

Concurrent Error Detection in Linear Analog Circuits Using State Estimation

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Abstract

We present a novel methodology for concurrent error detection in linear analog circuits. We develop a rigorous theory that yields an error detection circuit of size that is, in general, much smaller than a duplicate of the circuit under test. The error detection circuit monitors the input and some judiciously selected observable internal nodes of the examined circuit to produce an estimate of its output. In error-free operation, this estimate converges to the actual output value in a time interval that can be controlled to be sufficiently small. From then onwards, it follows exactly the output. The estimate is constructed such that it does not converge in the presence of errors and, thus, concurrent error detection is performed by comparing the two signals through an analog checker. The derived theory is validated through representative simulations on two filter examples.

1. Introduction

Analog test solutions are essential to the success of modern systems that comprise analog interfaces [1, 2]. While off-line test methods are capable of detecting manufacturing faults, wear-and-tear faults and transient errors require additional care. In high safety applications, a circuit should monitor itself and report potential deviations from its correct functionality through a concurrent error detection mechanism.

The objective of concurrent error detection in analog circuits is to examine whether the output signals remain within a predefined band around their nominal values and flag any deviation that is unacceptably large. The nominal band stems from a transient worst-case analysis [3]. Concurrent error detection is achieved by continuously examining whether an inherent or imposed invariant property of the circuit is satisfied. This property should reflect valuable information regarding the operational health of the circuit under test and, ideally, should deviate from its nominal state whenever an error occurs in the circuit response.

Processing analog signals and accurately measuring their values constitute the main sources of difficulty in realizing an invariant property. Additionally, to optimize error detection, this property should exhibit the same level of sensitivity to the parameter space as the actual outputs. Otherwise, the test output will not reflect accurately the compliance of the circuit

to its specifications. Thus, the probability of false alarms will become non-negligible. Apart from choosing a property that has this type of dependence on the circuit parameters, there are additional, equally important objectives that need to be considered. The error detection circuit should respond accordingly for all realistic signals and at the actual speed of operation. This implies that the test criterion should adapt automatically to the properties of the evaluated signals; also, if several signals are combined to set the test criterion, there should not exist a phase lag amongst them. Moreover, the added circuitry should neither interrupt the normal operation of the circuit under test nor degrade its performance. Any performance loss has to be negligible. Last, but not least, any method for concurrent error detection, by default, must result in a detection circuit of size less than that of the original circuit. Otherwise, duplication would be more palatable.

A comprehensive overview of previously proposed methodologies for concurrent error detection in analog circuits can be found in [4]. This topic has attracted much interest recently, particularly for the class of linear circuits. In [5], the time-invariant matrices of the state-variable equations are encoded into a single continuous checksum as coefficients of observable signals and their derivatives. The error signal is then generated by a cascade of voltage summing configurations and integrators. An extension of this work is presented in [6], where the authors derive an optimal design of the error detection circuit that minimizes the number of false alarms under specific fault assumptions. The optimization problem consists in finding appropriate values for the elements of the coding vector. Another algorithm, presented in [7], finds the coding vector which realizes the optimal error detection circuit in terms of hardware overhead. In [8], a projection technique which eliminates the unknown variables in the state linear equations is employed, resulting in a checksum of inputs, outputs and their derivatives up to the order necessary to generate the fault indicating signal. In [9], the nodal equations are combined in order to obtain an invariant signal.

In this paper, we present a novel method for designing an error detection circuit of size that is, in general, much smaller than a duplicate of the circuit under test and in the worst possible case it does not exceed it. Additionally, this size is predictable by simple observation of the circuit under test. The circuit monitors a selected set of observable nodes and produces an estimate of the output. Provably, this estimate converges exponentially to the actual output value at a

rate that can be controlled to be arbitrarily small. Once convergence is accomplished, the estimate remains identical to the output for any input change. An analog checker may then be utilized to compare the output and its estimate and, thus, to report potential unacceptable deviations.

The paper is organized as follows. In the next section, we show a procedure to estimate a set of unobservable state-variables in a linear time-invariant system. Based on this theory, in section 3 we present the proposed concurrent error detection method. In section 4, we discuss design issues related to analog checkers. In section 5, we illustrate two examples which validate the theory and, furthermore, reveal the advantages of our method in terms of hardware overhead.

