The Changing Factor Structure of Equity Returns*  

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Abstract  

We examine the factor structure of equity security returns over the time period 1952-1999 from two different perspectives—differences over time and differences over portfolio size in terms of the number of stocks in a portfolio. After extracting out the idiosyncratic returns for individual stocks based on an eight (or five) factor model, we find: (1) the explanatory power of the first “practical” (statistical) factor has decreased dramatically while that of the other “practical” factors have had moderate increases in recent decades and (2) the total number of “practical” factors needed to explain 95% of the systematic returns decreases rapidly with the number of stocks in a portfolio. The first result shows a time varying factor structure. This suggests that factor sensitivity has become more variable across assets while the average factor sensitivity of individual stocks has decreased in recent years. Our second result leads to the notion of automatic “factor hedging”, where the resulting single “practical” factor can be approximated using an equally weighted stock market index.

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Abstract

We examine the factor structure of equity security returns over the time period 1952-1999 from two different perspectives—differences over time and differences over portfolio size in terms of the number of stocks in a portfolio. After extracting out the idiosyncratic returns for individual stocks based on an eight (or five) factor model, we find: (1) the explanatory power of the first “practical” (statistical) factor has decreased dramatically while that of the other “practical” factors have had moderate increases in recent decades and (2) the total number of “practical” factors needed to explain 95% of the systematic returns decreases rapidly with the number of stocks in a portfolio. The first result shows a time varying factor structure. This suggests that factor sensitivity has become more variable across assets while the average factor sensitivity of individual stocks has decreased in recent years. Our second result leads to the notion of automatic “factor hedging”, where the resulting single “practical” factor can be approximated using an equally weighted stock market index.

Key Words: APT, CAPM, Factor Hedging, Practical Factors, Maximum Explanatory Component Analysis.
As an alternative to the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1964), the Arbitrage Pricing Theory (APT) of Ross (1976) starts with a return generating process which excludes arbitrage opportunity. However, the structure of APT is largely unspecified including the number of pricing factors. Unlike the CAPM where the endogenous market factor can be approximated by an index portfolio, the unspecified exogenous factors used in APT are commonly estimated using a statistical procedure, such as factor analysis (Roll and Ross, 1980; Jones 2001), principal components analysis (Connor and Korajczyk, 1988, 1993), and maximum explanatory component (MEC) analysis (Xu, 2001). This means that the factors thus obtained are method dependent and do not necessarily map into commonly acceptable risk factors. In order to provide a better interpretation, some researchers have tried prespecified macroeconomic variables (Chen, Roll, and Ross, 1986) and proxies to fundamental variables (Fama and French, 1993) as pricing factors. The difficulty with this approach is that the explanatory power is not very high, especially for individual stocks. More recently, Merville, Hayes-Yelken and Xu (2001) have tried to reconcile these two approaches by interpreting the statistical factors using fundamental and macro-factors. Several papers have appeared that relate to the temporal behavior of factors (or factor proxies).\footnote{An incomplete list includes Kho (1996), Barbar and Lyon (1997), Pontiff and Schall (1998), Lewellen (1999), Dechow and Sloan (1997), Liew and Vassalou (2000), and Lynch (2001).}

No matter whether the statistical method or macro-variables approach is used, it is very important to know the number of useful or “practical” factors (a rotation of fundamental factors) in pricing a security. Depending on different methodologies, sample data, and return frequency, the existing literature differs in the number of factors driving equity security returns as well as their meaning. For example, Roll and Ross show that there are at least three but most likely no more than four “priced” factors in APT. Connor and Korajczyk, using an asymptotic principal components (PC) technique, assert that while neither APT or CAPM is a perfect model of asset pricing, the APT with more than one factor is more consistent than CAPM when
studying the monthly returns of portfolios of stocks drawn from the NYSE and AMEX exchanges. At the same time, Fama and French use multiple regression analysis for characteristic-sorted portfolios over long time periods and conclude that adding two additional variables—one for a size effect (SMB) and one for a market-to-book ratio effect (HML) greatly enhances the $R^2$ of the regressions of security returns over that found when just using a proxy for total market returns.

We believe the key to understanding the above empirical test differences lies in the notion of a changing factor structure, both over time and over portfolio size. It has been documented already that both the industry structure matters and the stock risk premium changes over time. Evidence of changing betas and expected returns can be found in Campbell (1996), Ferson and Harvey (1991, 1997), Shanken (1992), and in Jaganathan and Wang (1996). The study by Jaganathan and Wang uses the cross-sectional approach of Fama and French (1992) but only on NYSE/AMEX stocks over the same time period 1962-1990. They empirically demonstrate that the risk premiums and betas for the CAPM vary over time. Thus, they conclude that a conditional CAPM holds rather than an unconditional CAPM. Effects of industry classification on equity returns trace their origins to King (1966). That is, if we are to choose a reference set of assets which captures much of the investment opportunities available to firms, i.e., equity securities, then industry groupings can help do that function for common stock returns. Chen and Knez (1996) likewise use industry portfolios to study the performance of mutual funds. A more recent study by Campbell, Lettau, Malkiel, and Xu (2001) suggests that the volatility structure has also changed in recent decades. Thus, if the risk premium can vary over time, one might expect that the factor structure itself changes over time. In fact, we show in this study that the characteristics of “practical” factors—an orthogonal transformation of the fundamental factors that determine individual asset returns—\(^2\) are very different over different sample periods.

\(^2\)Technically, the estimated factors in the empirical sections do not fit this definition of “practical” factors. However, as shown in Xu (2001), factors estimated using the MEC approach will converge to linear combinations of the fundamental factors, i.e., the “practical” factors. Therefore, we use the
In particular, the explanatory power of the first “practical” factor has decreased dramatically while that of other “practical” factors have had moderate increases in the most recent decade. Moreover, we show that this could be due to the fact that factor sensitivity has become more variable across assets while the average factor sensitivity of individual stocks has decreased in recent years if we hold the variability of factors constant.

Although it is reasonable to suggest that there are many fundamental risk factors influencing individual equity returns, from a user perspective, there may only be a few “practical” factors that matter. Moreover, individual stocks can still have different beta sensitivities in terms of both magnitudes and signs to certain of these “practical” factors. That is, when more stocks are added to a portfolio, the beta sensitivities of the individual stocks may well cancel out one another. Thus, the number of $K$ “practical” factors with nonzero beta sensitivities drops as the number of stocks included in a portfolio increases. In other words, some of the “practical” factors are automatically “hedged out” of the portfolio. In this sense, the multifactor APT model may well converge to a single-factor model for a well diversified portfolio. This leads to our notion of automatic “factor hedging”—a relationship between portfolio size and the number of “practical” factors.

Existing statistical tests may lose power when there are both a diversification effect for idiosyncratic risk and automatic “factor hedging” for systematic risk simultaneously. Therefore, we investigate the systematic return only after extracting the idiosyncratic returns for individual stocks using a statistical approach. Based on simulations performed on NYSE/AMEX stocks only, we find the total number of “practical” factors needed to explain 95% of the systematic returns decreases rapidly with the number of stocks in a portfolio. This evidence supports our notion of automatic “factor hedging”. In other words, holding a well diversified portfolio will also automatically hedge against some of the “practical” factors. That is, our result validates the use of a single-factor same terminology in the empirical sections for simplicity.
APT model to describe the returns of a portfolio of even a small number of stocks. However, this does not invalidate the CAPM because the single “practical” factor used in APT may only do a good job in capturing the common variations\(^3\) as found in a prior study.

Although the APT model is silent about the behavior of investors, it could be potentially consistent with individual investors being heterogeneous. That is, even though average investors may still be “passive”, there are some “active” investors who wish to hedge their risks only in some dimensions. Therefore, they care about the sensitivities of stocks to different factors. At the same time, they need to hold a large number of stocks in order to diversify away idiosyncratic risks according to modern portfolio theory. If automatic “factor hedging” occurs at a similar speed as idiosyncratic risk diversification, all diversified investors will be subject to the same aggregated risk that is represented by the first “practical” factor. In other words, “active” investors face a dilemma. On the one hand, they need to hold a sufficient number of stocks in order to reduce their exposure to idiosyncratic risks. On the other hand, this will greatly reduce the possibility of exploring different risk factors according to their personal preferences. Therefore, for “active” investors, they must balance the two in choosing their portfolios.

