.

Related to this paper is an equilibrium model proposed by De Long, Shleifer, Summers, and Waldmann (1990), which relies crucially on assumptions of constant dividends and misperceptions of noise traders. This is a popular model and offers many important insights. Yet, Brauer (1993) finds that only 7% of the variance of a standardized measure of weekly changes in discounts and premiums
can be attributed to noise-trading activity. Besides, DeLong, et al’s model implies that investors must be pessimistic most of the time, since closed-end funds usually sell at discounts. Also relevant is the static model studied by Oh and Ross (1994) that explains discounts from the perspective of asymmetric information with noisy supply. Despite problems in the simultaneous determination of a fund price and its underlying security prices in derivative pricing, it is an important alternative to rationalize noisy traders’ behavior in DeLong, et al’s model. From investors’ perspectives, one can interpret Oh and Ross’s asymmetric information assumption in the spirit of our stochastic turnover assumption. Yet, our assumption is weaker in the sense that we only assume investors are uncertain about the future portfolio composition of a closed-end fund in our dynamic model. Researchers have also studied the discount issue from an option perspective, such as the open-ending option proposed by Brauer (1988), and Chen, Merville and Won (2000), and the tax-timing option of Brickley, Manaster, and Schallheim (1991), and Kim (1994). These alternatives can partially account for the empirical observations.

Stochastic turnover adds a new dimension to the understanding of the pricing of closed-end funds as it identifies a new source of risk in the closed-end fund market. Both observed large discounts and occasional premia in the secondary market can be reconciled in this framework. Our dynamic model is also useful in studying time series properties of closed-end funds, such as weak return autocorrelations, the “excess” volatility of returns, and positive cross correlations between current discounts and future returns. The Swaminathan’s (1996) result on discount predictability can be understood in this framework too. As implied by our model, we further document that the positive cross correlations decrease both with return volatilities of the underlying assets and with the persistence of discounts. The structure of the model can be used to study other issues including corporate spin-offs and the “excess volatility” of stock markets, topics that will be explored later in the paper.

In order to motivate the model, we offer detailed discussions about some of the model assumptions in the next section, followed by an analysis of the pricing mechanism using a static model in section 1. We then introduce our dynamic rational expectations equilibrium model in section 2. The different roles played by the stochastic turnover and underlying assets in the determination of discounts are studied in sections 3 and 4, respectively. Empirical tests of our model implications are offered in section 5. Section 6 discusses some possible applications of the model. Section 7 concludes.
1 Understanding discounts from a static model perspective

In order to gain intuition, we first study discounts or premia on closed-end fund prices using a static counterpart of our dynamic model that is offered in the next section. Although we have motivated the use of the “limited market participation” assumption and a “stochastic turnover” assumption in the introduction, they have not been explored in the closed-end fund literature from the perspectives emphasized in this paper. For this reason, we take another look at these assumptions first.

1.1 Discussion of key assumptions

In the absence of arbitrage opportunities, it is reasonable to assume that investors cannot completely invest in the same portfolio of securities (whose supplies are infinitely elastic) as in a closed-end fund without directly holding a share of the fund (which has a fixed supply). As discussed earlier, a fund portfolio can be decomposed basically into three groups—the illiquid securities, the observable liquid securities and the unobservable securities at any point in time. In addition to illiquid holdings, a typical closed-end fund manager invests continuously in some securities for long-term investment purposes or to satisfy the fund’s investment objectives. The composition of these assets is reasonably stable and is, thus, observable. Investors can also freely invest in these observable individual securities. The unobservable securities are those assets that fund managers buy or sell frequently. By doing so, managers intend to chase short-term speculative gains or to take advantages of frequent changes in relative security prices. Since these securities move into and out of the portfolio on a regular basis, investors do not have knowledge about them. That is, investors only have limited access to the underlying securities held by closed-end funds. Therefore, even without illiquid assets in the portfolio of a closed-end fund, the existence of unobservable securities can still produce a similar effect. In other words, it applies to both diversified equity closed-end funds as well as to country funds. This important observation makes the “limited market participation” assumption a weak assumption.

We also note that the portfolio turnover among the underlying securities of a fund is stochastic from the shareholders’ perspectives. In the case of mutual funds, Day, Wang, and Xu (2000) have shown that there is a negative relationship between turnover ratios and the before expense performance. In the case of closed-end funds, we obtained composition data from a large closed-end fund traded on the New York Stock Exchange for illustrative purposes. The fund was traded at an average discount of 20%. Over the three-month period in mid-1996 for which we have data, the portfolio was turned

\[\text{3A report that discloses portfolio composition is usually sent to shareholders several weeks after the close of every quarter, and the aggregate premiums (or discounts) are published weekly. After a quarterly disclosure, the composition will likely change due to the turnover. Thus, continuously observing the composition of a closed-end fund portfolio is impossible.}\]
over significantly from month to month. In particular, turnover was as high as 60% in one of the
months, among which about 50% of the assets were bought for the first time or totally liquidated.
However, excess turnover for this particular fund does not seem to add value. Over the three months,
the portfolio suffered a total loss of 6.02%. In contrast, if there were no turnover activities in the
subsequent two months, the static portfolio would have had a loss of 5.86%. In the presence of taxes
and transactions costs, the advantages of a passive strategy are even more apparent. This illustrates
the claim that excess turnover is unlikely to enhance portfolio value, and can be characterized as an
exogenous stochastic process. The stochastic turnover assumption and the unobservability argument
are mutually supportive.

Some variations of both the “limited market participation” and the “stochastic turnover” assump-
tions have been used in the literature. For example, previous empirical studies have focused on the
negative side of the liquidity issue, which only implies discounts. Our model, however, stresses its effect
on the optimal portfolio choices of investors, and thus, can explain both discounts and premia. As to
the turnover issue, distinct from the existing literature that argues merely from increasing transactions
costs, our model explores the unique risk created by turnover. Since no explicit transactions costs or
taxes are assumed in the model, we focus on the unique channel created by stochastic turnover and
its effect on discounts of a closed-end fund. Moreover, stochastic turnover also allows the model to
generate many interesting dynamic structures.

With respect to turnover, it is not unique to closed-end funds. Therefore, one might expect that
stochastic turnover story should also apply to open-end mutual funds. We know, by law, that open-end
mutual funds have to be sold or redeemed at their net asset value. In other words, there is no discount
or premium on open-end mutual funds. However, if the stochastic turnover risk is also a systematic
risk in mutual funds, one should be able to argue that the net asset value does not measure the true
value of a mutual fund. This is unlikely to be the case given that there are thousands of open-end
mutual funds. In fact, by holding enough mutual funds in a portfolio, we will be able to trace the
“market” performance easily. Since the composition of a “market” portfolio is known and stable, any
stochastic turnover risks in individual mutual funds can be diversified away, which makes such risks
“idiosyncratic” in the mutual fund market. Nevertheless, since there are thirty-seven equity closed-
end funds in the U.S. with significant holdings in illiquid assets at the end of 1999, it is impossible to
diversify away stochastic turnover risk. This is shown in Figure 1.

Insert Figure 1 Approximately Here

The solid line in the graph represents the weekly returns of the S&P500 index from January 1990
to December 1996, while the solid dots represent differences between the weekly S&P500 index returns
and the average returns from net asset values of the hyper-portfolio of the twenty closed-end funds
that survived over the same period. Relative to the index returns, the differences are huge. In fact, the correlation between the two series is only about 0.7. Therefore, the stochastic turnover risk is still a systematic risk in the closed-end fund market.\footnote{One might further argue that investors should hold both closed-end and open-end funds to diversify the stochastic turnover risk. By doing so, we only make the closed-end funds negligible relative to the open-end funds in the basket. The risk is still there with respect to the closed-end fund market. Otherwise, there exists an arbitrage strategy that explores the discounts on closed-end funds.} In other words, it is reasonable to specify the turnover process as exogenously given.

### 1.2 A first look at discounts in a static model

In order to gain intuition, we first study discounts or premia on closed-end fund prices using a static counterpart of our dynamic model that is discussed in the next section. For simplicity, the group of observable underlying securities is modeled as one stock. Since fund investors cannot directly invest in either illiquid or unobservable underlying securities for the reasons discussed above, these assets are modeled as another stock. That is, we assume that there are two stocks in a two-period world with prices $P_1$ and $P_2$, respectively. Their random payoffs in the second period will be, $F_1 = \bar{F}_1 + \eta_1$ and $F_2 = \bar{F}_2 + \eta_2$. While fund managers observe both $P_1$ and $P_2$ to calculate the net asset value (NAV), investors only observe $P_1$. The shocks $\eta_1$ and $\eta_2$ are jointly distributed as a bivariate normal with zero means and a variance-covariance matrix of

$$
\begin{bmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2
\end{bmatrix}
$$

This distributional information is common knowledge to both investors and the fund managers. In addition, there is a risk-free bond that yields $R (= 1 + r)$ units for each unit of investment. Therefore, a unit of investment in, say, the first risky stock (with zero financing) yields an excess return of

$$\pi_1 = F_1 - RP_1 = \bar{F}_1 - RP_1 + \eta_1. \quad (1)$$

Investors are assumed homogeneous and have a Constant Absolute Risk Aversion (CARA) utility with a risk aversion parameter $\tau$, i.e.,

$$u(W) = -e^{-\tau W}, \quad (2)$$

where $W$ is an investor’s wealth. Given such a utility function, it is well known that the optimal portfolio is mean-variance efficient. In other words, if a representative investor were able to hold the equilibrium allocations in the first and the second stock $[a, b]'$, given their prices, then

$$
\begin{bmatrix}
a \\
b
\end{bmatrix} = \text{Var}^{-1}(\tau \begin{bmatrix}
\pi_1 \\
\pi_2
\end{bmatrix}) E\left( \begin{bmatrix}
\pi_1 \\
\pi_2
\end{bmatrix} \right) = \begin{bmatrix}
\frac{F_1 - RP_1}{(1 - \rho^2) \sigma_1^2} - \rho \frac{F_2 - RP_2}{(1 - \rho^2) \sigma_1 \sigma_2} \\
\frac{F_2 - RP_2}{(1 - \rho^2) \sigma_2^2} - \rho \frac{F_1 - RP_1}{(1 - \rho^2) \sigma_1 \sigma_2}
\end{bmatrix}.
\quad (3)
$$
derivative securities, we focus on the pricing of the closed-end fund given the prices of underlying securities. Therefore, \(P_1\) and \(P_2\) are assumed to be exogenously given. The possible influence on the prices of the underlying securities from portfolio turnover in the closed-end fund will be ignored here. In fact, the closed-end fund market is much smaller than the overall stock market and is substantially smaller than the open-end mutual fund market, where it is commonly argued that inflows into open-end funds influence the prices of their underlying securities.

As is convention in the closed-end fund literature, we do not model in this paper how a closed-end fund is introduced into the economy. As documented by Lee, Shleifer, and Thaler (1991), in the primary market, closed-end funds are issued at premia of nearly 10%. But these premia quickly turned into discounts in four months. This is even more challenging using a rational expectations framework. Denote \(r_m\) as the benchmark market return, \(r_{NAV}\) as the return from the net asset value, and \(NAV_1\) and \(NAV_{120}\) as the net asset values at the first date of an IPO and at the 120th date, respectively. In order for the closed-end fund to perform at least as well as the market, we should have,

\[
0.9NAV_{120} - 1.1NAV_1 = r_m,
\]

or simply

\[
r_{NAV} = 1.2r_m + 22%.
\]

This means that an average closed-end fund’s net asset value will out perform the benchmark by 20% plus an extra 22% return within four months, which is highly implausible. Since the IPO stage is a relatively short period compared with the life of the funds, we focus instead on the equilibrium share price afterwards in the secondary market when markets are more liquid. Unlike open-end mutual funds, the total supply of a closed-end fund is fixed. Therefore, its price will be determined endogenously. In general, the fund price will depend on the structure of the portfolio and the investment opportunities.

**Case I: Limited investment opportunities with a fixed portfolio**

Suppose that the closed-end fund holds a fixed portfolio with \(\alpha\) shares of the first stock and \(\beta\) shares of the second stock. When the fund investors are free to allocate their wealth among the two stocks in addition to investing in the closed-end fund, the fund price should exactly equal its net asset value \(NAV = \alpha P_1 + \beta P_2\). Otherwise, arbitrage profits can be realized through buying or selling short the two underlying stocks and making opposite transactions in the closed-end. This implies that there is no discount or premium.