2. Reduced-size state estimation

A linear time-invariant analog circuit with m inputs and ℓ outputs has the following state-variable representation

$$\begin{aligned}\dot{x} &= Ax + Bu \\ g &= Mx + Du\end{aligned}\quad (1)$$

where u is the $m \times 1$ input vector, x is the $n \times 1$ state-variable vector, g is the $\ell \times 1$ output vector and A , B , M and D are $n \times n$, $n \times m$, $\ell \times n$ and $\ell \times m$ real-valued matrices, respectively. Suppose that p state variables are observable, namely, we can monitor their value without degrading the normal operation of the circuit. Let

$$y = Cx \quad (2)$$

be the $p \times 1$ reduced-size state variable vector. C is of dimension $p \times n$. Every row has an one in the appropriate position and the rest of the elements are all zero. Hence, $\text{rank}C = p$. We show how to generate an *asymptotic estimate* of the remaining $n-p$ state variables by using another linear dynamic equation that accepts as inputs the inputs of the circuit, u , and the reduced-size state variable vector, y [10]. The method for producing the estimate also makes use of the coefficient matrices in (1).

Since $\text{rank}C = p$, there exists a $(n-p) \times n$ matrix P_b such that the matrix P defined below is nonsingular

$$P^{-1} = \begin{bmatrix} C \\ P_b \end{bmatrix} \quad (3)$$

This definition results to the following matrix equality

$$I = \begin{bmatrix} CP \\ P_bP \end{bmatrix} \quad (4)$$

where I is the identity matrix. In the following, we rearrange the state-variable vector such that the first p rows correspond to the observable state-variables. For this purpose, letting $z(t) = P^{-1}x(t)$, the state equations (1)-(2) become

$$\begin{aligned}\dot{z} &= Fz + Gu \\ y &= CPz\end{aligned}\quad (5)$$

where $F = P^{-1}AP$ and $G = P^{-1}B$. Using (4), (5) can be rewritten in the form

$$\begin{aligned}\begin{bmatrix} \dot{z}_\alpha \\ \dot{z}_\beta \end{bmatrix} &= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} z_\alpha \\ z_\beta \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} u \\ y &= \begin{bmatrix} I_{p \times p} & 0_{p \times (n-p)} \end{bmatrix} \begin{bmatrix} z_\alpha \\ z_\beta \end{bmatrix}\end{aligned}\quad (6)$$

where the matrices F_{11} , F_{12} , F_{21} , F_{22} , G_1 and G_2 are $p \times p$, $p \times (n-p)$, $(n-p) \times p$, $(n-p) \times (n-p)$, $p \times m$ and $(n-p) \times m$, respectively, and the vectors z_α and z_β are $p \times 1$ and $(n-p) \times 1$, respectively. Clearly, $y = z_\alpha$.

Our aim now is to define a linear observer of the general form

$$\begin{aligned}\dot{z}_\gamma &= Hz_\gamma + Kz_\alpha + \Gamma u \\ \hat{w} &= z_\gamma + Nz_\alpha\end{aligned}\quad (7)$$

in such a way that along any trajectory of the combined linear system (5)-(7), \hat{w} converges to z_β in the limit $t \rightarrow \infty$, that is

$$\lim_{t \rightarrow \infty} [\hat{w} - z_\beta] = 0 \quad (8)$$

We define the difference

$$e_\beta = z_\beta - \hat{w} \quad (9)$$

For \hat{w} to asymptotically estimate z_β it is, therefore, adequate to ensure that e_β converges to zero as $t \rightarrow \infty$. Using (6) and (7) and after some simple algebraic manipulations we obtain the following error differential equation

$$\dot{e}_\beta = He_\beta \quad (10)$$

provided that the conditions below are satisfied

$$\begin{aligned}H &= F_{22} - NF_{12} \\ K &= F_{21} - NF_{11} + HN \\ \Gamma &= G_2 - NG_1\end{aligned}\quad (11)$$

The solution of (10) can be written as

$$e_\beta(t) = \Phi_H(t)e_\beta(0) \quad (12)$$

where $\Phi_H(t) = e^{Ht}$ is the transition matrix of H . Hence, if we choose a matrix H *exponentially stable*, i.e. a matrix for which there exist positive constants μ and λ such that

$$\|e^{Ht}\| \leq \mu e^{-\lambda t}, \forall t \geq 0 \quad (13)$$

then

$$e_\beta(t) = e^{Ht}e_\beta(0) \rightarrow 0, \quad t \rightarrow \infty \quad (14)$$

Therefore, if we denote the estimated state-variable vector by \hat{x} , the following limiting behavior is achieved

$$x - \hat{x} = P \begin{bmatrix} 0 \\ e_{\beta} \end{bmatrix} \rightarrow 0, \quad t \rightarrow \infty \quad (15)$$

Note that by selecting a matrix H with such a property, the linear system (7) that produces the estimate is stable itself.