In order to study the factor structure, we need a framework not only to extract the factors but also determine the number of “practical” factors. Thus, principal components analysis has many advantages over factor analysis. Chamberlain and Rothschild (1983) have shown that it can be applied to an approximate factor structure. Unlike factor analysis, factors from principal component analysis can be uniquely defined up to a scalar transformation without prespecifying the number of factors. Furthermore, the order of factors from principal components analysis is also unique. Shukla and Trzcinka (1990) explicitly compare principal components analysis against factor analy-

\(^3\)More precisely, this single “practical” factor could actually be the sum of all the fundamental factors which we later show mathematically.
sis and conclude that principal components is at least as good as factor analysis in studying security returns. However, principal components analysis has one serious disadvantage. Since the idiosyncratic risk is not explicitly estimated, one may include too many factors in the result. In order to deal with this issue, we apply Xu’s (2001) Maximum Explanatory Component (MEC) methodology in which factors are constructed from stock returns with maximum explanatory power.

The paper is organized as follows. In Section I, we detail our strategy and approach to study a changing factor structure. We also formally define the notation of automatic “factor hedging.” The specific data included in the study is given in Section II. Section III addresses the issue of a time-varying factor structure. The significance of portfolio size on the number of “practical” factors $K$ which drive stock returns is studied in more detail in Section IV. Concluding comments are provided in Section V.
I. Analyzing the Factor Structure

Due to overwhelming evidence, researchers have begun to accept the fact that individual asset returns are explained by a multifactor model. If a multifactor model is true, it is likely to suggest that individual investors are heterogeneous. For example, a promising young assistant professor is more interested in building up his personal capital quickly after going through the hardship of graduate school. He worries less about a possible recession and retirement. Therefore, he will focus more on “growth” type of stocks. In contrast, a middle age construction worker who is facing retirement in several years is very vulnerable to recession risk. Such kinds of people will be more interested in “value-preserving” types of stocks. This suggests that some individual investors have the need to hedge along different directions of factor risks. Otherwise, their holdings will be similar and we return to the CAPM paradigm. Stocks with different sensitivities to different risk factors will suit this need. Thus, it is important to know if the factor structure itself changes.

A changing factor structure could mean both a time varying risk premium and a change in the ability to describe the covariance structure of returns. In this study, we focus on the latter for two reasons. First, there is no consensus with respect to interpretations of useful factors found in current empirical studies. Moreover, those factors may not be very robust. Second, as a parallel to a multifactor model such as the Merton’s ICAPM, we can study the APT model, which focuses more on portfolios’ or individual securities’ covariance structure. Most of the prior APT studies rely on statistical methodologies, such as factor analysis and principal components analysis. The factors thus extracted represent a rotation of the true underlying factors. The following analysis helps us to establish a connection between the two.

As a starting point, let there be $K$ fundamental (true) factors $\mathbf{F}$ for the universe of $N$ stocks, i.e., the $\mathbf{F}$’s drive the returns of each individual stock. Each fundamental factor is common to all stocks but what distinguishes one stock from another is its beta
loadings on the factors as well as its idiosyncratic return risk. Applying a multifactor APT model, return \( \mathbf{R} \) on the \( N \) stocks can be written as,

\[
\mathbf{R} = \mathbf{\alpha} + \mathbf{B}\tilde{\mathbf{F}} + \mathbf{\epsilon} \tag{1}
\]

where \( \mathbf{\alpha} \) is the \( N \times 1 \) vector of expected returns which satisfy the no arbitrage condition of Ross (1976), \( \mathbf{\epsilon} \) is the \( N \times 1 \) idiosyncratic component of the total return \( \mathbf{R} \), and \( \mathbf{B} \) is an \( N \times K \) matrix of the factor loadings or sensitivities of the returns to factors \( \mathbf{F} \). Not to lose generality, we further assume that \( E(\tilde{\mathbf{F}}) = \mathbf{0} \) and \( \text{Cov}(\mathbf{F}, \mathbf{F}') = \mathbf{I}_K \). Equation (1) imposes no structure on the correlation between return \( \mathbf{R} \) and factor \( \mathbf{F} \). For ease of exposition, we follow Brown (1989) to impose the following structure on the factor loadings

\[
\beta_{i,k} \sim N(\bar{\beta}, \sigma_k^2), \tag{2}
\]

where \( N(.,.) \) is the normal distribution function with \( \sigma_k \geq \sigma_{k+1}, \forall k = 1, \cdots, K \). This assumption is very general. When each actual factor has a different \( \bar{\beta} \), we can rescale the corresponding factor returns so that the mean of factor loadings are equal. Moreover, equation (2) is much weaker than the one used by Brown (1989) since we do not assume a constant cross-sectional variance of the factor loadings. This simple economy is not ruled out by an APT model or any other equilibrium pricing models.

Although in the pricing world of equation (1), each of the \( K \) factors are equally important because of assumption (2), we can rotate the fundamental factors in the following way such that some of the factors are more important than others:

\[
\mathbf{R} = \mathbf{\alpha} + \mathbf{B}'\mathbf{H}\tilde{\mathbf{F}} + \mathbf{\epsilon} \\
= \mathbf{\alpha} + \mathbf{B}'\tilde{\mathbf{F}}^r + \mathbf{\epsilon}, \tag{3}
\]
where

$$
H = \begin{bmatrix}
\frac{1}{\sqrt{K}} & \frac{1}{\sqrt{K}} & \frac{1}{\sqrt{K}} & \frac{1}{\sqrt{K}} & \ldots & \frac{1}{\sqrt{K}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & \ldots & 0 \\
\frac{1}{\sqrt{2^2}} & \frac{1}{\sqrt{2^2}} & -\frac{2}{\sqrt{2^3}} & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{\sqrt{(K-1)^2}} & \frac{1}{\sqrt{(K-1)^2}} & \frac{1}{\sqrt{(K-1)^2}} & \frac{1}{\sqrt{(K-1)^2}} & \ldots & -\frac{K-1}{\sqrt{(K-1)^2}}
\end{bmatrix},
$$

is the Helmert matrix with the property $H' = H^{-1}$. In other words, we can transform the fundamental factors into the following “practical” factors,

$$
\tilde{F}_k = \begin{cases}
\frac{1}{\sqrt{K}} \sum_{j=1}^{K} \tilde{F}_j, & k = 1, \\
\frac{1}{\sqrt{(k-1)K}} \sum_{j=1}^{k-1} (\tilde{F}_j - \tilde{F}_k), & k = 2, \ldots, K.
\end{cases}
$$

(5)

It is easy to see that these “practical” factors are still orthogonal to one another when the fundamental factors ($\tilde{F}_i$) are orthogonal. At the same time, we still have unity factor variance, i.e. $\text{var}(\tilde{F}_i^r) = 1$. The corresponding factor loadings will take the following form,

$$
\beta_{i,k}^r = \begin{cases}
\frac{1}{\sqrt{K}} \sum_{j=1}^{K} \beta_{i,j}, & k = 1, \\
\frac{1}{\sqrt{(k-1)K}} \sum_{j=1}^{k-1} (\beta_{i,j} - \beta_{i,k}), & k = 2, \ldots, K.
\end{cases}
$$

(6)

That is, the distributions of the factor loadings for the “practical” factors are different. In particular, we have,

$$
\beta_{i,k}^r \sim \begin{cases}
N(\sqrt{K} \beta, \sigma_{r,1}^2), & k = 1 \\
N(0, \sigma_{r,k}^2), & k = 2, \ldots, K
\end{cases}
$$

(7)

where $\sigma_{r,1}^2 = \frac{1}{K} \sum_{j=1}^{K} \sigma_j^2$ and $\sigma_{r,k}^2 = \sigma_k^2 + \frac{1}{(k-1)K} \sum_{j=1}^{k-1} (\sigma_j^2 - \sigma_k^2), \forall k = 2, \ldots, K$. One can further show that the cross-sectional variance of factor loadings is again in descending order,

$$
\sigma_{r,k}^2 - \sigma_{r,k+1}^2 = (1 - \frac{1}{k})(\sigma_k^2 - \sigma_{k+1}^2) + \sum_{j=1}^{k-1} \frac{(\sigma_j^2 - \sigma_k^2) + (\sigma_j^2 - \sigma_{k+1}^2)}{(k^2 - 1)k} > 0.
$$

(8)
Clearly, the first “practical” risk factor positively covaries with the returns of most stocks, if not all, which produces positive beta loadings. Other factors can be either positively or negatively correlated with different stocks. This gives rise to positive and negative beta loadings, which leads to the following analysis.