In practice, however, this is rarely the case. As discussed in the previous section, the existence of either illiquid assets or unobservable assets limits investment opportunities. Not to lose generality, we assume that a representative closed-end fund investor can only invest in the closed-end fund and the first stock. The optimal portfolio with respect to the second stock is unachievable given the fixed composition and the fixed total supply of the funds in general, forcing the fund price to deviate from its net asset value (NAV). It is shown in Appendix A that at equilibrium, the discount \(\Delta_I\) of the
closed-end fund from its net asset value (NAV) is,

\[ \Delta^I = NAV - P^c = (1 - \rho^2)\bar{\beta}(\bar{\beta} - b)\frac{\tau\sigma^2}{R}, \tag{4} \]

where \( P^c \) is the closed-end fund price and \( \rho \) is the correlation coefficient between the two underlying assets. Since investors do not observe the price of the second stock, they do not necessarily know their implied optimal holdings with respect to the second stock \( b \). But as long as they understand the statistical distribution of the closed-end fund payoffs and the fund manager correctly calculates the net asset value, equation (4) suggests that the \textit{ex post} discounts depend on the implied optimal portfolio holding \( b \) with respect to the second stock. When the fund happens to supply the exact number of shares that an investor would desire, i.e., \( \bar{\beta} = b \), there is no discount. In other words, despite the constraint in the direct investment of the second stock, by holding shares in the closed-end fund, investors can still realize their overall optimal portfolios. However, when the closed-end fund provides more shares of the second stock than the aggregate closed-end fund investors’ optimal portfolios require, i.e., \( \bar{\beta} > b \), there will be a discount.\(^5\) By the structure of closed-end funds, it is impossible to reduce holdings in the second stock for the fund investors as a whole. The equilibrium fund price has to be lower than its net asset value.

The size of the discount depends on various factors including the risk premium of the underlying asset, \( \tau\sigma^2 \), the correlation coefficient \( \rho \), and the number of shares of the second stock in the portfolio \( \bar{\beta} \). Clearly, other things being equal, discounts are large when either the volatility of the second stock is high or investors are very risk averse. The correlation coefficient \( \rho \), however, has an inverse relationship with discounts. When the correlation between the two underlying assets increases, offsetting the inaccessibility to the second stock by directly investing in the first stock becomes much easier than before. Discounts decrease as the effectiveness of the investment opportunity constraint eases.

Similarly, when \( \bar{\beta} < b \), the implied excess demand in the second stock results in a premium on the closed-end fund instead. Therefore, under the limited market participation assumption, a premium or a discount can occur naturally. This can be labeled as the \textit{portfolio effect}. Together with the fact that \( \frac{\partial\Delta^I}{\partial\bar{\beta}} > 0 \) when \( \bar{\beta} > .5b \), we have the following implication:

**Model Implication 1** A closed-end fund that holds illiquid securities (or unobservable securities) could sell at either a premium or a discount depending on the total supply relative to the implied demand. When a discount occurs, it increases with the holding size of these underlying securities.

In other words, when a fund has “over” invested in these illiquid assets, divestiture may reduce discounts. We should emphasis that such a portfolio effect does not depend on whether a fund holds

\(^5\)It appears that a Pareto improvement is possible by changing the fund’s holding. Since we are not modeling the behavior of the fund managers, the portfolio composition is exogenous in the model.
illiquid assets or not, since any closed-end fund portfolio has the unobservable assets. Most studies have focused on holdings of illiquid securities as an expectation of discounts. In our model, however, when investors are favoring restricted or foreign securities, an increase in the holdings of these securities could potentially lower the fund’s premia too. In fact, there have been periods of time when “emerging market” closed-end funds sold at a premium over net asset value.

In the current setting, discounts or premia are independent of the first stock directly. However, since the two underlying stocks are correlated, there is an indirect effect. Equation (3) reveals that an increase in the volatility of the first asset will raise the implied optimal portfolio holding, \( b \), in the second stock. In other words, the second stock becomes desirable when the volatility of the first stock increases. The discount will then shrink. Applying the same argument, a decrease in the payoff or an increase in the price of the first stock will have a similar effect.

**Case II: Limited investment opportunities with a stochastic portfolio**

In order to incorporate the stochastic turnover phenomenon into our framework, we model it explicitly as stochastic weights of the fund’s portfolio. Suppose there is one share of a closed-end fund that holds \( \bar{\alpha} \) shares of the first stock and \( \bar{\beta} \) shares of the second stock, at time 0, with a net asset value of \( \text{NAV} = \bar{\alpha}P_1 + \bar{\beta}P_2 \). The fund managers change the composition of the fund stochastically for exogenous reasons after individual investors submit their demand first. In other words, the weight on the first stock is assumed to be \( \alpha = \bar{\alpha} + \epsilon \). Due to the fixed supply of the fund, the weight on the second stock will be \( \beta = \bar{\beta} + K \epsilon \), where \( K = -\frac{P_1}{P_2} \) and \( \epsilon \) has a normal distribution with zero mean and variance \( \sigma^2 \). The total payoffs that investors receive at time 1 from the closed-end fund is \( F_c = \alpha F_1 + \beta F_2 \). Because of the multiplication of random variables, we show, in Appendix A, that investors’ optimal portfolios are no longer mean-variance efficient. In particular, the expected equilibrium discount \( \Delta^{II} \) can be written as,

\[
\Delta^{II} = \xi \Delta^I + (\xi \Delta^I \frac{RK}{\beta})^2 \frac{\tau \sigma^2}{R} + \frac{(\sigma_1 + \rho K \sigma_2)^2 + (1 - \rho^2) \sigma_2^2 K^2}{1 - \tau^2 \sigma_2^2 (\sigma_1 + \rho K \sigma_2)^2} \xi \frac{\tau \sigma^2}{R},
\]

where \( \xi = \frac{1 - \rho^2}{1 - (\rho^2)(\tau \sigma_2 K)^2} > 1 \). Although equation (5) appears to be complicated, each term can be analyzed separately. The first term represents primarily the portfolio effect as discussed in the previous case, except that it has been strengthened by a factor \( \xi \). This is because the stochastic turnover effect makes the fund portfolio even less accessible. It is also intuitive that the multiplier \( \xi \) increases with the turnover volatility \( \sigma \) and decreases with the correlation coefficient \( \rho \).

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6 In this sense, the model can be reduced to a single risky asset model. Essentially the closed-end fund investors can achieve their optimal portfolios with respect to the first stock by directly adjusting holdings in that stock. This is the motivation to assume that the first asset is risk-free in our dynamic model in order to simplify the analysis.

7 The structure of the model and the statistical characteristics of \( \epsilon \) are assumed to be common knowledge.
The last term in equation (5) remains in the absence of a portfolio effect, i.e., $\beta = b$. Therefore, it represents a pure stochastic turnover effect on discounts. Since the closed-end fund investors have to bear the additional stochastic turnover risk, the associated discount simply rewards investors. As we have suggested earlier, stochastic turnover risk is a systematic risk in the closed-end fund market and is uncorrelated with the risks inherent in the underlying stocks. This is,

**Model Implication 2** Less portfolio turnover or a more transparent portfolio composition reduces discounts.

The empirical evidence is mixed. Boudreaux (1973) finds strong evidence that fund discounts are related to turnover while Malkiel (1977) finds no significant association. In any case, these empirical studies only provide indirect evidence since they do not control for the portfolio effect.

The last term in equation (5) can be further analyzed by recognizing the risk premium associated with stochastic turnover $\frac{1}{R} \sigma^2$ and the term in front of it. As the uncertainty in the portfolio composition ($\sigma$) increases, both the risk premium and $\xi$ become large, which increases the discount. Similarly, the term in front of the risk premium is increasing in $\sigma_1$, $\sigma_2$, and $\xi$. When underlying stocks become more volatile, the stochastic turnover effect is more significant, which increases the discount. The first stock will thus have a direct effect on discounts in this case. If we consider both the indirect portfolio effect due to $\sigma_1$ and the direct portfolio effect due to $\sigma_2$ as discussed in the previous case, together with the stochastic turnover effect from the volatilities of the underlying stocks, then,

**Model Implication 3** Changes in the volatilities of observable underlying assets have a weaker effect on discounts than that of unobservable securities, other things being equal.

The stochastic turnover effect and the portfolio effect also interact with each other, which is represented by the second term in equation (5). Clearly, no matter whether the fund is in “excess supply” or in “excess demand” with respect to the second underlying stock, the interactive effect always results in a discount. It is also useful to examine the correlation structure among underlying stocks. The volatility of turnover in one underlying stock will have great impact on the other if the two stocks are closely related. In other words, discounts due to stochastic turnover always increase with $\rho$. At the same time, the portfolio effect will be weak if the two assets become close substitutes, i.e., the absolute value of $\Delta I$ decreases with $\rho$. In general, the discount and the correlation between observable and unobservable assets will likely have a “hump” shaped relationship. Finally, it is unambiguous and intuitive to observe that the discount increases with the degree of risk aversion of fund investors.

The overall effect is undetermined, however, since the portfolio effect can be positive or negative depending on the parameter values of the model. In the case of excess supply of the second stock,
both the portfolio effect and the stochastic turnover effect work in the same direction, which leads to a large discount. In the case of excess demand, we may observe a premium if the portfolio effect dominates. Again, since the stochastic turnover effect is likely to be large, discounts prevail most of the time.

In summary, based on the institutional structure of closed-end funds, our static model has offered some new insights into why most closed-end funds sell at prices below their net asset value. In our framework, such differential pricing phenomenon could be attributed to differences between the implied investors’ optimal portfolios and the portfolio offered by a closed-end fund, which makes the overall fund less or more favorable. Such a portfolio effect is sustainable as long as the effective investment opportunities are limited, which we have argued is likely to be the case. There are some discussions in the literature about discounts as a result of the liquidity effect. In contrast, our model does not depend on a liquidity argument and is also consistent with occasional premia on closed-end funds. In addition, stochastic turnover not only enhances the portfolio effect but also adds additional risk for holders of a closed-end fund, which makes discounts more likely to be observed in reality. While the turnover issue studied in the literature focuses only on the tax and the transactions cost effects, we have approached the issue from the perspective of inherited risk and inadequate information about portfolio compositions.
2 A dynamic model for pricing closed-end funds

After discussing the cross-sectional properties and determinants of the closed-end fund discounts from the perspective of our static model, it is equally important to investigate the time series behavior of discounts. We extend our static model to a dynamic rational expectations equilibrium framework. In general, the behavior of both the observable and unobservable underlying securities of a closed-end fund are important in determining the dynamics of discounts. However, there is no closed form solution if both underlying securities are stochastic.\(^8\) Relatively speaking the second stock is more critical than the first stock in determining discounts and in order to emphasize the dynamic features, we build a simple model in this section using a stronger assumption about the underlying securities than the one used in the static model.

2.1 The basic model

The basic building blocks for the dynamic model are in the tradition of Campbell, Grossman, and Wang (1993), and De Long, Shleifer, and Waldmann (1990). In each period, a representative closed-end fund investor invests her current wealth in the financial market to maximize the expected utility given in equation (2) derived from the net wealth in the next period.\(^9\) As in the static model, we bundle the unobservable securities and illiquid securities as a single dividend paying stock. Its demeaned dividend \(\tilde{D}_t = D_t - \bar{D}\) is assumed to follow an AR(1) process of the following form,

\[
\tilde{D}_t = \alpha \tilde{D}_{t-1} + \eta_t, \tag{6}
\]

where \(\eta_t \sim N(0, \sigma^2_{D_t})\). Since closed-end funds are derivative securities, as usual, the supply of this underlying stock is assumed to be infinitely elastic. In other words, the stock price is exogenously given as\(^10\)

\[
P_t = \theta + \phi \tilde{D}_t, \tag{7}
\]

where \(\theta = \frac{\bar{D}}{r} - \varphi^2 \frac{\sigma^2_D}{r}, \phi = \frac{\alpha}{R - \alpha}, \text{ and } \varphi = 1 + \phi.\) Using the same logic, we continue to assume that closed-end fund investors cannot directly invest in this stock since it represents the unobservable assets. However, in order to focus on the dynamics and be able to obtain a closed-form solution, we simply model the observable securities as a riskless bond. In fact, discounts are more related to the unobservable securities than the observable securities in the fund portfolio.

---

\(^8\)With some simplifying assumptions, we are able to find an equilibrium solution. Details are available from the author.

\(^9\)This can be rationalized in an “Overlapping Generation Model” where the next period wealth for an investor is the same as her consumption.