Using the Jordan Normal Form of H , it can be shown [11] that H is exponentially stable if and only if all its eigenvalues have negative real parts, i.e. if $Re(\lambda_i) < 0, i = 1, \dots, n - p$. Moreover, the rate at which $\|e^{Ht}\| \rightarrow 0$ depends on the magnitude of the largest eigenvalue λ_m . The convergence rate in (15) decreases with λ_m .

The following theorem states that we can choose the eigenvalues of matrix H so that, under the specified conditions, the convergence rate in (15) is arbitrarily fast. The proof is outlined in the Appendix.

Theorem: *Suppose the time-invariant linear state equation (1) is observable¹ and $rank C = p$. Given any degree $n - p$ monic polynomial $q(\lambda)$ there exists gain H such that the reduced-dimension observer defined by (7) has an error state equation (10) with characteristic polynomial $q(\lambda)$.*

In the next section, we explain how the described state-variable estimation methodology can be used to render concurrently testable linear analog circuits.

3. Observer-based concurrent error detection

The objective of concurrent error detection is to continuously examine whether the output of the circuit under consideration remains within a tolerance band around its nominal value. Hence, we should choose an invariant property that is highly correlated to the output of the circuit. Ideally, this property should be violated in case one or more circuit specifications are failed. This implies that the property needs to depend on all circuit parameters.

Based on the discussion of the previous section, the proposed concurrent error detection methodology relies on examining the invariant difference $\hat{y} - y$, where \hat{y} is an estimate of the output. Ideally, $\hat{y} - y = 0$, yet in reality, due to transistor mismatches and other non-idealities that are always present, this comparison cannot be made exact. Hence, to compensate for process variations, the absolute difference $|\hat{y} - y|$ has to be compared to a threshold $V_{\delta} > 0$, in order to realize a tolerance window within which the two signals are deemed equal. This issue will be addressed later in section 4.

The circuit that produces the output estimate monitors continuously the circuit input as well as a set of observable

¹An n -dimensional time invariant system (1) is observable if and only if

$$rank \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$

Other equivalent definitions may also be listed [10], for example one may consider the eigenvalue test given in the Appendix.

nodes internal to the circuit. Monitoring an internal node should not affect the performance of the circuit. For example, in an active linear analog circuit, the outputs of the operational amplifiers may be directly monitored without loading the circuit, provided that the amplifier's output stages can supply and sink the necessary amount of current. Actually, even non-observable nodes may be monitored if an additional buffer is utilized; however, the use of this voltage test point structure depends on the hardware that can be potentially saved, as compared to estimating these voltages.

In the following, we discuss issues regarding output estimation, size of the estimation circuit, and reliability of the proposed method.

3.1. Output estimation

An estimate of the output can be obtained as follows. If the output is a state-variable itself, then the method discussed earlier is directly applicable. The estimate vector, z_{β} , will consist of the output as well as all the state-variables that cannot be directly observed.

If the output is not a state-variable but rather a linear combination of state-variables and inputs, i.e. it is of the form $Mx + Du$, then it is sufficient to estimate just one state variable that appears in the expression of the output. In case all state variables that appear in the output expression are observable, one has to be treated as non-observable and its value has to be estimated by monitoring the rest of the state variables. If more than one state variables that appear in the output expression are non-observable, then all of these non-observable state variables need to be estimated. The output estimate can then be produced by substituting the actual state variables by their estimated values in the output expression.

In case the circuit under test can be partitioned into a cascade of stages that do not communicate through feedback, special attention must be paid in order to ensure that all parameters are taken into account in the design of the estimator. For example, consider a circuit with two cascaded stages. If the output of the first stage is included in the set of observable nodes, then the contribution of all other selected observable nodes within the first stage will be nullified in the construction of the estimation circuit. Therefore, the output estimate will only depend on the components of the last stage and any information for the rest of the circuit will be lost. A remedy for this is to avoid including the connection node between two cascaded stages into the set of observable variables or to apply the method separately for each stage.