We now study a changing factor structure in the framework of a multifactor APT model with respect to the portfolio size of individual stocks. In particular, we consider what we call automatic “factor hedging”, which is formally defined next,

**Definition 1** A factor is automatically hedged if returns of a portfolio have a zero beta sensitivity to the factor when there are sufficient number of securities in the portfolio.

Note, for individual securities, the beta loadings for any factor described above is assumed to be nonzero. Automatic “factor hedging” only applies to portfolios. Presumably, if the first factor is an equally weighted index, the factor loadings of an equally weighted portfolio are zeros except for the first factor by construction. Thus, it is important to know under what conditions and for what portfolio size we observe such a phenomenon.

Automatic “factor hedging” is possible since factor loadings of individual stocks are randomly distributed with mean zero except for the first factor. Let a portfolio \( p(n) \) consist of \( n \) equity securities drawn from a universe of stocks which contains an infinite number. The portfolio \( p(n) \)'s (weighted) beta loadings on each “practical” factor \( \tilde{F}_k \) will converge to,

\[
\lim_{n \to \infty} \beta_{p(n),k} = \lim_{n \to \infty} \sum_{i=1}^{n} w_{p(n),i} \beta_{i,k}^{r} \rightarrow \begin{cases} \sqrt{K} \bar{\beta} 
eq 0, & k = 1, \\ 0, & k = 2, \ldots, K \end{cases} \tag{9}
\]

where \( w_{p(n),i} \) is portfolio \( p(n) \)'s weight on the \( i \)-th stock. In other words, these “practical” factors (except for the first factor) are hedged (or approximately hedged) out of the portfolio when there are sufficient number of stocks in the portfolio. In general, equation (9) may not hold exactly in a finite sample. However, since \( \sigma_{r,k} < \sigma_{r,l} \) if \( k > l \),
and $k, l > 1$, from equation (8), it is easy to see using a basic statistical argument that $\beta_{p(n),k}$ is closer to zero than $\beta_{p(n),l}$ for a given portfolio size $n$. That is, when the size of a portfolio $n$ gets larger, the number of “practical” factors that matters (as evidenced by nonzero beta loadings) gets smaller. When a portfolio grows even larger, we may only observe a single “practical” factor driving returns instead of the $K$ fundamental factors driving the returns of each stock in the portfolio.

The importance of this result again lies in the dilemma it creates. APT implies that individual investors hold large portfolios in order to reduce pricing errors due to idiosyncratic risks. Holding a well-diversified portfolio is also suggested by modern portfolio theory. However, if automatic “factor hedging” occurs very quickly, that is the number of “practical” factors needed to account for “most” of the variations of a portfolio return is smaller than the number of stocks need to achieve risk diversification, the benefit of holding a large portfolio will be reduced for those investors who want to hedge their consumption risk in different dimensions.

Although in reality we may only need a single-factor model to price a portfolio, a multifactor model still applies to individual stocks. Moreover, other “practical” factors still exist. Only a large sized portfolio may have zero beta sensitivities to these other “practical” factors. Therefore, it is important to emphasize that we do not suggest that the other $K - 1$ fundamental factors themselves vanish in a portfolio. In fact, each fundamental factor may have an equal contribution to the remaining “practical” factors as shown in equation (6). That is, if each individual investor holds an adequately diversified portfolio, the only remaining “practical” factor that determines the portfolio returns will represent the aggregate of all the fundamental factors.

In a different direction, we can also study a changing factor structure over time.
Because $\sigma_k > \sigma_{k+1}$, the following relationship holds,

$$
\begin{align*}
\sigma^2_{r,1} &\leq \sigma^2_1, \\
\sigma^2_{r,k} &\geq \sigma^2_k, \\
\sum_{j=1}^k \sigma^2_{r,j} &\leq \sum_{j=1}^k \sigma^2_j.
\end{align*}
$$

(10)

The equality holds when $\sigma_1 = \sigma_2 = \cdots = \sigma_K$. Because $\text{var}(\tilde{F}_r^i) = 1$, the average variations explained by each “practical” factor converges to,

$$
\frac{1}{N} \sum_{i=1}^N \text{Var}(\beta^r_{i,k} \tilde{F}_r^i) \xrightarrow{N \to \infty} \begin{cases} 
\sigma^2_{r,1} + K \beta^2, \\
\sigma^2_{r,k},
\end{cases} \quad \forall k > 2.
$$

(11)

A change in the factor structure can be in both the average loadings and the variability of loadings. The above result suggests that when the average loading changes, it is more likely reflected in the explanatory power of the first “practical” factor. The variability of factor loadings will influence the explanatory power of high order factors. Therefore, by studying the explanatory power of high order “practical” factors, we should be able to infer the variability of factor loadings on individual assets. From the above discussion of automatic “factor hedging,” hedging is less likely to occur for a portfolio with the same number of stocks when the loading variability increases. In other words, we can also study the loading variability through “factor hedging” as an alternative.

Certainly, the above analysis hinges on the structure we have assumed in equation (2). Since the normal distribution assumption can be replaced with any additive symmetric distribution, such a structure on factor loadings is very weak. It also depends on the Helmert transformation of equation (4). However, as shown in Brown (1989), this transformation is equivalent to Principal Components Analysis when factor loadings are independently distributed across factors. Moreover, the empirical results in the next two sections suggest that this structure is a good representation of reality. Therefore, we suggest the following properties with respect to the “practical” factors:

1. Loadings of practical factors will continue to exhibit the same level of variability across individual securities if those of the fundamental factors do the same.
2. When the loading variabilities of fundamental factors are different, the loading variability of the “practical” factors tends to be larger than that of the corresponding fundamental factors except for the first factor.

3. The explanatory power of higher order “practical” factors is directly associated with the variability of the corresponding factor loadings. Thus, it can contribute to changes in the variability of factor loadings of fundamental factors.

4. When the average loading of the first “practical” factor decreases, the average loadings of all the fundamental factors must have reduced. Therefore, when the factor variability increases or is unchanged, the explanatory power of the first “practical” factor is likely to decrease.

5. The number of factors needed to explain the same level of return variability of a portfolio decreases with the portfolio size.

Automatic “factor hedging” is useless if it only holds in the limit, i.e., for a portfolio with a very large number of stocks. We demonstrate in the next section that a single “practical” factor is adequate in practice even when the number of stocks in a portfolio is small to moderate. In other words, automatic “factor hedging” seems to occur at a similar speed as idiosyncratic risk diversification. From a practical perspective, our analysis also associates portfolio “diversity” with the number of factors. In other words, one can be guided in choosing the number of factors given the degree of diversity of the portfolio.
II. The Methodology and Data

A. Factor Construction

We investigate the changing factor structure in two dimensions—the behavior of each “practical” factor in terms of explanatory power as well as the distribution of beta loadings in the time dimension and “automatic” factor hedging in the portfolio size (n) dimension. For this purpose, methods based on principal components (PC) analysis is very useful since it does not require a pre-specification of the number of explanatory factors (variables). However, one drawback to the PC methodology is that it may put too much emphasis on stocks with extremely large idiosyncratic volatilities. In order to mitigate this potential problem, we base our analysis on Xu’s (2001) maximum explanatory component (MEC) analysis. MEC modifies PC analysis by using the correlation structure of the security returns. The first factors extracted can be shown to capture the maximum average explanatory power in describing the individual asset returns. Such extracted factors will converge to a linear combination of underlying fundamental factors when the number of securities goes to infinity.

This methodology is different from that of Connor and Korajczyk (1988) in that it focuses on the correlation structure instead of the covariance structure. This not only allows us to choose factors that maximize the explanatory power of individual asset returns, but also offers a clearer interpretation of the eigenvalues from the analysis. Furthermore, it excludes the possibility of selecting a security that has a huge idiosyncratic return as a factor.