\(^10\)The price is consistent with an equilibrium price in a market with a large number of investors endowed with one unit of the stock per capita.
A representative investor can transfer the period $t$ wealth into the period $t + 1$ wealth through:

- the same riskless bond that pays interest $r$ per period (or with a terminal value of $R = r + 1$); and/or
- a closed-end fund with one unit supply per capita.

The closed-end fund holds $\omega_{1,t}$ units of the riskless bond (with a unit price) and $\omega_{2,t}$ shares of the risky stock at time $t$. The net asset value (NAV) computed by the fund managers and the dividend of the fund can be written as,

\[
\text{NAV}_t = \omega_{1,t} P_{1,t} + \omega_{2,t} P_{2,t} = \omega_{1,t} + \omega_{2,t} P_t, \quad \text{and} \quad (8)
\]
\[
D_{t+1}^c = \omega_{1,t} r + \omega_{2,t} D_{t+1}, \quad (9)
\]

respectively. The timing is as follows. At time $t$, after the total dividends of the fund computed using the portfolio composition $w_{1,t-1}$ and $w_{2,t-1}$ are distributed, managers reset the portfolio composition to $w_{1,t}$ and $w_{1,t}$. The equilibrium price $P_t^e$ then prevails. For ease of exposition, we let the demeaned weight (or turnover) $\tilde{\omega}_{2,t} (= \omega_{2,t} - \bar{\omega}_2)$ associated with the underlying stock also follow an AR(1) process of form,

\[
\tilde{\omega}_{2,t} = \nu \tilde{\omega}_{2,t-1} + \varepsilon_t, \quad (10)
\]

where $0 \leq \nu < 1$ and $\varepsilon_t \sim N(0, \sigma^2)$. Since stochastic turnover risk is an independent risk, $\varepsilon_t$ and the innovation in the dividend process $\eta_t$ are uncorrelated with each other.

Once a closed-end fund is established, there is no additional investment flowing into or out of the fund. The total value of the fund thus remains constant (except for transactions costs, which are ignored here) right before and after turnover occurs. This implies the following constraint,

\[
\tilde{\omega}_{1,t} + \tilde{\omega}_{2,t} P_t = \tilde{\omega}_{1,t-1} + \tilde{\omega}_{2,t-1} P_t, \quad \text{or,} \quad \tilde{\omega}_{1,t} = \tilde{\omega}_{1,t-1} - P_t (\tilde{\omega}_{2,t} - \tilde{\omega}_{2,t-1}). \quad (11)
\]

Therefore, turnover $\tilde{\omega}_{1,t}$ on the observable asset depends on both the turnover $\tilde{\omega}_{2,t}$ associated with the risky stock and the price $P_t$, which in turn depends on the dividend process $D_t$.

### 2.2 Description of equilibrium in the closed-end fund market

After receiving dividend $D_t^c$ at time $t$ from the investment made in the previous period, closed-end fund investors reallocate their portfolios between the closed-end fund and the risk-free asset in order to maximize their expected utilities. The rational expectations are formed based on information set $\mathcal{I}_t$.
that includes current dividends, prices, and turnover information, i.e., $\mathcal{I}_t = \{P^c_t, D_t, D^c_t, \omega_{1,t}, \omega_{2,t}\}$. For consistency, we now define the equilibrium notion in this economy.

**Definition 1** A rational expectations equilibrium in the closed-end fund market is a stochastic price path $P^c_t$ that clears the closed-end fund market, given that investors’ optimal holdings in the closed-end fund are determined from maximizing their expected utility of wealth $E[u(W_{t+1})|\mathcal{I}_t]$.

In general, the price of a security in a perfect market is determined by its fundamental value, i.e., the expected value of discounted future dividends. Because of the additional uncertainty in turnover, the equilibrium fund price is determined by equating demand defined in Definition 1 to supply. This equilibrium is summarized in the following proposition.

**Proposition 1** There exists a stationary equilibrium in the closed-end fund market with the following quadratic price path,

$$P^c_t = -p_0 + \tilde{\omega}_{1,t} + b\tilde{D}_t + c\tilde{\omega}_{2,t} + d\tilde{D}^2_t + e\tilde{\omega}_{2,t}^2 + f\tilde{\omega}_{2,t}\tilde{D}_t,$$

(12)

where coefficients $p_0 (> 0), b, c, d (\leq 0), e (< 0)$, and $f$ are constants (defined in Appendix B) and depend on the model parameters.

**Proof:** See Appendix B.

Based on Proposition 1, we can easily compute returns for the closed-end fund. Under the normality assumption and CARA utility, the closed-end fund dollar return is defined as $R^c_{t+1} = P^c_{t+1} - D^c_{t+1} - RP^c_t$, i.e., the excess return.

$$R^c_{t+1} = g_0(\tilde{D}_t, \tilde{\omega}_{2,t}) + g_1(\tilde{D}_t, \tilde{\omega}_{2,t})\epsilon_{t+1} + g_2(\tilde{D}_t, \tilde{\omega}_{2,t})\eta_{t+1} + F\eta_{t+1}\epsilon_{t+1} + d[e\epsilon_{t+1}^2 - (R - \alpha^2)\tilde{D}_t^2] + e[e\epsilon_{t+1}^2 - (R - \nu^2)\tilde{\omega}_{2,t}^2] + F(R - \alpha

\nu^2)\tilde{\omega}_{2,t}\tilde{D}_t.$$

(13)

where $g_0(\ldots), g_1(\ldots),$ and $g_2(\ldots)$ are linear functions in the first and the second arguments. Since $d \leq 0, e < 0, R - \nu^2 > 1 - \nu^2,$ and $R - \alpha^2 > 1 - \alpha^2,$ the distribution of the closed-end fund returns is positively skewed, while the distribution for the returns from the fund’s net asset value is symmetric because,

$$R^c_{t+1} = NAV_{t+1} + D^c_{t+1} - NAV_t = p^0_{NAV} + \tilde{d}\tilde{\omega}_{2,t} + \varphi(\tilde{\omega}_2 + \tilde{\omega}_{2,t})\eta_{t+1},$$

(14)

where $p^0_{NAV} = \tilde{d}\tilde{\omega}_2$ and $\tilde{d} = \varphi^2\sigma_D^2$. Since most empirical studies have found skewness in high frequency portfolio returns, we should observe the following by comparing the distribution of a closed-end fund returns with that of returns from its $NAV$.

11 Although investors do not observe the stock price directly, they can infer its current dividends from the dividend payment of the closed-end fund.

12 In a model with a log-normal distributions and CRRA utility, it is natural to define returns in terms of dollars per dollar. However, under current model assumptions, it is common to define returns in terms of dollars per share.
Testable Implication 1 In general, returns of closed-end funds tend to be less skewed than returns from their net asset value.

This prediction is unique to our model. We later provide further evidence to support the claim.

Analogous to the definition of returns, we can define the closed-end fund discount $\Delta_t$ to be the difference between its net asset value and the market price, i.e.,

$$
\Delta_t = NAV_t - P^c_t = p_0^c + (\varphi \tilde{\omega}_2 - B) \tilde{D}_t - C \tilde{\omega}_{2,t} + (-d) \tilde{D}^2_t + (-e) \tilde{\omega}^2_{2,t} - F \tilde{\omega}_{2,t} \tilde{D}_t, \tag{15}
$$

where $p_0 = p_0 + E(NAV_t)$, $B = b + \tilde{\omega}$, and $F = f - \phi$. Equation (15) shows that the equilibrium discount also has a quadratic form in $\tilde{\omega}_{2,t}$ and $\tilde{D}_t$. Moreover, since $\tilde{\omega}^2_{2,t}$ and $\tilde{D}^2_t$ have $\chi^2$ distributions, the distribution of the discount will exhibit significant skewness. In other words, discounts tend to move in one direction far away from their means. Therefore, our model is not only consistent with the observed discounts on average but also predicts that discounts are likely to have a skewed distribution as well.

Clearly, the equilibrium fund price has a quadratic form both in the dividend process of the underlying stock and in the portfolio weights. Moreover, the coefficients $p_0$, $b$, $c$, $d$, $e$, and $f$ are themselves determined from a system of nonlinear equations. Therefore, as shown in Appendix B, two sets of equilibria emerge. These two equilibria are all rational and self-confirming in the sense that they can be interpreted separately.

When investors perceive the fund as an alternative supply of the risky stock, the fund price can be determined in two steps. Since the total supply of the closed-end fund is fixed, the total supply of the risky stock from holding the closed-end fund is $\omega_{2,t}$, which is determined exogenously by the fund managers. It is equivalent for the fund investors first to determine the value of the stock with a random supply of $\omega_{2,t}$. As shown in Campbell and Kyle (1992), the stock price is linear in $\omega_{2,t}$. Therefore, the total portfolio value of the fund, i.e., the fund price, is a quadratic function in $\omega_{2,t}$. Indeed, such equilibria are sustainable in our model as shown in Appendix B with the simplified equilibrium price of,

$$
P^c_t = NAV_t - p_0^c + C \tilde{\omega}_{2,t} + e \tilde{\omega}^2_{2,t},
$$

where, $p_0^c = \frac{1-v_w}{2v^2} + C^2 \frac{1+v_w}{2sv} \frac{\sigma^2}{r}$, $C = c - \theta$, and $v_w = 1 + 2e\sigma^2$. Although these equilibria can be understood in a noisy equilibrium framework, the implications are quite different. In a conventional model, noisy supply affects equilibrium price only at the aggregate level since individual investors can hold any number of shares of a stock. In contrast, stochastic turnover influences the equilibrium fund price at both the aggregate and individual levels. As each share of the fund has effectively $\omega_{2,t}$ shares of the risky stock, there will be a direct effect of $\omega_{2,t}$. At the same time, there is an indirect effect through the portfolio balancing.
The second set of sustainable equilibria occurs when investors literally treat the fund as one risky asset with a fixed supply. The equilibrium thus achieved has a price that is quadratic in both stochastic turnover and stock dividend as shown in equation (12). The non-linearity in the price is primarily due to interactions between the stochastic turnover effect and the optimal portfolio effect. The dynamics of the underlying stock thus directly enter into discounts.

The special structure of the two sets of equilibria allows us to focus on different aspects of the model. In particular, the first set of equilibria enables us to study the pure stochastic turnover effect alone, while the second set of equilibria allows us to analyze the dynamic effect of the dividend process of the underlying stock.
3 Intertemporal turnover effect

Due to the structure of the first set of equilibria, we focus on the stochastic turnover effect in this section in light of the stylized facts about closed-end funds. These stylized facts include an average discount of 10%, substantial discount fluctuations over time as documented by Lee, Shleifer, and Thaler (1991), and positive correlation between current discounts and future returns shown by Pontiff (1995) and Thompson (1978). Numerical calibrations are also offered to demonstrate the consistency of our model.

3.1 About discounts

The level of discounts can be studied by taking an unconditional expectation of equation (15),

\[ E(\Delta_t) = \frac{1}{r v_w} + \frac{1}{1 - \nu^2} \frac{1 - v_w}{2 \tau} + \frac{1 + v_w}{2 v_w^2} C^2 \frac{\tau \sigma^2}{r} + \bar{\omega}_2 (\bar{\omega}_2 - 1) \varphi^2 \frac{\tau \sigma_D^2}{r}, \]

where \( C = \frac{(2 \bar{\omega}_2 - 1) \varphi^2 \sigma^2}{\nu - \bar{R}_w}. \) The first two terms in equation (16) are positive, while the sign of the last term depends on the average size of the risky stock, \( \bar{\omega}_2. \) Since the net effect is indeterminate, it is consistent with the fact that most, but not all, closed-end funds sell at discounts. Similar to our static model, we can interpret the last term in equation (16) as the optimal portfolio effect, the first term as the stochastic turnover effect, and the second term as the interactive effect. The portfolio effect is independent of the existence of stochastic turnover. However, the stochastic turnover effect is only sustainable when there is a limited investment opportunity. Otherwise, investors can undo changes made by fund managers through taking opposite transactions in the underlying stock.

The discount effect due to stochastic turnover is decreasing in \( v_w. \) This is because an investor’s utility is low in the presence of stochastic turnover risk. Part of the utility loss needs to be compensated through the nonlinear response of the stochastic turnover shock, which can be measured by \( v_w. \) Relatively speaking, the overall utility loss will be large when \( v_w \) is small. Other things being equal, investors require a large discount to compensate for the extra risk from the stochastic turnover.