For ease of exposition, denote $\tilde{R}$ as the demeaned $N$ asset returns over $T$ period, $V$ as a diagonal matrix of standard deviations of asset returns, and $R^* = V^{-1}\tilde{R}$ as a standardized returns. The correlation matrix of the asset returns can be estimated by $\hat{\Omega} = Corr(\tilde{R}, \tilde{R}') = \frac{1}{T-1}R^*R^{**}$ when the number of time periods exceeds the number of stocks. $\hat{\Omega}$’s $i$-th eigenvector and eigenvalue are denoted as $\alpha_i$ and $\lambda_i$, respectively.
By convention, eigenvectors are assumed to be orthogonal to one another with a unit norm. In most cases, we have more stocks than time periods. We base our analysis on the “cross-correlation” matrix \( \hat{G} = \frac{1}{N-1} R^* R^* \). The corresponding \( i \)-th eigenvalue and eigenvector are \( \eta_i \) and \( \xi_i \), respectively. For each of the scenarios, the following methods can be applied to construct the MEC factors:

**Case 1:** When \( N < T \), the \( i \)-th factor return can be constructed by letting \( f_i = \tilde{R} \alpha_i \).

The average \( R^2_{f_i} \) for that factor can be estimated by \( \frac{1}{T} \lambda_i \).

**Case 2:** When \( N > T \), the \( i \)-th factor return can be constructed by letting \( f_i = \sqrt{T-1} \xi_i \). The average explanatory power for that factor is \( R^2_{f_i} = \frac{1}{T} \eta_i \).

**B. Methodology**

Equation (6) implies that the number of “practical” factors should be exactly the same as the number of fundamental factors. Although the MEC approach estimates those “practical” factors effectively, it is not easy to determine the number of underlying fundamental factors or “practical” factors, especially when idiosyncratic returns changes over time as documented by Campbell, Lettau, Malkiel, and Xu (2001). As noted by Brown (1989), higher order statistical factors from a PC analysis could be indistinguishable from idiosyncratic risk, thus, statistical tests will be less powerful to determine whether the number of fundamental factors has changed over time. On the other hand, from an application perspective, it is more useful to know the importance of each “practical” factor when statistical approaches are used in extracting factors. Users can then choose the number of factors according to their needs in practice. One way to measure the importance of a “practical” factor is by examining its explanatory power over time. This task is conveniently accomplished by the MEC approach used in our analysis.

As discussed in the last section, after normalizing the factor returns, changes in
factor structure will be reflected in changes in the distributions of each factor loading. In particular, the mean loading of the first “practical” factor and the variance of all the “practical” factors can vary over time. An alternative measure which simultaneously considers changes in both mean and variance is the $t$-statistic. We also examine the percentage of individual stocks which have statistically significant loadings at a 5% level on a particular factor.

Our results will not be robust if the estimated “practical” factors are sample dependent. In order to be free from such a potential problem, we randomly divide our sample of stocks into two groups. We then analyze each group of stocks for both explanatory power and factor loadings of the extracted factors. This process will be repeated for 100 times.

In studying automatic “factor hedging”, we adopt a similar approach as above. Since both factor hedging and diversification occur simultaneously, it is difficult to detect a pure “factor hedging” effect. Therefore, we need to separate the two effects first. In other words, we should only study the systematic part of individual stock returns. In order to show the effect of automatic “factor hedging”, we use a simulation approach similar to a diversification graph. In particular, we form portfolios with $n$ stocks by randomly drawing from the stock universe after excluding idiosyncratic returns, where $n$ is chosen ranging from 2 to 40 for each subsample period. The process is repeated 100 times. We then ask the question: how many “practical” factors does it take to represent 95% of the total variations in the systematic returns of portfolios. If that number decreases with portfolio size $n$, then we have found evidence to support the notion of automatic “factor hedging.”

Whether or not we are able to provide convincing evidence also hinges on our ability to decompose individual stock returns. This would be relatively easy had we known \textit{ex ante} how many factors there are. In order to be safe, we construct the “systematic” returns for individual stocks using returns from the first eight $MEC$ factors. Undoubt-
edly, the constructed “systematic” returns are contaminated by idiosyncratic returns to some degree. Therefore, our results should be seen as conservative.

C. Data

We focus on the factor structure for the post world war period. In particular, as in most studies of factor structure, our sample starts in 1961 and ends in 1999. This also corresponds to an era of changing interest rate regimes. In order to avoid microstructure problems in daily stock returns, we use monthly stock returns for individual stocks, which are exclusively from the CRSP tape. We also limit our sample to NYSE/AMEX stocks for two reasons. First, CRSP tapes started to include NASDAQ stocks after 1972. It would bias our sample if these stocks were included. Second, NASDAQ stocks tend to exhibit more idiosyncratic risk than NYSE/AMEX stocks as well as tend to represent more small stocks.

As discussed above, since the risk premiums seem to change over time, the factor structure itself may not be stable over the whole sample period. In order to investigate the dynamic behavior of the factor structure, we evenly divide the whole sample period into the following thirteen subsample time periods: 1961-1963, 1964-1966, 1967-1969, 1970-1972, 1973-1975, 1976-1978, 1979-1981, 1982-1984, 1985-1987, 1988-1990, 1991-1993, 1994-1996, and 1997-1999. We then apply the MEC approach as described before to each of the two randomly selected groups of stocks in each subsample period. Since the number of time period observations is much less than the number of stocks, we use the method described in Case 2 of Section A above to construct factors. Due to the special requirement of implementing the Connor and Korajczyk (1993) test, we compute their statistics over four subsample periods. In other words, it is reported for the three subsample periods of 1952-1963, 1964-1975, 1976-1987, and 1988-1999.
III. Has the Factors Structure Changed Over Time?

The number of “practical” factors $K$ affecting security returns is a function of both the number of stocks $n$ in a portfolio and time $t$, i.e., $K = K(n, t)$. In this section, we examine the temporal nature of $K(1, t)$ to see if the maximum number of factors explaining individual equity returns is changing over time. Then in Section IV we turn our attention to the cross-sectional nature of $K$ as the number of stocks added to a portfolio is increased. That is, if $K(n, \cdot)$ varies with $n$, the notion of automatic “factor hedging” is supported.

A. Changing Characteristics of Individual Stock Returns

We first provide summary statistics for the thirteen sub-subsample time periods in Table 1. In each sub-subsample period, stocks are selected if they have had continuous records on returns for the period. The total number of stocks for each sample period is reported in the second column. Although the total sample size fluctuates over time, it has an increasing trend from 1021 stocks in the early 60’s to 2669 in the second half of the 90’s. The next two columns report the mean and volatility of market returns. The mean returns in the 90’s are much higher than those in any other previous periods, while volatilities are relatively low. For individual stock returns, we find (not reported in the table) that the average returns are symmetrically distributed in the early 60’s. However, average returns are positively skewed in the later sub-subsample periods, except during the early 70’s and mid 80’s, where the average returns are negatively skewed. Thus, we see that return distributions vary over time.

[ Insert Table 1 here ]

More importantly, idiosyncratic volatilities, estimated using the root-mean-squared errors from regressing individual stock returns on an equally-weighted market index,
exhibit an increasing trend. This is consistent with Campbell, Lettau, Malkiel, and Xu’s (2001) finding. Idiosyncratic volatilities are about 6.5% per month at the beginning of our sample and rise to about 10.3% per month at the end of our sample period. There is also indirect evidence pointing to the same direction. The average beta coefficients for the CAPM model have been steadily decreasing over time. It was about 1.25 in the 60’s and is 0.85 now as shown in the fifth column of Table 1. This suggests that there are less stocks covarying tightly with the market. Consistent with this trend, we observe a significant drop in the explanatory power of the market factor throughout 90’s. Despite this, the average $R^2$ for the market model fluctuates around 28% over time, it is only about 13% now. One possible cause for the low explanation of a single factor model in recent decades may lie in the changing nature of the number of pervasive factors.

B. Analysis of the Factor Structure for Individual Stocks

In addition to the above evidence on the changing characteristics of individual stock returns, a large body of evidence has suggested that both the market risk premium is time-varying and a market model cannot explain most of the variations in expected returns across individual securities. It is, therefore, reasonable to believe that the factor structure itself may also have changed over time when a multi-factor model is applied. In this study, we put more emphases on the correlation structure over different sample periods. Moreover, we avoid answering the question of the number of “practical” factor for individual stocks directly. Instead, we investigate the changing characteristics of factor loadings and the explanatory power of each “practical” factor over the thirteen subsample periods.

Since factors are not directly observable, the volatility of factor returns and factor loadings cannot be separately identified. As in other methods, the MEC approach normalizes each factor volatility to one. Therefore, the first step in studying the changing
factor structure is investigating the distribution of factor loadings in different sub-sample periods. The average factor loading and standard deviation of factor loadings across individual stocks are reported in Table 2 for the first six extracted “practical” factors over the thirteen sample periods. Note that these averages can be positive or negative. For comparison, we report an absolute value in Table 2. Similar to a Helmert transformation, the average factor loadings are large for the first factor, while those of the second factor onward are close to zero as shown in Panel A.

[ Insert Table 2 here ]

In order to see if there is a trend in the average factor loading of each factor, we plot these averages over time for each factor in Figure 1. Although the average loading for the first “practical” factor is largest in the mid 70’s, it fluctuates over time with a clear decreasing trend. This is consistent with our observation for the beta coefficient of a market model. If we believe that all the fundamental factors are equally important, the analysis in the last section help us to interpret this result as suggesting that individual stock returns have become less sensitive to all the fundamental factors over time. The average for the second factor loading fluctuates more around zero in recent decades, although still very small. The average loadings for high order “practical” factors are virtually zero. We offer two implications. First, If the importance of the systematic part of the return remains in terms of the proportion of its volatility in the total return volatility, we need more “practical” factors to explain the systematic volatility. Second, it could also due to the fact that the idiosyncratic volatility has gone up while the systematic part of the return volatility remains the same. This means that the total volatility of a typical stock has gone up. Therefore, even when the covariance between a stock return and a particular fundamental factor is unchanged, the correlation will decrease.

[ Insert Figure 1 ]
It is equally important to study the variability of factor loadings across individual stocks. Presumably, a large deviation in a particular factor loading suggests that the factor is useful in differentiating the return difference among different stocks. It will directly affect the accuracy of factor premium estimates. At the same time, it is largely responsible for determining the explanatory power of high order factors as discussed in the previous section. Therefore, we report the standard deviations of loadings for each factor over different subsample period in panel B of Table 2.

In general, loading variabilities have been greatest during the oil shock period of the mid 70’s and lowest in the beginning of the 60’s and in the mid of 90’s for all the factors. During other subsample periods, the loading variabilities are similar. Therefore, there is no apparent trend. When we compare the loading variabilities across different factors for the same sample period, they appear relatively similar. As discussed in the last section, if the variabilities are indeed equal, the loading variabilities of the “practical” factors should be equal to the corresponding fundamental factors. We first test for the equality among the first three factor variabilities and among the second three factor variabilities based on all the stocks using the Thompson and Merrington’s (1943) M-test. From Table 3 we see that for most time periods, we reject the equal variability hypothesis. We then test the pairwise equality hypothesis among any two adjacent factors. Since the number of stocks is very large, we assume the variability estimates are normally distributed. We can compute the pairwise difference for each random sampling. These differences form the base for a t test which is reported in Table 3. Again, we reject the hypothesis in most cases. However, we also observe that the variability of the first “practical” factor is smaller than that of the second factor. From the second to the sixth factors, it decreases gradually.

[ Insert Table 3 here ]

As mentioned before, a changing in factor structure can also be seen in the explanatory power of each factors. Since the eigenvalues obtained using the MEC approach is
directly related to the average explanatory power of each factor, we report the first 15 average coefficient of determination for the three subsample periods in Table 4. The $R^2$’s for the first factor are around 30% for the first two subsample periods (1964-1975 and 1976-1987) but it decreases to 16% (a 40% drop) in the most recent subsample period (1988-1999). At the same time, as expected, the $R^2$’s for the second factor are substantially smaller than their corresponding first factor. Moreover, the explanatory power for the second factor has increased from 3.60% in the first subsample period to 6.08% in the last subsample period. In general, the explanatory power decreases less rapidly from factor to factor for the last subsample period than for those in the first three subsample periods. These structural differences lead to the conclusion that one need more “practical” factors to explain the same variability of returns in recent decades.

[ Insert Table 4 here ]

When we shorten each subsample period, a similar conclusion emerges as shown in Figure 2. The solid line shows the total explanatory power for the first six “practical” factors over time. The downward trend is apparent. This trend is largely due to drops in the explanatory power of the first “practical” factor as shown by the broken line. At the same time, there is a slightly increasing trend in the explanatory power of the second factor. However, it cannot offset the decrease in the first factor.

[ Insert Figure 2 here ]

An alternative measure for changing factor structure is the relative importance of each “practical” factors. In particular, we study the number of stocks which have statistically significant loading estimates for a particular factor. If a “practical” factor is pervasive, it should be able to explain individual stock returns significantly for most stocks. We plot the percentage numbers in Figure 3 for each “practical” factor over
time. For the first “practical” factor, the percentages are over 95% up until the mid 80’s except for two subsample periods. However, the percentages decrease substantially to around 60% starting in the late 80’s. In contrast, there are about 30% of individual stocks loaded significantly on the second “practical” factor. Although the percentage numbers are relatively stable over time, the second “practical” factor is much more important relative to the first “practical” factor in recent time period. Similarly, the number for the third factor is around 22%. For high order factors, such as the fifth and the sixth, not only is the number around 10%, but also there are only a few cases where the loadings are significant. Therefore, these high order “practical” factors are largely capturing idiosyncratic returns.

[ Insert Figure 3 ]

Our observation of a changing factor structure can be also confirmed from a Connor and Korajczyk’s (1993) mispricing perspective. In particular, after regressing individual portfolio returns on factors, we compute the idiosyncratic volatility difference $\Delta$ and the Connor and Korajczyk test statistics $t_{CK}$. In Table 4 we report the $t$-statistics for additional factors over different subsample periods.\(^4\) A similar pattern exists, namely the $t$-scores decrease much faster in the first two subsample periods than that in the most recent subsample period. For example, only after including 14 factors does an additional factor drive the $t$-score below 1.68, which corresponds to a 10% significance level. This number is much lower than in any of the first two subsample periods. At a 10% significance level, their approach indicates that we need 5 and 6 factors for the first two subsample periods, respectively. Although the test statistics seem to aggressive, the conclusion of an increased number of factors is robust to different methodologies.

As Brown’s (1989) study indicates that one should observe a significant drop in the eigenvalue from the last factor to idiosyncratic risk in $PC$ analysis,\(^5\) we can also

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\(^4\)Due to the requirement of this particular test, longer sample periods are used.
\(^5\)Since the rest will be idiosyncratic risk, the high order eigenvalues should have similar magnitude and small.
examine the pattern in the explanatory power of each addition “practical” factor in
determining the number of factors. In particular, we require each additional factor
should at least increase the overall explanatory power by 2% and a significant drop
afterwards. With these criteria, we reexamine $R^2$ in Table 4 and conclude that 5 factors
are needed for the first two subsample periods. An additional factor is needed for the
last subsample period.
IV. Factor Hedging

We now study *empirically* the idea that the number of “practical” factors with nonzero beta loadings associated with a portfolio return is associated with the degree of portfolio diversity or size \( n \).\(^6\) In other words, a well diversified portfolio cannot only diversify away idiosyncratic risk, but also can reduce the beta influence of some of the “practical” factors that influence individual stock returns. We use the words “factor hedging” to mean that a “practical” factor has zero influence on a portfolio return. Unlike idiosyncratic risk that only affects returns of a single stock and is independent across stocks, the “practical” factors which can be hedged influence the returns of most individual stocks, if not all.