It is interesting to note that, although counter intuitive, discounts could decrease with the persistence (\( \gamma \)) of the turnover process. When \( \nu \) increases, any shocks to the turnover will have a prolonged effect, which depresses the price further or widens the discount. However, what is more significant is that changes in the two underlying assets will also be more tightly connected through the portfolio balancing condition. The responsiveness of non-linearity in the price becomes strong. That is, the utility compensation \( v_w \) increases, which makes discounts drop. It can also be understood from the perspective of the portfolio effect. When the stochastic turnover process becomes more persistent, the relative role of the risky stock in the portfolio becomes less significant, which weakens the discount
At the same time, an increase in $\nu$ makes a shock to the turnover behave more like a permanent effect on discounts. Therefore, discounts will be more persistent. This can be further illustrated by calculating the autocorrelation for discounts. Applying equation (15), we obtain the following (see Appendix C),

$$\text{Corr}(\Delta t, \Delta_{t+1}) = \frac{\nu}{2(1-\nu^2)} \frac{2(1-\nu^2)\nu^2 + \nu(1-v_w)^2}{2(1-\nu^2)\nu^2 C^2 + (1-v_w)^2}. \tag{17}$$

Researchers have found persistent and time-varying patterns of discounts (see, e.g., Lee, Shleifer, and Thaler, 1991; and Pontiff, 1995). In our model, the persistence in the discount is largely driven by the persistence in stochastic turnover. Indeed, equation (17) reveals a tight relationship between the persistence of discounts and the persistence in stochastic turnover.

The volatility of discounts can also be computed from equation (15) as,

$$\sigma^2_{\Delta} = \text{Var}(\Delta_t) = C^2 \left( \frac{\sigma^2}{1-\nu^2} + \frac{1}{2\tau^2} \frac{(1-v_w)^2}{(1-\nu^2)} \right), \tag{18}$$

which is influenced by several factors. Equation (18) suggests that discounts fluctuate more when either the persistency or the volatility in the stochastic turnover process increases. In addition, since $C$ is proportional to $\sigma^2_D$, discounts are more volatile when the volatilities of underlying securities increase. Considering the effects of increases in either the persistence or the volatility of stochastic turnover to the level of discounts, we have the following testable implication:

**Testable Implication 2** The level of discounts of a closed-end fund decreases with the persistence and the volatility of the discount process.

As shown in equation (16), discounts are independent of the levels of the dividend streams of underlying securities while asset prices increase with their dividends in general. This implies that the relative (or percentage) discounts move in opposite directions of the dividends of the underlying assets. Pontiff (1995) has found supportive evidence.

### 3.2 The dynamics of closed-end fund returns

Perhaps the most interesting time series property concerning a closed-end fund is the negative (positive) correlation between premia (discounts) and future returns, which was first documented by Thompson (1978) and confirmed in Pontiff (1995). Our model is useful in offering alternative explanations. In particular, we examine the following properties: (1) positive correlations between current discounts and future returns, (2) weak autocorrelations of closed-end fund returns, and (3) low correlations between net asset value and discounts.
Excess volatility

First, let us examine the return volatility. In Appendix C, we show that,

\[ \sigma_{Rc}^2 > \sigma_{NAV}^2 + \frac{1 + R^2 - 2R\nu^2}{2\tau}(1 - \nu^2)^2. \]  

This relationship indicates that returns of a closed-end fund are more volatile than the returns from its NAVs, which is consistent with the empirical evidence documented by Pontiff (1995). It can be interpreted as the “excess volatility” in the spirit of Shiller (1989). However, such an “excess volatility” is not due to the non-synchronous trading, or distorted net asset values, or the small firm risk. In our framework, it is driven by the stochastic turnover risk.

Return autocorrelation

When investors only emphasize the mean and variance of returns, there should be no return autocorrelation even when dividends are highly autocorrelated. Closed-end fund investors in our model, however, primarily care about the fund returns. Returns from the net asset value could thus be weakly autocorrelated especially in the presence of stochastic turnover risk as shown in equation (14). In contrast, the closed-end fund returns are negatively autocorrelated by examining equation (13). Explicitly the autocorrelation can be expressed as,

\[ \text{Corr}(R_{ct}, R_{c(t+1)}) = -\frac{1}{\sigma_{Rc}^2} \left[ C(RC - \bar{d})(1 - \nu)^2 - (RC - \bar{d})^2\nu \sigma^2 + [R - (\frac{r\nu}{1 - \nu^2})^2](1 - \nu^2)^2 \right]. \]  

As \( \bar{d} \) is relatively small, the autocorrelations are negative in general. Since investors’ portfolios will not be mean-variance efficient in this case, fund returns could be autocorrelated. Intuitively, the return autocorrelation rewards investors for taking the stochastic turnover risk. In summary,

**Testable Implication 3** Returns of closed-end funds are more “predictable” than the returns from their net asset value.

Cross correlation

We can also study the cross correlation between current discount and future returns. Applying equations (15) and (13), we can show the following,

\[ \text{Corr}(\Delta_{t}, R_{c(t+1)}) = \frac{1}{\sigma_{Rc}^2\sigma_{\Delta}^2} \left[ C^2\sigma^2 + \frac{\sigma^2C(\bar{d} - RC)}{1 + \nu} + \frac{R - \nu^2}{2\tau^2}(1 - \nu^2)^2 \right]. \]  

The first and last terms in equation (21) are positive for sure. Although the second term could be negative when \( C \) is small, this term is likely to be small, so that, in general, the cross correlation will be
positive. This is because a stochastic turnover shock depresses the current price, which in turn lowers current return. Future returns, however, are positioned to move high due to negative autocorrelation. At the same time, since the shock will have little effect on the NAV’s, the current discount is large, which results in a positive cross correlation between current discounts and future returns. In equation (21), we also observe that the cross correlation depends on the persistence of stochastic turnover. In particular, when \( \bar{d} \) is relatively small,

**Testable Implication 4** The cross correlation between current discounts and future returns decreases with the persistence of discounts.

Finally, the relationship between the net asset value and the discount can also be analyzed from equations (9) and (15). Since \( \text{Cov}(\tilde{\omega}_{1,t}, \tilde{\omega}_{2,t}) = -\theta \frac{\sigma^2}{1 - \nu^2} \), it is relatively easy to show that returns from NAV’s are uncorrelated with discounts.

### 3.3 A numerical example

In this section, we construct a numerical example to illustrate the quantitative nature of our model in light of the stylized facts. In particular, we assume that the risk aversion parameter (\( \tau \)) is 1; the risk-free rate (\( r \)) is 5%; the average number of shares for the underlying assets is \( \bar{\omega}_1 = 10 \) for the bond and \( \bar{\omega}_2 = 1 \) for the stock; the persistence parameter (\( \alpha \)), the mean (\( \bar{D} \)), and the innovation variance (\( \sigma^2_D \)) of the stock dividend process are .8, 3.5, and .035, respectively. The persistence parameter (\( \nu \)) and the innovation variance (\( \sigma^2 \)) for the turnover process are .65 and .022, respectively. Calibration results are reported in Table 1. There are three equilibria in this example including one that is discussed in the next section.

Insert Table 1 Approximately Here

In the first equilibrium with \( v_w = .9536 \), the utility compensation is relatively high. The resulting discount is moderate (about 3%), and so is the return volatility. In the second equilibrium, where the utility loss is substantial (\( v_w = .6599 \)), a larger discount (about 10%) is observed. This level of the discount is consistent with the empirical evidence. The volatilities of both returns and discounts in the second equilibrium are large. These two equilibria are perfectly rational. They correspond to two sets of self-confirming expectations. For example, in the equilibrium with small \( v_w \), investors expect a high volatility in returns (see equation (19)). Since small \( v_w \) also implies that investors have suffered a large loss in utility, the expected return needs to adjust substantially to compensate for the stochastic

\[13\text{These numbers correspond to an expected stock price of 58. In principle, these numbers can be estimated from historical closed-end funds’ compositional data.}\]
turnover, which in turn implies that returns will indeed have a high volatility. In equilibrium, a high volatility requires a large discount.

One can also perform a comparative static analysis of discounts by varying other parameters numerically. It is confirmed that discounts decrease with the level of stock dividends, but increase with the volatilities of both the turnover process and the underlying stock due to increased uncertainty. The relationship between discounts and the persistence in the turnover is monotonically decreasing as discussed in Testable Implication 2.

Table 1 also shows various calibrated statistics. For example, the volatility for the closed-end fund returns is higher (.7893 in the first equilibrium and .8946 in the second equilibrium) than the volatility of the net asset value (.6554). This demonstrates the “excess volatility” discussed before. The cross correlation is also substantial. It is about 30% in the “low volatility” equilibrium and about 23% in the “high volatility” equilibrium, which replicates the empirical finding of Pontiff (1995). It seems that future returns could be “predicted” from current discounts. Yet, this does not suggest inefficiency here. In fact, the possible “abnormal” returns are generated to compensate the closed-end fund investors for assuming the extra risk associated with stochastic turnover. One can, therefore, consider a discount as a partial measure of the fund’s exposure to stochastic turnover risk. The comparative static results suggest that such cross correlations decrease with the persistent parameter $\nu$, which is also consistent with an inverse relationship between discounts and $\nu$. Finally, there are small autocorrelations for the closed-end fund’s returns in both equilibria.\footnote{Due to the persistency in the turnover, there is autocorrelation in the Net Asset Value. But it is very small (about 2% in this example), and is consistent with the empirical results.} This weak autocorrelation is in turn consistent with the positive cross correlation.
4 The persistence effect from underlying securities

We have discussed the return and the discount dynamics from the perspective of our model when the only driving force is stochastic turnover. The second set of equilibria (i.e., the third equilibrium in our numerical example) is equally sustainable. Under such circumstances, investors perceive the fund as a single risky asset, so that stochastic turnover can serve as a bridge between the two groups of underlying securities through a portfolio balancing condition. Therefore, the dynamics of the dividend process of the risky stock will play an important role in the determination of discounts.

Despite its complex feature for the expected discount in current equilibria (see equation (28) in Appendix B), it is still possible to distinguish the portfolio effect from the stochastic turnover effect. Different from the first set of equilibria, the persistence measure of the underlying stock dividends (\( \alpha \)) plays a significant role. In particular, closed-end fund discounts change with \( \alpha \) in a non-monotonic way since \( \alpha \) influences both the optimal portfolio and turnover risk. When the dividend process becomes more persistent, an innovation will have a prolonged effect, which tends to increase the discount indirectly through portfolio balancing. In other words, the stochastic turnover process will appear to be more volatile. At the same time, investors will increase their preference toward the observable assets when the dividends of unobservable assets become more persistent, which lowers the discount. The combined effect is an U-shape relationship with \( \alpha \).

Discounts also increase with the riskiness of the underlying stock. For the same amount of risk inherited from a stock, the fund investors will discount its effect on the fund price more than the discount already reflected in the stock price. This is because the volatility of the underlying stock will also feed through turnover to affect the risk of the observable part of the portfolio as well (even though it only consists of a riskless bond in our model). A testable implication can be stated as,

**Testable Implication 5** Discounts tend to increase with the return volatilities of underlying assets.

The numerical example discussed in the previous section can also be applied to demonstrate the quantitative features of our model. In Appendix B, we have used an additional measure \( v_D \) that is similar to \( v_w \) to describe the equilibrium. For the same reason, \( v_D \) can be interpreted as a measure of utility compensation to the dividend effect. In the first set of equilibria, \( v_D \) is always one, which means there is no direct dividend effect in discount. In the current equilibrium, however, \( v_D \) is strictly less than one. In other words, discounts could be partially attributed to the dividend effect. From Table 1 we see that the discount and the return volatility of the fund are close to that of the second equilibrium discussed before.\(^{15}\) Moreover, the autocorrelation in discounts is much high because of the

\(^{15}\)Even though \( v_w \) is bigger than that in the second equilibrium, part of the utility compensation is now coming from the dividend effect.
direct effect from the persistence in the dividends of underlying stock. However, from static analysis perspective, volatilities of underlying assets will have an adverse impact to predictability for obvious reasons. In particular, we observe that,

**Testable Implication 6** Cross correlations between current discounts and future returns decrease with the return volatilities of underlying assets.

The most interesting feature of the current equilibrium is that the dynamics of discounts not only depend on stochastic turnover effect, but are also influenced by the characteristics of the underlying stock directly. This is useful in understanding Swaminathan’s (1996) empirical result that closed-end fund discounts forecast only the small firm factor returns. Although a typical closed-end fund holds a diversified portfolio, it is reasonable to believe that the observable assets in a fund mainly consist of large firms, while the unobservable assets are more likely to be small firm stocks. As documented by Lo and McKinlay (1990), current small firm and large firm returns only forecast future small firm returns. Therefore, Swaminathan’s finding could be driven by Lo and McKinlay’s results since discounts in our model are tied more closely to the dynamics of unobservable assets.

We can also study how the dynamic price and how the discount respond to fundamentals. Since the coefficients on $\tilde{D}_t$, $\tilde{D}_t^2$, and $\tilde{\omega}_t\tilde{D}_t$ in the closed-end fund price of equation (12) are all negative in this example, the fund price is more sensitive to fundamentals than the underlying assets. In other words, the existence of stochastic turnover makes it impossible for a fund’s investors to respond only to the fundamentals of the underlying stocks. They have to take extra steps to adjust for the stochastic turnover effect. If we believe that the institutional investors are mostly interested in the underlying assets and individual investors buy closed-end funds, institutional investors are more responsive to fundamental changes than individual investors. This is exactly the empirical finding of Sia (1997).
5 Empirical evidence

Since there is no existing evidence to support our model, an empirical study is inevitable. At the same time, however, a full-scale study on stochastic turnover requires a comprehensive portfolio composition data, which is unavailable to us. As a preliminary test, we will focus on the testable implications discussed above. Moreover, these implications may also be consistent with other theories. The related empirical evidences by themselves will be important. We have collected equity closed-end fund data from Monday issues of the Wall Street Journal from 1990 to 1996.\(^\text{16}\) There are 20 general domestic equity funds that have continuous records and 8 randomly selected specialized domestic equity funds.\(^\text{17}\) We have also randomly selected 28 international equity funds to match the number of domestic funds. Therefore, in our database, we have 56 funds with 361 time periods. The monthly dividends and capital gains distribution information are from Yahoo Finance. In order to fully utilize the data set, we will apply both cross-sectional regressions and time series analysis.

We first report summary statistics for our data set in Table 2. During the sample period, the average discount seems to be higher for the domestic closed-end funds (about 5\%) than that of the international funds (about 3\%). As noted by other researchers, not every fund in our sample sells at a discount. In fact, there are about a quarter of funds with premia in both domestic and international fund categories. Although average discounts vary substantially across funds with a standard deviation of 10\% for both categories, discounts of international funds seem to fluctuate much more over time (about 11\%) than that of domestic funds (about 6\%). Table 3 also shows that discounts are about twice as volatile as the closed-end fund returns. For all funds, the average volatility of discounts is more than twice the average discount. While the average domestic fund return is comparable to the average S&P500 index return (about .223\% per week) over the same period, international funds performed much worse than the benchmark. However, no matter which category we examine, returns from funds are more than 30\% more volatile than those from their net asset values, which confirms the so-called “excess volatility” implications of the model. Table 2 also shows that discounts for the domestic funds as a whole are a little more persistent (about 91\%) than that of the international funds (about 89\%). At the same time, the persistence of discounts seems to vary much less across international funds (about 3.2\%) than that of the domestic funds (about 7.8\%).

Insert Table 2 Approximately Here

Despite all the differences across the two categories of funds, the characteristics of the cross-correlations between current discounts and future returns are very similar. All funds have a positive

\(^{16}\)Although these funds are traded daily, the net asset value information is only available weekly. I am grateful to Chaehwan Won for collecting the data from The Wall Street Journal.

\(^{17}\)The number is proportional to the number of available funds in that category.
one-week cross-correlation with an average of 16%. This number quickly drops to 6% for the four-week cross-correlation. It is even smaller if we examine the quarterly horizon. Therefore, the dividend taxation and the dividend yield arguments mentioned in the introduction could not account for such a large weekly cross-correlation. Otherwise, cross-correlations should be stronger over longer horizons.

5.1 The cross-sectional results

Since closed-end funds are actively managed by professionals, returns from the net asset values resemble those of the open-end mutual funds. In particular, Table 3 summarizes some of the characteristic differences between the closed-end fund returns and returns from their net asset values (or mutual funds). For both domestic and international funds, the fund returns seem to be positively skewed (0.439) while returns from the net asset values are negatively skewed (−0.214). This confirms the Testable Implication 1. Although the differences in skewness are much more severe for the international funds than for the domestic funds, these differences are all statistically significant. Moreover, on an individual fund level, there are more than 64% of the domestic funds and more than 85% of the international funds exhibiting larger skewness in fund returns than that of returns from net asset values. Table 3 also shows that the fund returns are more predictable than returns from the net asset values as described in Testable Implication 3. The average $R^2$ from an AR(1) model fitted to closed-end fund returns is 4.61% for domestic funds (or 2.46% for international funds), while the comparable $R^2$ for the net asset returns is only 2.53% for domestic funds (or 1.59% for international funds). Despite the fact that the difference in $R^2$s is statistically insignificant for the international fund group due to the low level of predictability, the $R^2$ difference for the domestic funds are significant at a 1% level. When examining the individual funds, we find again that more than two-thirds of the funds exhibit large predictability in fund returns than the corresponding returns from the net asset values.

Insert Table 3 Approximately Here

We further investigate the closed-end fund discounts from cross-sectional regressions. Two methods have been implemented in particular. In the first method we run a single cross-sectional regression based on estimates for individual funds. Although we use heteroscedastic consistent variance estimates in the cross-sectional regression, we will likely suffer an error-in-variable problem if these estimates are time varying. In order to partially account for the problem, in the second method, we run cross-sectional regressions based on individual fund estimates using 26 weeks (i.e. six months) of data, and aggregate these cross-sectional results using a Fama-McBeth approach. Since volatility in the net asset value reflects both the volatilities of the underlying assets and the volatility of stochastic turnover, we use it as a proxy for the aggregate underlying asset volatilities and/or for the stochastic turnover volatility. As discussed earlier that the discount process is very persistence, we use the innovation
volatility of the discount process in our cross-sectional regressions in order to separate the persistence and the volatility effects of discounts. In particular, we first estimate an AR(1) model for each fund’s discounts to obtain the persistence measure \( \nu^d \) and the volatility measure \( \sigma_\Delta \). Table 4 shows the cross-sectional regression results on the determination of discounts.\(^{18}\)

The first and the third regression equations in Table 4 implement \textit{Testable Implication 2}. We first regress the level of discounts on the persistence of discounts and the volatility of discounts separately. As predicted, discounts decrease with either the persistence or the volatility of discounts. This is a new result although the persistence variable is more significant for domestic funds while the volatility variable is more important in international funds. In order to implement \textit{Testable Implication 5}, we use the return volatility for the net asset value \( \sigma_{NAV} \) as a proxy. As discussed before, this measure summarizes volatilities for both stochastic turnover and returns of underlying assets. The reported results in the second equation of Table 4 are mixed. On the one hand, we find strong empirical support for the positive relationship between discounts and return volatilities of the net asset values for the domestic funds at a 1% significance level. On the other hand, we find an opposite significant relationship (at 5% level) for the international funds. This could be attributed to the fact that \( \sigma_{NAV} \) and \( \sigma_\Delta \) are more highly correlated and the latter has much more significant effect for international funds.

Insert Table 4 Approximately Here

The three factors—the persistence of discounts and the volatilities of both discounts and returns from the NAVs seem to be very important in explaining the cross-sectional difference in discounts for the domestic funds. In the last equation in Table 4, we consider the three factors simultaneously. Not only are the signs of the regression coefficients correct and the estimates significant, but the \( R^2 \) is as high as 57% as well. The results are weaker for the international funds with only the discount volatility variable being significant. But the \( R^2 \) from the regression is close to 40%.

One can also investigate how persistence of the stochastic turnover process and volatilities of underlying assets will affect the cross-correlations between current discounts and future returns as suggested in \textit{Testable Implication 4} and 6. The first equation in Table 5 shows a significant negative relationship between cross-correlations and persistence in discounts for both groups of funds, which supports \textit{Testable Implication 4}. The \( R^2 \) for the cross-sectional regression is as high as 75% for the domestic funds and 24% for the international funds. In order to study \textit{Testable Implication 6}, we again use \( \sigma_{NAV} \) as the proxy for the volatilities of underlying assets. However, this variable is only significant for the international fund group with an \( R^2 \) of 57% under Method I, but is significant for both groups under Method II. The discrepancy under the two methods may be attributable to the

\(^{18}\)Note that we have taken logarithm on all the volatility measures since volatility can only be positive.
time-varying estimates. When both factors are used in the cross-sectional regression, we are able to explain more than 60% of the cross-sectional variations in the cross-correlations for all funds.

As discussed earlier, the volatility of discounts and the volatility of net asset value will be correlated with each other. We provide evidence in the last block of Table 5. In particular, we regress the volatility of discounts on the volatility proxy for both stochastic turnover and returns of underlying assets. The positive relationship is highly significant with an $R^2$ of 52%.

5.2 A further look at discounts and the predictability of returns

We now turn to the time series behavior of discounts. In our sample, there are many more observations over time than across funds. In order to accommodate that, we apply the following two approaches to summarize test results. In the first approach, we pool the data across funds and estimate relationship between discounts and various factors. Such regressions are adjusted for cross fund heteroscedasticity using an iterative procedure. The second approach aggregates estimates from regressions performed on individual funds. As suggested by Swamy (1971), one can use the variance-covariance matrix of estimates from individual regressions to weight the corresponding estimates. This approach allows us to use Newey-West variance-covariance matrix to adjust for autocorrelation and heteroscedasticity over time, i.e.,

$$\bar{\beta} = [\sum_i \hat{v}_i^{-1}]^{-1} \sum_i \hat{v}_i^{-1} \hat{\beta}_i,$$

where $\hat{\beta}_i$ and $\hat{v}_i$ are parameter estimates and Newy-West variance-covariance matrix of parameter estimates, respectively.

The first regression in Table 6 shows that discounts are very persistent for both domestic funds and international funds. In addition, the equation also reveals that interest rates are a significant factor in affecting the dynamics of discounts. This is due to the fact that stochastic turnover risk will be relatively large when interest rates are high. If we believe stochastic turnover risk should be higher for international funds than for domestic funds in general, interest rates should be more responsive to international funds. Indeed the regression coefficients on interest rates are .184 for international funds and .032 for domestic funds. In the same equation, we also include the future return of closed-end fund since Thompson (1981) has suggested the cross-correlation between current discounts and future fund returns. As expected, we see that the relationship is highly significant. However, as discussed above, a similar relationship between discounts and future returns from the net asset value should not be observed, which is confirmed (not shown in the table).

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19Since the cross group residual correlations are very small (about 5%), it is ignored in the estimation.

20Pontiff (1995) argued that interest rate should be positively related to discounts if one interprets it as the cost to do arbitrage.
Cross-correlation can also be attributed to tax implications on dividend distribution or the dividend yield effect. When a fund pays out dividends, its yield will be higher if it sells at a discount than if it sells at a premium, other things being equal. In order to disentangle these known effects from our model implications, we further investigate the effect due to anticipated future movement in returns versus surprises in returns. Presumably these tax effects and dividend yield effects should largely be anticipated if dividends are stable, while the stochastic turnover effect should mainly be unanticipated. Here the anticipated returns are computed as the predicted returns from an AR(4) model and the surprises are the residuals from the same model. The last equation in table 6 shows that both the expected and unexpected future returns are significantly correlated with current discounts. Therefore, stochastic turnover effect is also important in generating positive cross-correlations.

As a final note, we study the predictability issue of closed-end fund returns, since discounts could serve as an important predictor due to the cross-correlations. The first equation in Table 7 indicates a moderate $R^2$s of 4.34% for the domestic funds and 2.46% for the international funds when only the first lag of returns is included. The $R^2$s are comparable if discounts alone are used as predictors (2.98% for the domestic funds and 3.19% for the international funds). The two predictors are largely independent. When both the lagged returns and discounts are included as predictors, each variable acts significantly with $R^2$s of 6.71% for the domestic funds and 5.09% for the international funds. We can further investigate the source of predictability by separating the predictors into expected components and surprises. The last regression in Table 7 shows that the surprises in either lagged returns or discounts are very significant, while only the expected discounts are significant in predicting returns. Furthermore, such decompositions of predictors have greatly improved the predictability. In particular, the $R^2$s are as high as 10% for the domestic funds and 8% for the international funds.
6 Possible applications

In many aspects, the fundamental structure of a closed-end fund resembles that of a public corporation. On the one hand, a public corporation issues a fixed number of shares traded on a stock exchange, and regularly distributes dividends to its shareholders just like a closed-end fund. On the other hand, a corporation engages in a broad line of business or in several different projects. These projects can be considered as the underlying assets of the firm. Since the two basic assumptions used to build our closed-end fund model are applicable in the case of a corporation, valuing a firm is similar to valuing a closed-end fund. In other words, some of the implications regarding closed-end funds carry over to the case of publicly traded corporations.