It is natural to observe automatic “factor hedging” for a portfolio that is close to a market portfolio. However, this is not necessarily so for small \( n \) as shown in this section. It is a stylized fact that idiosyncratic risk can be mostly diversified away for a portfolio with a randomly selected set of 30 stocks in the early sample periods. Recently, Campbell, Lettau, Malkiel, and Xu (2001) show that the number of stocks needed to reach an adequate level of diversification has increased to 50 in recent decades due to an increase in idiosyncratic volatilities. In the case of factor hedging, the number could be different. Our analysis in Section III suggests that automatic “factor hedging” hinges on the distribution of factor loadings. We first examine how the distribution of portfolio loadings change with the portfolio size. We then investigate the existence and the speed of factor hedging through a simulation on all the CRSP stocks.

A. Factor Hedging and the Distribution of Factor Loadings

If the influence of some of the “practical” factors can be “automatically” hedged away in a portfolio, it has to be true that the average beta estimates for the same factor across

\(^6\)Note, we use the term portfolio size to refer to \( n \) instead of market capitalization.
individual securities are statistically indifferent from zero. Except for the first factor, the average beta estimates for the other five factors are all zero as shown in Table 2. Also, beta estimates for the first factor are all positive in different subsample periods, which means that the first “practical” factor will exist in any portfolio. However, the second “practical” factor, onwards, has a fairly symmetric distribution in the beta estimates around zero. This is true for all the subsample periods. Randomly forming a portfolio is equivalent to randomly sampling individual stocks. Such a portfolio’s loading on a factor is just the mean of a random sample draw from the distribution of the corresponding beta loadings of individual stocks. Since we observe a zero mean distribution for higher order loadings, the portfolio loadings approach zero when the sample size is sufficient. In other words, a sufficient condition for “factor hedging” holds.

Factors that influence the speed of factor hedging include variance and kurtosis of the loading distribution. When a loading variance is large, a relatively large size portfolio is needed to achieve hedging. In addition, a large kurtosis means a fat-tail distribution. It will also be more difficult to hedge away a specific factor even though the loading variances are the same. In Figure 4 we show kurtosis for each “practical” factor. For individual stocks, the first factor loading has a large kurtosis, 5.2 in the most recent sample period. In contrast, the kurtosis of the first factor loading for other sample periods is about 3.5. For higher order factor loadings, kurtosis measures are even larger, ranging from 6.5 to 8.5 in recent sample periods. Kurtosis measures for higher order factor loadings during other sample periods are also relatively high.

[ Insert Figure 4 here ]

It is also useful to examine the kurtosis for portfolio factor loadings in order to understand how easy it is to achieve automatic “factor hedging” for portfolios. In each of the 13 subsample periods from 1961 to 1999, we form the number of portfolios equal to the available stocks. Each portfolio consists of $n$ stocks randomly drawn from all
the available stocks. Returns from the constructed portfolios, each with 36 months of data, are analyzed using the MEC approach. This process is repeated for portfolio sizes of \( n = 2, 3, \ldots, 100 \). For a better visual effect, in Figure 4 we consolidate the 13 subsample periods into four subsample periods by taking averages of each of the three subsample periods’ results.

Kurtosis decreases with portfolio size quickly in the first three subsample periods. For a three-stock portfolio, kurtosis is below 4 for most cases. In other words, kurtosis for the early subsample periods are close to that of a normal distribution. Moreover, they stay close to 3 no matter how many stocks are included in a portfolio. In contrast, those during the most recent sample period decrease relatively slower. Kurtosis drops below 4 for a portfolio with six stocks. This evidence suggests that factor loadings of individual stocks are not normally distributed. At the same time, loadings of portfolios can be approximated very well by a normal distribution even with a small size \( n \) except for the most recent sample period. Therefore, automatic factor hedging can occur quickly when a portfolio size increases.

**B. Factor Hedging from an Explanatory Power Perspective**

Similar to the traditional way of showing the diversification phenomenon, we next study factor hedging based on NYSE/AMEX stocks from the CRSP tape using a simulation method. As mentioned before, both the diversification effect and the automatic “factor hedging” effect might coexist as we change the portfolio size. Furthermore, as shown in the last section there is a decrease in the explanatory power of each “practical” factor over time. This might be attributed to an increase in the idiosyncratic volatility as measured by \( R^2 \). Even when variation in the systematic returns remain constant over time, \( R^2 \) will decrease if idiosyncratic returns become more volatile. Therefore, it is important to examine the explanatory power of each factor relative to the systematic return only. Since individual stock systematic returns are unobservable, we estimate
them using returns from the first six factors in order to be conservative. Specifically, during each of the 13 subsample periods from 1961 to 1999 we first construct the systematic returns of individual stock returns by using the first six factor returns for each individual stock. We then form $N$ portfolios that equals the number of available stocks. Each portfolio consists of $n$ stocks randomly drawn from all the available stocks. We equally weight each stock’s systematic returns to compute the portfolio return.

We investigate the automatic “factor hedging” issue by positing the following question: How many “practical” factors are needed to capture 95% of the total variation in the systematic returns that are captured by the six factors for individual stocks? We first compute the average $R^2$ for each portfolio of size $n$ by regressing each portfolio return on one, two, ···, and 6 factors, respectively. We then set the number of factors to be $\hat{K}$ for a portfolio size of $n$ when the first $\hat{K}$ factors start to produce an $R^2$ of 95%. Again, in order to be more concise, we aggregate each three-adjacent subsample periods. Figure 5 is similar to a diversification graph where we plot the relationship between $\hat{K}$ and $n$ over the four subsample periods.

[ Insert Figure 5 here ]

Figure 5 is our “factor hedging” graph if we accept that there are six factors for individual stocks. It shows the relationship between the number of factors and portfolio size. Two interesting observations emerge from the plot. First, for the first three subsample periods examined, the number of factors drops to one rapidly. By randomly selecting 10 stocks in the 70’s and 15 stocks in the 80’s in a portfolio, 95% of the total systematic variation in that portfolio can be explained by the first two “practical” factors.$^7$ When additional 5 stocks and 15 stocks are added to the portfolio in the 70’s and 80’s, respectively, we only need one “practical” factors to explain the same level of systematic variation, which illustrates the speed of automatic “factor hedging”. In

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$^7$Note that the “factor hedging” lines are clustered together during the first and the third subsample periods.
contrast, that of the last subsample period (1991-1999) is far above the other three. It steadily decreases to two factors when there are 45 stocks in the portfolio and further decreases to one factor after adding another 20 stocks to the portfolio. This result suggests that there are changes in the factor structure as discussed above but from a different perspective. When Figure 5 is compared with the diversification graph in Campbell, Lettau, Malkiel, and Xu (2001), we see that automatic “factor hedging” is achieved with a slower speed than idiosyncratic risk diversification.

One might consider that “factor hedging” is just related to industry effects. This argument would be true if there are pervasive industry factors which co-move with some groups of industries in one direction and other industry groups in the opposite direction. However, the conventional definition of an industry effect is that the effect only occurs within one particular industry group. In fact, when we include industry indexes for each industry, the additional explanatory power does not seem to vary with the portfolio size (not shown in the paper). Therefore, we believe that an industry effect has nothing at all to do with our automatic “factor hedging” effect.

V. Concluding Comments

In this research we first study the changing characteristics of each “practical” factor in explaining stock returns over time. We find empirical evidence from stock returns included in the CRSP tapes that we need more “practical” factors to explain most of the systematic variations in stock returns in recent decades. Thus, we conclude that in a multifactor model, not only does the distribution of return sensitivities to factors change over time, but also does the number of important or significant “practical” factors which drive equity returns.

Our second important finding relates to the notion of automatic “factor hedging.” That is, the number of “practical” factors which have a nonzero beta loading is a func-
tion of the number of securities in the portfolio. Through a simulation on the universe of NYSE/AMEX stocks, we also demonstrate that the speed of “factor hedging” is a little slower than that of idiosyncratic risk diversification. Therefore, the traditional diversification curve should still be valid even in the framework of a multifactor model.