In general, there are many projects a firm takes on at any time. In order to survive in a competitive environment, a firm must have its own uniqueness, which can be characterized by some well-known projects. These projects are in the spirit of “illiquid” assets in the portfolio of a closed-end fund. Other projects may be highly substitutable in nature by other firms’ products. These projects resemble the “observable” assets in a closed-end fund. Still, there are some projects that are less well known and may be classified as “unobservable” assets. In any case, from an investor’s perspective, investing separately in each underlying project of a firm without actually buying a share of the firm is impossible. This means investors do not have a full investment opportunity regarding the underlying projects. Furthermore, while most shareholders may have an idea about the long-term investment plan of a firm, daily investment decisions are solely at the managers’ discretion. In other words, investment emphasis may be shifted from one project to another depending on short-term performance. This suggests that, from shareholders’ viewpoints, there is stochastic turnover among a firm’s projects. If one is willing to accept this analogy, our closed-end fund model is useful in understanding some of the issues in financial economics, such as the “excess” volatility of stock price and corporate spin-offs.

6.1 Excess volatility

In a seminal paper, Robert Shiller (1981) directly challenged the efficient market hypothesis using a so-called “excess volatility” puzzle.\(^\text{21}\) In his words, excess volatility means that “the very variability of price movements is too large to be justified in terms of efficient markets models, given the relatively low variability of fundamentals.” In other words, stock prices are much more volatile than their fundamentals. This result is very robust and survives careful econometric analysis, though the explanations that Shiller has given relies on the existence of fads and changing fashions, i.e., changes in prices are largely due to changes in investors’ opinion or psychological motion.

\(^{21}\)This claim was also independently made by Stephen LeRoy and Richard Porter.
In our model, excess volatility does not necessarily suggest market inefficiency. As pointed out in section 4, a closed-end fund exhibits excess volatility due to stochastic turnover effect. Since a corporation can have a similar structure as that of a closed-end fund, the excess volatility in stock market can be justified using our closed-end fund model. In particular, by the same argument and from the perspectives of shareholders, there may exist stochastic turnover among the investment projects of corporations. The observed stock prices that incorporate this added risk should be more volatile than the volatility of the underlying projects or fundamentals. This argument yields the following implication,

**Testable Implication 7** *Firms with less diversified business should have less “excess” volatilities relative to their fundamentals.*

### 6.2 Corporate spin-offs

Corporate spin-offs refer to the phenomenon where a corporation separates its entity into two or more independent entities. The current shareholders receive pro rata distributions of separate equity claims on the assets of each new corporate entity. It is well documented that there are significant positive abnormal returns surrounding announcements of corporate spin-offs (see Hite and Owers, 1983; Miles and Rosenfeld, 1983; and Schipper and Smith, 1983). There are several theoretical arguments that attempt to explain this phenomenon, for example, emanation of negative synergies, agency costs (see Myers, 1977), and market incompleteness (see Hakansson, 1982). Based on these arguments, researchers have tested related hypotheses including wealth transfers from senior claimholders (Hite and Owers, 1983; Schipper and Smith, 1983), relaxation of regulatory or tax constraints (Schipper and Smith, 1983), enhanced contracting efficiency between the parent and the subsidiary (Hite and Owers, 1983), and improving managerial structure (Schipper and Smith, 1983). Nevertheless, the empirical support for these hypotheses is weak.

Our model is also useful in understanding the spin-off issue if one accepts the analogy we have drawn between a corporation and a closed-end fund. Just like a closed-end fund, a firm can be undervalued. In our model, this is likely to happen when stochastic turnover among underlying investment projects is high and/or the allocation of investment projects is not optimal from shareholders’ perspectives. In both cases, a feasible solution to this problem is via a separation, that is a spin-off of the firm. As a result, the value of each part appreciates through a reduction in the stochastic turnover risk. At the same time, shareholders are able to allocate their assets more efficiently to achieve optimal portfolios. This is consistent with the empirical findings of increasing shareholders’ wealth after spin-offs. Finally, our analysis on the return volatility and potential stochastic turnover risk implies that,

**Testable Implication 8** *Return volatility should decrease after a spin-off, other things being equal.*
7 Concluding comments

Different from the popular sentiment argument in the closed-end fund literature, we have been able to add some new insights in understanding the pricing of closed-end funds using a rational expectations equilibrium model. This framework is capable of explaining both cross-sectional and time series properties of closed-end funds. As a result of imperfect asset allocation, either a premium or a discount can be observed depending on the composition of a closed-end fund portfolio. In addition, stochastic turnover adds an additional risk to a closed-end fund. In order to compensate for this type of risk, a discount on the price of a closed-end fund is generally required. Stochastic turnover, interacting with the uncertainty inherited in the underlying assets, also generates some interesting dynamics. For example, our model suggests a strong negative correlation between current discounts and future returns, a week negative autocorrelation of returns, and “excess” volatility, which are consistent with empirical studies on closed-end funds. From a cross-sectional perspective, this model implies that discounts tend to be high when a fund has a large percentage of unobservable securities, has a risky portfolio of the underlying assets, and has a low dividend level. Discounts also tend to increase when interest rates are high, when the turnover process is less persistent, and when investors are very averse to risk.

Distinct from the existing literature that studies the turnover issue merely from increasing transactions costs, our model suggests that there is a unique risk created by turnover. Since no explicit transaction costs are assumed in the model, we are able to focus on the effectiveness of turnover on discounts. Moreover, stochastic turnover also allows the model to generate many interesting dynamic structures. At the same time, our model does not crucially depend on the existence of illiquid securities in closed-end fund portfolios. Therefore, it applies to both diversified equity closed-end funds as well as to country funds. While some discussions regarding the liquidity issue can be found in the literature, these studies focused on the negatives, which only imply discounts. Our model, however, stresses the optimal portfolios of investors, and thus, can explain both discounts and premia.

Many interesting testable implications have been suggested throughout the paper. Empirical results using collected data support most of these implications. In addition, we documented that the closed-end fund returns are both more predictable and more positively skewed than those of the returns from the corresponding net asset value. We have also shown that the level of discounts decreases with both the persistence and the volatility of discounts, while it increases with the volatility of returns from the net asset value. At the same time, cross correlations decrease with return volatilities of underlying assets and the persistence of discounts. Based on an analogy drawn between a corporation and a closed-end fund, our model also shed lights in understanding other issues in financial economics, such as “excess” volatility of stock prices and corporate spin-offs.
Appendix A

Case I derivation

In this case, we can first write down her budget constraint when investing in \( x \) shares of the first stock and \( y \) shares in the closed-end fund as

\[
W_1 = W_0 + (F_1 - RP_1)x + (F_c - RP_c)y,
\]

where \( F_c = \bar{\alpha}F_1 + \bar{\beta}F_2 \) is the payoff for the closed-end fund. With the CARA utility model of equation (2), one can show that the expected utility, \( E(W_1) \), is proportional to,

\[
(\bar{\alpha}F_1 - RP_1)x + (\bar{\alpha}F_1 + \bar{\beta}F_1 - RP_c)y - \frac{\tau}{2}[(x + \bar{\alpha}y)^2\sigma_1^2 + 2\rho(x + \bar{\alpha}y)\bar{\beta}y\sigma_1\sigma_2 + \bar{\beta}^2y^2\sigma_2^2].
\]  

(22)

The representative investor’s optimal portfolio is the one that maximizes her expected utility. Together with the market clearing condition of \( y = 1 \), the first order conditions for equation (22) lead to the expression (4)

Case II derivation

As usual, we will first write down the budget constraint for a representative investor in order to determine the equilibrium price of a closed-end fund,

\[
W_1 = W_0 + \bar{F}_1(x + \bar{\alpha}y) + \bar{F}_2\bar{\beta}y - R(P_1x + P_cy)
\]

\[
+ (x + \bar{\alpha}y)\eta_1 + \bar{\beta}y\eta_2 + (\bar{F}_1 + K\bar{F}_2)y\epsilon + (\eta_1 + K\eta_2)y\epsilon.
\]

This leads to the following expected utility

\[
E[u(W_1)] \propto -\frac{1}{\sqrt{f_2(y)}}e^{g_2(x,y)} \propto \frac{1}{2} \log[f_2(y)] - g_2(x,y),
\]  

(23)

where

\[
f_2(y) = 1 - (\tau\sigma_1)^2y^2 - (\tau\sigma_2K)^2y^2 - 2\rho\tau^2\sigma_2\sigma_1K^2y^2,
\]

\[
g_2(x,y) = -\tau[\bar{F}_1(x + \bar{\alpha}y) + \bar{F}_2\bar{\beta}y - R(P_1x + P_cy)] + \frac{(\tau\sigma_1)^2}{2}(x + \bar{\alpha}y)^2
\]

\[
+ \frac{(\tau\sigma_2)^2}{2}\bar{\beta}^2y^2 + \rho\tau^2\sigma_1\sigma_2(x + \bar{\alpha}y)\bar{\beta}y + \frac{(\tau\sigma_2)^2}{2}[\bar{F}_1 + K\bar{F}_2 - \tau\sigma_1(\sigma_1 + \rho K\sigma_2)(x + \bar{\alpha}y) - \tau\sigma_2(\sigma_1 + K\sigma_2)\bar{\beta}y^2y^2]
\]

\[
\frac{2}{1 - (\tau\sigma_1)^2y^2 - (\tau\sigma_2K)^2y^2 - 2\rho\tau^2\sigma_2\sigma_1K^2y^2}.
\]

Therefore, the equilibrium price for the closed-end fund can be obtained by optimizing the expected utility function of equation (23) and applying the market clearing condition, \( y = 1 \). The expected premium in equation (5) is just the expected difference between the equilibrium price \( P_c \) and the net asset value.
Appendix B

Proof of Proposition 1:

Under a CARA utility assumption, there is a subset of models for which closed form solutions exist. In particular, they are models that produce the following excess returns,

\[ \pi_{t+1} = P[v_t, \zeta_{t+1}]_n \quad n = 1, 2 \]

where \( P[.]_n \) denotes an \( n \)-th order polynomial, and \( v_t \) is a vector of random process and \( \zeta_{t+1} \) is a vector of innovation to \( v_t \). This is the reason for using the risk-free asset in the model. Since \( n = 2 \) in our model, there is no linear equilibrium. Nevertheless, from the insight of mean variance analysis and by trial and error, we can conjecture a quadratic equilibrium price in the form of equation (12). Correspondingly, we can write the excess return in the following form,

\[
\pi_{c,t+1}^e = P_{c,t+1}^e + D_{c,t+1}^e - R \pi_{c,t}^e \\
= p_{c,t}^e + p_{c,t}^e \eta_{t+1} + p_{c,t}^e \varepsilon_{t+1} + F \eta_{t+1} \varepsilon_{t+1} + d \eta_{t+1}^2 + e \varepsilon_{t+1}^2 \quad (24)
\]

where,

\[
p_{c,t}^e = -rp_0 + \omega_1 r + \omega_2 \tilde{D}_2 + |B(\alpha - R) + R\tilde{D}_1| \tilde{D}_t + |C(\nu - R) + \varphi^2 \tau \sigma^2| \tilde{\omega}_{2,t} + d(\alpha^2 - R) \tilde{D}_t^2 + e(\nu^2 - R) \tilde{\omega}_{2,t}^2 + F(\alpha \nu - R) \tilde{\omega}_{2,t} \tilde{D}_t \\
p_{\eta,t}^e = \tilde{B}(\varphi + F \nu) \tilde{\omega}_{2,t} + 2 \alpha \omega \tilde{D}_t \\
p_{\varepsilon,t}^e = C + 2 \alpha \omega \tilde{\omega}_{2,t} + F \alpha \tilde{D}_t, \quad and
\]

where \( B = b + \omega_2 \), \( C = c - \theta \), and \( F = f - \phi \). Since the excess return will respond to the underlying shocks asymmetrically, the excess return will be weakly serially correlated even when both the dividend process and the turnover process are uncorrelated. This is not a contradiction to the efficient market hypothesis. It is this serial correlation that rewards investor taking the stochastic turnover risk.