Finally from a theoretical perspective, since the first “practical” factor represents an aggregation of the fundamental pricing factors used in the multifactor APT model, our results do not suggest that there is only one fundamental factor that determines a portfolio return as a result of automatic “factor hedging.” We only mean to say that the differential role for each fundamental factor vanishes as more stocks are included in a portfolio. As confirmed in other studies, the single factor can be approximated well using an equally weighted market index.
References


Table 1: **Subsample Summary Statistics**

This table shows the number of stocks, the distribution of average returns, the beta coefficients, the average idiosyncratic volatilities, and the coefficient of determinations for each of the eight subsample period from 1952 to 1999. “Idio. Vot.” represents the root mean square error estimated from a single factor model using an equally weighted index. Each NYSE/AMEX stock is selected into our sample if it survived over the corresponding subsample period.

<table>
<thead>
<tr>
<th>Subsample Period</th>
<th>Number of Stocks</th>
<th>Market Return</th>
<th>Beta Coeff.</th>
<th>Idio. Vot.</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>St.D</td>
<td>Mean</td>
<td>St.D</td>
</tr>
<tr>
<td>1961-1963</td>
<td>1021</td>
<td>0.766</td>
<td>4.148</td>
<td>1.096</td>
<td>0.428</td>
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<tr>
<td>1964-1966</td>
<td>1743</td>
<td>0.241</td>
<td>2.592</td>
<td>1.272</td>
<td>0.653</td>
</tr>
<tr>
<td>1967-1969</td>
<td>1790</td>
<td>0.395</td>
<td>4.050</td>
<td>1.422</td>
<td>0.693</td>
</tr>
<tr>
<td>1970-1972</td>
<td>2090</td>
<td>0.561</td>
<td>4.408</td>
<td>1.284</td>
<td>0.552</td>
</tr>
<tr>
<td>1973-1975</td>
<td>2337</td>
<td>-0.984</td>
<td>6.106</td>
<td>1.226</td>
<td>0.516</td>
</tr>
<tr>
<td>1976-1978</td>
<td>1902</td>
<td>0.418</td>
<td>4.139</td>
<td>1.336</td>
<td>0.669</td>
</tr>
<tr>
<td>1979-1981</td>
<td>2036</td>
<td>0.455</td>
<td>4.888</td>
<td>1.116</td>
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<tr>
<td>1982-1984</td>
<td>1953</td>
<td>0.493</td>
<td>4.379</td>
<td>1.059</td>
<td>0.575</td>
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<tr>
<td>1985-1987</td>
<td>1816</td>
<td>0.888</td>
<td>6.074</td>
<td>1.027</td>
<td>0.409</td>
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<tr>
<td>1988-1990</td>
<td>2002</td>
<td>0.450</td>
<td>3.982</td>
<td>0.990</td>
<td>0.651</td>
</tr>
<tr>
<td>1994-1996</td>
<td>2653</td>
<td>1.014</td>
<td>2.752</td>
<td>0.845</td>
<td>0.669</td>
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<tr>
<td>1997-1999</td>
<td>2669</td>
<td>1.630</td>
<td>4.950</td>
<td>0.767</td>
<td>0.570</td>
</tr>
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</table>
Table 2: Summary Statistics for Factor Loadings

This table shows the average beta loadings and standard deviation of beta loadings across individual stocks for all the replications and over each subsample period. Each NYSE/AMEX stock is selected into our sample if it survived over the corresponding subsample period. In particular, during each subsample period, stocks are divided randomly into two equal numbered groups. The MEC approach of Xu (2002) is used to extract the first six factors for each group. All the factor returns have been normalized to have variance of one. We then run multiple regressions to estimate the beta loadings for individual stocks. This process is repeated for 100 times. The reported number has been multiplied by 10.

<table>
<thead>
<tr>
<th>Subsample Period</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
<th>Factor 4</th>
<th>Factor 5</th>
<th>Factor 6</th>
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<tr>
<td>Panel A: The absolute value of mean loadings</td>
<td></td>
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<tr>
<td>1961-1963</td>
<td>0.467</td>
<td>0.0036</td>
<td>0.0109</td>
<td>0.0078</td>
<td>0.0098</td>
<td>0.0064</td>
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<tr>
<td>1964-1966</td>
<td>0.379</td>
<td>0.0515</td>
<td>0.0094</td>
<td>0.0099</td>
<td>0.0092</td>
<td>0.0116</td>
</tr>
<tr>
<td>1967-1969</td>
<td>0.620</td>
<td>0.0163</td>
<td>0.0429</td>
<td>0.0116</td>
<td>0.0066</td>
<td>0.0050</td>
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<tr>
<td>1970-1972</td>
<td>0.627</td>
<td>0.0547</td>
<td>0.0071</td>
<td>0.0027</td>
<td>0.0036</td>
<td>0.0041</td>
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<td>1973-1975</td>
<td>0.875</td>
<td>0.0084</td>
<td>0.0288</td>
<td>0.0089</td>
<td>0.0111</td>
<td>0.0121</td>
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<td>1976-1978</td>
<td>0.592</td>
<td>0.0538</td>
<td>0.0053</td>
<td>0.0071</td>
<td>0.0059</td>
<td>0.0074</td>
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<td>1979-1981</td>
<td>0.583</td>
<td>0.0242</td>
<td>0.0029</td>
<td>0.0193</td>
<td>0.0105</td>
<td>0.0076</td>
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<td>1982-1984</td>
<td>0.486</td>
<td>0.0463</td>
<td>0.0161</td>
<td>0.0045</td>
<td>0.0037</td>
<td>0.0049</td>
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<tr>
<td>1985-1987</td>
<td>0.630</td>
<td>0.0371</td>
<td>0.0105</td>
<td>0.0143</td>
<td>0.0062</td>
<td>0.0127</td>
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<tr>
<td>1988-1990</td>
<td>0.410</td>
<td>0.0240</td>
<td>0.0516</td>
<td>0.0238</td>
<td>0.0136</td>
<td>0.0192</td>
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<td>1991-1993</td>
<td>0.311</td>
<td>0.0671</td>
<td>0.0568</td>
<td>0.0146</td>
<td>0.0189</td>
<td>0.0073</td>
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<tr>
<td>1994-1996</td>
<td>0.220</td>
<td>0.0938</td>
<td>0.0231</td>
<td>0.0120</td>
<td>0.0156</td>
<td>0.0212</td>
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<tr>
<td>1997-1999</td>
<td>0.428</td>
<td>0.0503</td>
<td>0.0073</td>
<td>0.0092</td>
<td>0.0151</td>
<td>0.0183</td>
</tr>
<tr>
<td>Panel B: The standard deviations of loadings across individual stocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1961-1963</td>
<td>0.185</td>
<td>0.172</td>
<td>0.164</td>
<td>0.155</td>
<td>0.150</td>
<td>0.143</td>
</tr>
<tr>
<td>1964-1966</td>
<td>0.209</td>
<td>0.239</td>
<td>0.228</td>
<td>0.218</td>
<td>0.208</td>
<td>0.191</td>
</tr>
<tr>
<td>1967-1969</td>
<td>0.306</td>
<td>0.266</td>
<td>0.244</td>
<td>0.226</td>
<td>0.221</td>
<td>0.216</td>
</tr>
<tr>
<td>1970-1972</td>
<td>0.280</td>
<td>0.300</td>
<td>0.220</td>
<td>0.209</td>
<td>0.205</td>
<td>0.201</td>
</tr>
<tr>
<td>1973-1975</td>
<td>0.419</td>
<td>0.392</td>
<td>0.317</td>
<td>0.291</td>
<td>0.263</td>
<td>0.255</td>
</tr>
<tr>
<td>1976-1978</td>
<td>0.308</td>
<td>0.265</td>
<td>0.217</td>
<td>0.197</td>
<td>0.205</td>
<td>0.196</td>
</tr>
<tr>
<td>1979-1981</td>
<td>0.253</td>
<td>0.255</td>
<td>0.247</td>
<td>0.235</td>
<td>0.214</td>
<td>0.215</td>
</tr>
<tr>
<td>1982-1984</td>
<td>0.260</td>
<td>0.253</td>
<td>0.253</td>
<td>0.218</td>
<td>0.215</td>
<td>0.210</td>
</tr>
<tr>
<td>1985-1987</td>
<td>0.249</td>
<td>0.261</td>
<td>0.245</td>
<td>0.234</td>
<td>0.221</td>
<td>0.212</td>
</tr>
<tr>
<td>1988-1990</td>
<td>0.260</td>
<td>0.230</td>
<td>0.256</td>
<td>0.247</td>
<td>0.235</td>
<td>0.223</td>
</tr>
<tr>
<td>1991-1993</td>
<td>0.261</td>
<td>0.305</td>
<td>0.298</td>
<td>0.240</td>
<td>0.236</td>
<td>0.229</td>
</tr>
<tr>
<td>1994-1996</td>
<td>0.165</td>
<td>0.219</td>
<td>0.199</td>
<td>0.185</td>
<td>0.183</td>
<td>0.183</td>
</tr>
<tr>
<td>1997-1999</td>
<td>0.282</td>
<td>0.281</td>
<td>0.284</td>
<td>0.258</td>
<td>0.246</td>
<td>0.243</td>
</tr>
</tbody>
</table>
This table shows test statistics for the equality of variances of beta loadings. The “M test” is proposed by Thompson and Merrington (1943) to test the homogeneity among several variance estimates. The reported “M-test” statistics are based on estimated loadings for all the stocks. The critical value at 5% significance level is 6. The pairwise $t$-statistics are based on loading variances estimated from repeated sampling. In particular, during each subsample period, stocks are divided randomly into two equal numbered groups. The $MEC$ approach of Xu (2002) is used to extract the first six factors for each group. All the factor returns have been normalized to have a variance of one. We then run multiple regressions to estimate the beta loadings for individual stocks and the corresponding loading variances across individual stocks. This process is repeated 100 times. We only use NYSE/AMEX stocks that survived over the corresponding subsample period.

<table>
<thead>
<tr>
<th>Subsample Period</th>
<th>M test statistics $\sigma_{r,1}^2 = \sigma_{r,2}^2 = \sigma_{r,3}^2 = \sigma_{r,4}^2 = \sigma_{r,5}^2 = \sigma_{r,6}^2$</th>
<th>Pairwise $t$ statistics $\sigma_{r,1} = \sigma_{r,2} = \sigma_{r,3} = \sigma_{r,4} = \sigma_{r,5} = \sigma_{r,6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961-1963</td>
<td>21.4 7.14</td>
<td>31.9 15.5 11.8 10.5 16.0</td>
</tr>
<tr>
<td>1964-1966</td>
<td>31.0 59.9</td>
<td>-54.5 13.1 14.0 15.1 18.2</td>
</tr>
<tr>
<td>1967-1969</td>
<td>109. 0.58</td>
<td>34.6 12.3 17.1 9.23 9.20</td>
</tr>
<tr>
<td>1970-1972</td>
<td>223. 4.59</td>
<td>-40.5 129. 24.5 11.6 8.03</td>
</tr>
<tr>
<td>1973-1975</td>
<td>197. 42.9</td>
<td>39.7 121. 35.6 35.4 5.67</td>
</tr>
<tr>
<td>1976-1978</td>
<td>227. 4.61</td>
<td>65.6 55.3 17.4 -8.23 8.91</td>
</tr>
<tr>
<td>1979-1981</td>
<td>2.62 43.2</td>
<td>-6.53 14.8 17.1 28.5 -0.53</td>
</tr>
<tr>
<td>1982-1984</td>
<td>3.63 0.66</td>
<td>8.76 -0.34 48.1 4.63 9.55</td>
</tr>
<tr>
<td>1988-1990</td>
<td>38.9 37.1</td>
<td>58.3 -47.7 7.58 13.6 15.0</td>
</tr>
<tr>
<td>1991-1993</td>
<td>57.8 8.66</td>
<td>-49.4 8.08 75.1 3.79 8.00</td>
</tr>
<tr>
<td>1994-1996</td>
<td>203. 49.2</td>
<td>-132. 32.5 18.7 1.59 0.85</td>
</tr>
<tr>
<td>1997-1999</td>
<td>2.86 8.50</td>
<td>2.64 -4.72 16.8 9.47 2.37</td>
</tr>
</tbody>
</table>
Table 4: $R^2$ and Connor & Korajczyk’s $t$-Statistics for Individual Stocks
This table shows the average coefficient of determination ($R^2$) for each factor estimated using the MEC approach and Connor and Korajczyk’s $t_{CK}$-statistics for incremental changes in mispricing.

<table>
<thead>
<tr>
<th>Number</th>
<th>$R^2$ (%)</th>
<th></th>
<th></th>
<th></th>
<th>Connor &amp; Korajczyk $t_{CK}$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.24</td>
<td>30.40</td>
<td>16.82</td>
<td></td>
<td>2.781</td>
<td>2.308</td>
<td>3.100</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.38</td>
<td>4.99</td>
<td>6.08</td>
<td></td>
<td>2.584</td>
<td>2.453</td>
<td>3.099</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.42</td>
<td>3.19</td>
<td>3.54</td>
<td></td>
<td>1.966</td>
<td>2.106</td>
<td>2.813</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.25</td>
<td>2.48</td>
<td>3.05</td>
<td></td>
<td>1.869</td>
<td>1.810</td>
<td>2.296</td>
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</tr>
<tr>
<td>5</td>
<td>2.02</td>
<td>2.06</td>
<td>2.46</td>
<td></td>
<td>1.488</td>
<td>1.732</td>
<td>2.418</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.96</td>
<td>1.80</td>
<td>2.25</td>
<td></td>
<td>1.030</td>
<td>1.457</td>
<td>2.624</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.78</td>
<td>1.65</td>
<td>2.12</td>
<td></td>
<td>0.855</td>
<td>1.229</td>
<td>2.645</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.67</td>
<td>1.57</td>
<td>2.05</td>
<td></td>
<td>0.802</td>
<td>0.857</td>
<td>2.449</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1.56</td>
<td>1.52</td>
<td>1.87</td>
<td></td>
<td>1.057</td>
<td>0.615</td>
<td>2.053</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.51</td>
<td>1.42</td>
<td>1.72</td>
<td></td>
<td>1.066</td>
<td>0.728</td>
<td>2.251</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.44</td>
<td>1.39</td>
<td>1.65</td>
<td></td>
<td>0.592</td>
<td>0.782</td>
<td>2.454</td>
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</tr>
<tr>
<td>12</td>
<td>1.39</td>
<td>1.36</td>
<td>1.61</td>
<td></td>
<td>0.216</td>
<td>0.648</td>
<td>2.216</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1.34</td>
<td>1.30</td>
<td>1.55</td>
<td></td>
<td>0.042</td>
<td>0.720</td>
<td>2.112</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1.30</td>
<td>1.26</td>
<td>1.48</td>
<td></td>
<td>-0.033</td>
<td>0.728</td>
<td>1.626</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.29</td>
<td>1.21</td>
<td>1.43</td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>
Figure 1. The Average Beta Loadings for Each Factor Over Time
This figure shows the magnitude of average beta loading for each factor. Individual stocks’ beta loadings are estimated from regressing the returns of each stock on the six MEC factors. “61-63” on the horizontal axis represent the subsample period from 1961 to 1963.
Figure 2. The Overall and Each Factor’s Explanatory Power over Time
This figure shows the explanatory power of each factor in terms of average $R^2$s as well as the overall explanatory power for the first six factors. $R^2$s are estimated from regressing the individual stock returns on each of the MEC factor separately. “61-63” on the horizontal axis represent the subsample period from 1961 to 1963.
Figure 3. The Percentage of Stocks that Are Significantly Loaded on Each Factor Over Time
This figure shows the percentage of individual stocks that have significant t-statistics on each factor at a 5% level in different sample periods. “61-63” on the horizontal axis represent the subsample period from 1961 to 1963.
**Figure 5. Automatic Factor Hedging for Different Sample Period (with 8 Factors)**

This figure shows the number of factors needed to explain 95% of the variations in systematic returns of a portfolio consisting of n randomly selected individual stocks. The systematic returns are estimated using returns from the first six factors. 1 through 100 on the horizontal axis represent the number of stocks in a portfolio.
Figure 4. Kurtosis of Each Portfolio Factor Loadings for Different Sample Periods
This figure shows the kurtosis of each portfolio factor loadings. A portfolio of size n is constructed by randomly selecting n individual stocks. The number of portfolios is equal to the number of stocks in each of the subsample period.