A closed-end fund investor will choose \( z_t \) shares of the closed-end fund and the rest of her asset in the risk-free asset in order to maximize her expected utility. Based on the normality assumption, utility function (2), and excess return equations (24), some algebraic manipulations lead to the following expected utility function.

\[
E_t[u_{t+1}|I_t] \propto - \int \int \int_{-\infty}^{\infty} e^{-\tau \sigma \varepsilon_{t+1} z_t} e^{-\frac{1}{2} \tau \sigma \varepsilon_{t+1}^2} d \varepsilon_{t+1} d \eta_{t+1} d \eta_{t} \\
\times \left[ \frac{1}{2} \ln(\Xi_t + \tau z_t p_{c,t}^e) - \frac{(\tau \sigma z_t)^2 (1 + 2 \epsilon \sigma_0^2 z_t)}{2 \Xi_t} (p_{\eta,t}^e)^2 - \frac{(\tau \sigma_0 z_t)^2 (1 + 2 \epsilon \sigma^2 z_t)}{2 \Xi_t} (p_{c,t}^e)^2 + \frac{F \tau \kappa z_t^3}{2 \Xi_t} - p_{\eta,t}^e p_{c,t}^e \right], \quad (25)
\]

where

\[
\Xi_t = 1 + 2 \epsilon \tau \sigma_0^2 z_t + 2 \epsilon \sigma^2 z_t + (4 \epsilon d - F^2) \kappa^2 z_t^2.
\]

When a closed-end fund investor optimizes her expected utility \( E_t[u_{t+1}|I_t] \), she finds her optimal portfolio holding for the closed-end fund. Even so, the first order condition will be highly nonlinear with respect to \( z_t \).
Fortunately, in this problem, we have assumed that the closed-end fund investors are homogeneous and the total supply of the closed-end fund is 1. Therefore, we can set } \frac{z_i}{1} = 1 \text{ in equilibrium.}

\[ 0 = -\frac{\varepsilon \sigma^2}{\tau \Xi} + \frac{\tau \sigma^2}{\Xi^2}(1 + e \tau \sigma^2)(1 + e \tau \sigma^2 + d \sigma^2) + e \tau \sigma^2 \Xi(p_{e,t})^2 + \frac{\tau \sigma^2}{\Xi^2}(1 + 2d \sigma^2)(1 + e \tau \sigma^2 + d \sigma^2) + d \tau \sigma^2 \Xi(p_{e,t})^2 - \frac{F_r}{\Xi^2} \cdot 2[1 + e \tau \sigma^2 + d \sigma^2] + \Xi |p_{e,t}|^2 e \]

Using the method of undetermined coefficients, the unknown coefficients } B, C, d, e, \text{ and } F \text{ can be solved from the following system of equations:

\[
\begin{align*}
\alpha (\varphi + F \nu) \Xi - \alpha (v_w + v_D)(\varphi v_w + F \nu) + (RF + \alpha \varphi) \Xi^2 &= 0 \\
(\nu^2 + \varphi^2 \kappa^2) \Xi - (v_w + v_D)(\nu^2 v_D - \varphi^2 \kappa^2 + \frac{\nu^2 + F \nu}{1 - \nu^2} \kappa^2) + R \Xi^2 &= 0 \\
\frac{R}{\nu} \Xi^2 + \frac{1 + \nu v_D}{1 - \nu v_D} - \frac{v_w + v_D}{1 - \nu v_D} &= 0 \\
[\alpha \Xi B - \alpha (v_w + v_D)(v_w B - \varphi \kappa^2 FC) + R(B - \bar{w}_2) \Xi^2 = 0 \\
[(\nu - F \varphi \sigma^2) C - (1 - \nu \varphi) \sigma^2 B] \Xi - (v_w + v_D)[(\nu v_D + \varphi \kappa^2 F)C - (v_w \varphi + F \nu) \sigma^2 B] + (RC + \varphi^2 \sigma^2) \Xi^2 &= 0
\end{align*}
\]

where } v_w = 1 + 2e \tau \sigma^2 \geq 0 \text{ and } v_D = 1 + 2d \sigma^2 \geq 0. \text{ Since the last two equations are linear in } B \text{ and } C \text{ given } v_w, v_D \text{ and } F, \text{ we can concentrate on solving } v_w, v_D, \text{ and } F \text{ from the first three equations. Again this is a nonlinear system of equations. However, we can identify two sets of solutions. Since one can show that } 0 < v_D \leq 1, \text{ let us first examine the case where } v_D = 1. \text{ The third equation reduces to } F = 0. \text{ Therefore, } v_w \text{ can be determined from the second equation, i.e.,}

\[ R v_w^3 - (R - 2 \varphi^2 \kappa^2) v_w^2 - \nu^2 v_w + \nu^2 = 0 \]  

(26)

Since } v_w \text{ should be positive, there could only be two feasible solutions which satisfy } \frac{\nu}{\sqrt{R}} < v_w < 1 - \frac{2(\varphi \nu)^2}{R}.

When } 0 < v_D < 1, \text{ our strategy is to first solve for } \Xi \text{ from the third equation given } v_w \text{ and } v_D, \text{ i.e.,}

\[ \Xi = \frac{(1 + v_w) - \sqrt{(1 + v_w)^2 - 4Rv_w(v_D - 1)(v_w + v_D)/R}}{2R(v_D - 1)/\alpha^2} \]

(27)

After substituting } \Xi \text{ into the first two equations, we can numerically solve for } v_w \text{ and } v_D. \text{ Iterating this procedure until they converge.}

When we are in the second set of equilibria the expected discount can be expressed as,

\[ E(\Delta_t) = \frac{v_w + v_D}{2r \tau \Sigma} + \frac{v_w \beta + v_D \zeta - 2FC \tau \kappa^2}{2r \tau \Sigma} (1 + \frac{v_w + v_D}{\Sigma}) - \frac{d \sigma^2}{1 - \alpha^2} - e \frac{\sigma^2}{1 - \nu^2} - \frac{1}{r \tau} (1 + \frac{\beta + \zeta}{2 \Sigma} - \bar{w}_2 \varphi \frac{2 \sigma^2 \tau \kappa^2}{r}) \]

(28)

where } v_D = 1 + 2d \sigma^2 \text{, } \beta = (\tau \sigma \beta B)^2, \text{ } \zeta = (\tau \sigma C)^2, \text{ and } \Sigma = v_w v_D - (F \kappa)^2. \]
Appendix C

Calculations of variances and correlations:

Let \( \sigma^2 = \text{Var}(\tilde{\omega}_{2,t}) = \frac{\sigma^2}{1 - \nu^2} \) and \( \sigma^2_{DD} = \text{Var}(\tilde{D}_t) = \frac{\sigma^2_D}{1 - \nu^2} \). Since \( \tilde{\omega}_{1,t}, \eta_{1,t+1}, \eta_{2,t+1}, \) and \( \varepsilon_{t+1} \) are independently normally distributed, the cross correlations of these random variables are all zero. Furthermore, using the properties of the normal distribution, we have,

\[
E(\varepsilon^3_{t+1}) = 0, \quad \text{and} \quad \text{Var}(\varepsilon_{t+1}) = 3\sigma^4, \quad \text{and} \quad \text{Var}((\varepsilon^2_{t+1} - E[\varepsilon^2_{t+1}])^2) = 2\sigma^4,
\]

With these, it is just a matter of algebraic manipulation to show the followings,

\[
\text{Corr}(\Delta^e_t, \Delta^i_{t+1}) = (B - \varphi \tilde{\omega}_2)^2 \frac{\alpha \sigma^2_D}{1 - \alpha^2} + \nu C \sigma^2 \frac{\nu^2}{2\tau^2} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2 + \frac{\alpha^2}{2\tau^2} \left( \frac{1 - v_D}{1 - \alpha^2} \right)^2 + \frac{\nu \alpha F^2 \sigma^2 \sigma^2_D}{(1 - \alpha^2)(1 - \nu^2)}
\]

\[
\text{Corr}(R^e_t, R^i_{t+1}) = -[(1 - \nu \alpha R)F + (1 - \alpha^2)\nu \varphi] \frac{(R - \nu \alpha) F \sigma^2 \sigma^2_D}{(1 - \alpha^2)(1 - \nu^2)} - \frac{(1 - \alpha^2 R)(R - \alpha^2)}{2\tau^2} \left( \frac{1 - v_D}{1 - \alpha^2} \right)^2 - [(R - \alpha)B - R\tilde{\omega}_2] \frac{(R - \alpha)B + \alpha R\tilde{\omega}_2}{1 - \nu^2} \frac{\sigma^2_D}{1 - \alpha^2}
\]

\[
\text{Corr}(\Delta^e_t, \Delta^i_{t+1}) = (B - \varphi \tilde{\omega}_2)[(R - \alpha)B - R\tilde{\omega}_2] \frac{\sigma^2_D}{1 - \alpha^2} + [(R - \nu)C - \tilde{d}]C \frac{\sigma^2}{1 - \nu^2} + \frac{R - \nu^2}{2\tau^2} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2 + \frac{R - \alpha^2}{2\tau^2} \left( \frac{1 - v_D}{1 - \alpha^2} \right)^2 + \frac{(R - \nu \alpha) F^2 \sigma^2 \sigma^2_D}{(1 - \alpha^2)(1 - \nu^2)}
\]

\[
\text{Var}(\Delta_t) = (B - \varphi \tilde{\omega}_2)^2 \frac{\sigma^2_D}{1 - \alpha^2} + \frac{C \sigma^2 \sigma^2_D}{1 - \nu^2} + \frac{\nu^2}{2\tau^2} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2 + \frac{\alpha^2}{2\tau^2} \left( \frac{1 - v_D}{1 - \alpha^2} \right)^2 + \frac{F^2 \sigma^2 \sigma^2_D}{(1 - \alpha^2)(1 - \nu^2)}
\]

\[
\text{Var}(R^e_t) = [(\alpha - R)B + R\tilde{\omega}_2]^2 \frac{\sigma^2_D}{1 - \alpha^2} + [(\nu - R)C + \tilde{d}]^2 \frac{\sigma^2}{1 - \nu^2} + B^2 \sigma^2_D + \frac{2}{\nu \varphi} \left( \frac{1 - v_D}{1 - \alpha^2} \right)^2 + \frac{1 + R^2 - 2\nu^2}{2\tau^2} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2
\]

\[
\frac{\sigma^2 \sigma^2_D}{(1 - \alpha^2)(1 - \nu^2)}
\]

For the first set of equilibria, \( \nu_D = 1 \), and the return volatility simplifies to the following,

\[
\text{Var}(R^e_t) = (\tilde{\omega}_2^2 \varphi \sigma_D) + \frac{\nu \varphi^2 \sigma^2 \sigma^2_D}{1 - \nu^2} + \frac{C(R - \mu) - \tilde{d}^2}{1 - \nu^2} + C \sigma^2 + \frac{1 + R^2 - 2\nu^2}{2\tau^2} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2
\]

\[
> \sigma^2_{AV} + \frac{1 + R^2 - 2\nu^2}{2\tau} \left( \frac{1 - v_w}{1 - \nu^2} \right)^2
\]

(29)
References


[22] Oh, Gyutaeg, Stephen A. Ross (1993), ‘Asymmetric Information and the Closed-end Fund Puzzle,’ University of Iowa and Yale University.


Figure 1: Return differences between the S&P 500 Portfolio and average closed-end fund portfolios.
Table 1: **Numerical Results for Premia, Variances, and Correlations**

In this table, we calibrate our model using the following parameters. We set the risk aversion parameter $\tau = 1$; the risk-free rate $r = 5\%$; the average number of shares for the first asset $\bar{\omega}_1 = 10$ and for the second asset $\bar{\omega}_2 = 1$ in the fund portfolio. The persistence parameter ($\alpha$), the mean ($\bar{D}$), and the innovation variance ($\sigma^2_D$) of the stock dividend process are assumed to be $.8, 3.5, and .035$, respectively. The persistence parameter ($\nu$) and the innovation variance ($\sigma^2$) for the turnover process are $.65$ and $.022$, respectively. $v_w$ and $v_D$ are utility compensations for the stochastic turnover effect and the dividend effect, respectively, while $F$ measures the interaction between these two effects.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Equilibrium I</th>
<th>Equilibrium II</th>
<th>Equilibrium III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_w$</td>
<td>0.9536</td>
<td>0.6599</td>
<td>0.7843</td>
</tr>
<tr>
<td>$v_D$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9635</td>
</tr>
<tr>
<td>$F$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>2.8530</td>
</tr>
<tr>
<td>$E(\Delta_t) / E(NAV_t)$ (%)</td>
<td>3.149</td>
<td>10.48</td>
<td>10.03</td>
</tr>
<tr>
<td>$Var(R_t)$</td>
<td>0.7893</td>
<td>0.8946</td>
<td>0.8788</td>
</tr>
<tr>
<td>$Var(NAV_t)$</td>
<td>0.6554</td>
<td>0.6554</td>
<td>0.6554</td>
</tr>
<tr>
<td>$Corr(\Delta_t, R_{t+1}^e)$ (%)</td>
<td>30.01</td>
<td>22.45</td>
<td>37.29</td>
</tr>
<tr>
<td>$Corr(R_t^e, R_{t+1}^e)$ (%)</td>
<td>-1.342</td>
<td>-6.550</td>
<td>-7.596</td>
</tr>
<tr>
<td>$Corr(\Delta_t, \Delta_{t+1})$ (%)</td>
<td>64.45</td>
<td>49.07</td>
<td>69.00</td>
</tr>
</tbody>
</table>
Table 2: Summary Statistics for Domestic and International Closed-end Equity Funds

This table summarizes the statistics for the 28 domestic and 28 international equity closed-end equity funds. These results are based on weekly data from January 1990 to December 1996. $\bar{\Delta}$, $\nu$, and $\sigma_{\Delta}$ are the average, the persistent coefficient (AR(1) coefficient), and the volatility of discounts over time. $R^C$ and $\sigma_{R^C}$ denote the time series average and the volatility of a closed-end fund returns, while $R^{NAV}$ and $\sigma_{R^{NAV}}$ are the average and the volatility of returns from net asset values. $\text{Corr}(\Delta_t, R^C_{t+1})$ is the cross-correlation between current discount and future return. “Mean”, “StdDev”, “Min”, and “Max” are calculated across each group of funds. All numbers in the table are in percentage form.

<table>
<thead>
<tr>
<th></th>
<th>Domestic Funds</th>
<th>International Funds</th>
<th>All Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>StdDev</td>
<td>Min</td>
</tr>
<tr>
<td>$\bar{\Delta}$</td>
<td>4.93</td>
<td>9.90</td>
<td>-24.2</td>
</tr>
<tr>
<td>$\nu$</td>
<td>91.0</td>
<td>7.77</td>
<td>66.3</td>
</tr>
<tr>
<td>$\sigma_{\Delta}$</td>
<td>5.96</td>
<td>2.52</td>
<td>2.48</td>
</tr>
<tr>
<td>$R^C$</td>
<td>.204</td>
<td>.130</td>
<td>-.067</td>
</tr>
<tr>
<td>$\sigma_{R^C}$</td>
<td>2.61</td>
<td>.981</td>
<td>1.48</td>
</tr>
<tr>
<td>$R^{NAV}$</td>
<td>.190</td>
<td>.106</td>
<td>-.026</td>
</tr>
<tr>
<td>$\sigma_{R^{NAV}}$</td>
<td>2.00</td>
<td>.892</td>
<td>1.01</td>
</tr>
<tr>
<td>$\text{Corr}(\Delta_t, R^C_{t+1})$</td>
<td>15.5</td>
<td>7.55</td>
<td>2.64</td>
</tr>
<tr>
<td>$\text{Corr}(\Delta_t, R^C_{t+4})$</td>
<td>5.12</td>
<td>3.40</td>
<td>-.261</td>
</tr>
</tbody>
</table>

Table 3: Characteristic Difference Between Closed-end Fund Returns and Returns from Net Asset Value

This table shows distributional difference between a closed-end fund returns $R^C$ and the returns from the corresponding net asset values $R^N$. “Mean” and “StdDev” are calculated across each group of funds. $S$ denotes skewness.

<table>
<thead>
<tr>
<th></th>
<th>Domestic Funds</th>
<th>International Funds</th>
<th>All Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^C$</td>
<td>$R^N$</td>
<td>$R^C$</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.255</td>
<td>-0.065</td>
<td>0.625</td>
</tr>
<tr>
<td>$S_{R^C} - S_{R^N}$</td>
<td>0.320</td>
<td>0.985</td>
<td>0.652</td>
</tr>
<tr>
<td></td>
<td>(0.152)</td>
<td>(0.240)</td>
<td>(0.147)</td>
</tr>
<tr>
<td>$S_{R^C} &gt; S_{R^N}$ (%)</td>
<td>64.2</td>
<td>85.7</td>
<td>75.0</td>
</tr>
<tr>
<td>$R^2$ (%)</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>$R^2_{R^C} - R^2_{R^N}$</td>
<td>2.08</td>
<td>0.88</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.56)</td>
<td>(0.41)</td>
</tr>
<tr>
<td>$R^2_{R^C} &gt; R^2_{R^N}$ (%)</td>
<td>75.0</td>
<td>60.7</td>
<td>67.9</td>
</tr>
</tbody>
</table>
Table 4: Explaining Discounts from Cross-sectional Regressions

This table shows the cross-sectional evidence that supports the model implications on discounts. Under “Method I”, simple cross-sectional regressions using time-series average for each fund are used. “Method II” applies Fama and McBeth cross-sectional method. In particular, each cross-sectional regression utilizes averages based on 26 weeks (or 6 months) of non-overlapping observations. Numbers in the parentheses denote standard deviations. \( \Delta, \nu, \) and \( \sigma_{NAV} \) denote discount, persistence in discounts, and log of volatility in the returns from the net asset value, respectively. \( \sigma_{\Delta} \) is the log of innovation volatility in the discount process. “*” means a coefficient is significant at 5% level and “**” represents significance at 1% level.

| Method | Domestic Funds | | International Funds | | All Funds | |
|--------|----------------|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|        | \( \nu^d \) | \( \sigma_{NAV} \) | \( \sigma_{\Delta} \) | \( R^2 \) | \( \nu^d \) | \( \sigma_{NAV} \) | \( \sigma_{\Delta} \) | \( R^2 \) | \( \nu^d \) | \( \sigma_{NAV} \) | \( \sigma_{\Delta} \) | \( R^2 \) |
| I      | -.504* (.229) | 15.6 | -.956 (.563) | 10.0 | -.525** (.210) | 10.0 |
|        | .067** (.022) | 25.8 | -.051* (.023) | 16.4 | -.001 (.022) | 0.00 |
|        | -.035 (.036) | 3.50 | -.078** (.020) | 36.4 | -.030 (.012) | 11.1 |
| II     | -.400* (.179) | 57.4 | -.433 (.509) | 39.1 | -.558** (.189) | 33.0 |
|        | .089** (.020) | .011 (.024) | -.066** (.024) | .042** (.017) | -.061** (.015) | |
|        |-.106** (.028) | | | | | |
|        | -.502** (.047) | | | | | |
|        | .041** (.003) | | | | | |
|        | -.006* (.003) | | | | | |
|        | -.905** (.070) | | | | | |
|        | .050** (.006) | | | | | |
|        |-.058** (.007) | | | | | |

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Table 5: Cross-sectional Evidence on Implications of the Model

This table shows the cross-sectional evidence that supports the model implications on discount volatility and cross-correlation. Under “Method I”, simple cross-sectional regressions using time-series average for each fund are used. “Method II” applies Fama and McBeth cross-sectional method. In particular, each cross-sectional regression utilizes averages based on 26 weeks (or 6 months) of non-overlapping observations. Numbers in the parentheses denote standard deviations. $\Delta$, $\nu$, and $\sigma_{NAV}$ denote discount, persistence in discounts, and log of volatility in the returns from the net asset value, respectively. $\sigma_\Delta$ is the log of innovation volatility in the discount process. "*" means a coefficient is significant at 5% level and "**" represents significance at 1% level.

<table>
<thead>
<tr>
<th>Method</th>
<th>Domestic Funds</th>
<th></th>
<th>International Funds</th>
<th></th>
<th>All Funds</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu^d$</td>
<td>$\sigma_{NAV}$</td>
<td>$R^2$(%)</td>
<td>$\nu^d$</td>
<td>$\sigma_{NAV}$</td>
<td>$R^2$(%)</td>
</tr>
<tr>
<td>I</td>
<td>-.842**</td>
<td>75.1</td>
<td>24.4</td>
<td>-.876*</td>
<td>49.9</td>
<td></td>
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<tr>
<td></td>
<td>(.095)</td>
<td>(0.395)</td>
<td>(.119)</td>
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</tr>
<tr>
<td></td>
<td>.018</td>
<td>3.40</td>
<td>-.074**</td>
<td>57.4</td>
<td>-.014</td>
<td>3.20</td>
</tr>
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<td>(.012)</td>
<td>(.011)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>-.868**</td>
<td>75.8</td>
<td>66.3</td>
<td>-.958*</td>
<td>60.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.100)</td>
<td>(.279)</td>
<td>(.109)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.010)</td>
<td>(.012)</td>
<td>(.007)</td>
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</tr>
<tr>
<td>II</td>
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<td>29.4</td>
<td>.823**</td>
<td>51.8</td>
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</tr>
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<td>(.123)</td>
<td>(.108)</td>
<td></td>
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<tr>
<td></td>
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<td>.535**</td>
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<td></td>
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<tr>
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<td>(.09)</td>
<td>(.083)</td>
<td></td>
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<tr>
<td></td>
<td>-.524**</td>
<td>-.046**</td>
<td>-.610*</td>
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<tr>
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<td>(.228)</td>
<td>(.083)</td>
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<td>(.012)</td>
<td>(.009)</td>
<td>(.007)</td>
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<td>.377**</td>
<td>.667**</td>
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<td>(.123)</td>
<td>(.043)</td>
<td>(.046)</td>
<td></td>
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</table>
Table 6: **Times Series Evidence on Discounts**

This table shows aggregate time-series results on discounts. Numbers in the parentheses denote standard deviations. $R_c$ denote closed-end fund returns. $E_t(R_{t+1}^c)$ and $E_t^U(R_{t+1}^c)$ denote the predicted value and the residual from an AR(4) model on $R^c$. “Aggregation I” pools data for individual fund and adjusts for cross-sectional heteroscedasticity. “Aggregation II” aggregates estimates for individual fund using Newy-West variance and covariance matrix as weight. “Individual” counts the number of individual regression for each fund with significant coefficients at 5% level.

<table>
<thead>
<tr>
<th>Dep. Var. $\Delta_t$</th>
<th>$\Delta_{t-1}$</th>
<th>$r_{f,t}$</th>
<th>$R_{t+1}^c$</th>
<th>$E_t(R_{t+1}^c)$</th>
<th>$E_t^U(R_{t+1}^c)$</th>
<th>$R^2(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Aggregation I</td>
<td>0.980</td>
<td>0.032</td>
<td>0.182</td>
<td></td>
<td></td>
<td>85.0</td>
</tr>
<tr>
<td></td>
<td>(.0018)</td>
<td>(.013)</td>
<td>(.0077)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Aggregation II</td>
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<td></td>
</tr>
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<td></td>
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<td>(.011)</td>
<td>(.0079)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Individual (%)</td>
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<td>57.1</td>
<td>89.3</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(.0015)</td>
<td>(.011)</td>
<td>(.0079)</td>
<td></td>
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<tr>
<td></td>
<td>100.0</td>
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<td>89.3</td>
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<td>Aggregation I</td>
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<td>(.0313)</td>
<td>(.0067)</td>
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<tr>
<td>Individual (%)</td>
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<tr>
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<td>(.008)</td>
<td>(.0313)</td>
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</tr>
</tbody>
</table>

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Table 7: Closed-end Fund Return Predictability

This table shows aggregate predictability of closed-end fund returns. Numbers in the parentheses denote standard deviations. $R^c_t$ denote closed-end fund returns. $E_t(y_{t+1})$ and $E_U^t(y_{t+1})$ denote the predicted value and the residual from an AR(4) model on $y$. “Aggregation I” pools data for individual fund and adjusts for cross-sectional heteroscedasticity. “Aggregation II” aggregates estimates for individual fund using Newy-West variance and covariance matrix as weight. “Individual” counts the number of individual regressions for each fund with significant coefficients at 5% level.

<table>
<thead>
<tr>
<th>Dependent Variable $R_{t+1}$</th>
<th>Domestic Funds</th>
<th></th>
<th>International Funds</th>
<th></th>
</tr>
</thead>
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