In all cases described above, the output estimate can be realized as an active linear circuit which receives as inputs the inputs of the circuit under test and a subset of the observable internal nodes. The above propositions guarantee that the resulting estimation circuit depends implicitly on all parameters of the original circuit through the matrix coefficients in (7), provided that the elements of the real matrix N are all

non-zero. Moreover, from the conditions that are listed in (11), it is evident that the sensitivity of the estimate can be set by choosing an appropriate matrix N , always keeping in mind that any selection must necessarily lead to an exponentially stable matrix H .

3.2. Estimation circuit size

The number of nodes that need to be observed equals the largest degree of the denominators of the transfer functions \hat{w}/u_i , where u_i are the inputs of the observer (7). This is a very useful result since the size of the estimation circuit can be deduced by simple observation of the original circuit. It also indicates the potential of the proposed method; usually, observable nodes exist in the internal of the stages and hence the order of the estimation circuit will be smaller than that of a duplicate of the circuit under test. Smaller order generally implies smaller area overhead and less complexity, although area is also related to the placement of the components on the chip. Moreover, in the worst case where none of the internal nodes is observable and only the input is available, the size will equal that of a duplicate. This means that, in the worst case, the proposed strategy leads to the duplication method. In section 5, we present two examples where it is sufficient to estimate only one state-variable and hence the resulting estimation circuit is very simple and straightforward to implement. An example for which our method does not have any advantages over duplication is the Sallen-Key biquad [12], where all internal nodes are non-observable. It may be possible to set the coefficient matrices of the linear system that produces the estimate in such a way that its order is less than $n - p$ but special attention must be paid in order to incorporate all parameters into the invariant property.

The reason this method has the potential to result in a circuit of size less than the original one is that the output is estimated by means of present information that the circuit nodes possess. The estimate is delayed because of the lead time required to compute the initial value of the state-variable vector and the current state from this information. However, from the time the estimate converges to the actual output value and onwards, it follows accurately any change occurring at the output. Hence, the penalty we pay is the convergence time. As discussed in the previous section, the linear observer can be chosen so as to achieve arbitrarily fast convergence. Therefore, practically, the initial convergence interval does not constitute a drawback.

3.3. Reliability of the method

The estimation circuit along with the analog checker needed to examine continuously the difference $\hat{y} - y$ constitute the error detection circuit. We emphasize that this scheme does not assume any specific fault model. It is rather defined at an abstract level in terms of the correlation of

the output and its estimate. Since the values of the components of the estimation circuit depend on the whole parameter space of the original circuit, it is expected that any deviation from the nominal behavior will trigger the checker. The threshold of the checker sets the limits of acceptable behavior. Hence, the error detection circuit is code-disjoint [13], i.e. input values are mapped to the output code space if and only if they belong to the input code space. Equivalently, the observer has the capability to indicate its own faults, since a fault within it will cause the estimate \hat{y} to deviate from the correct output y . Therefore, the observer is also self-testing. In order to ensure the self-testing property for the entire error detection circuit we need to ensure that the analog checker is self-testing. This means that, after the occurrence of a fault within the checker, the checker must either indicate it or retain its ability to classify the input signals correctly.

4. Analog checker considerations

A checker is required to compare the two signals, \hat{y} and y . Comparators are extensively used in analog design [14], with their most important application occurring in analog-to-digital conversion. The sign of their output voltage indicates which of the input signals is larger. For test purposes, however, one is rather interested in the correlation between the two encoded signals. As an example, consider two signals that are expected to be identical in the absence of faults. Since deviations from nominal values attributable to process variations are certain to exist, what matters is the difference between the two signals. Therefore, instead of performing an exact comparison, checkers should take into account a tolerance window within which two signals are deemed equal.

In our particular case, the checker needs to examine whether the inequality $|\hat{y} - y| < V_\delta$ holds. V_δ is the assigned threshold, which is chosen so that the number of false alarms, both positive and negative, is minimized. Its optimal value can be found on the basis of a worst-case tolerance analysis, which aims to identify the worst case parameter sets.

Algorithms proposed for worst-case analysis fall into three major categories. *Monte Carlo* analysis usually underestimates the upper and lower bound of the output response since a finite number of simulations is, theoretically, never sufficient. Hence, the calculated response is enclosed in the actual response space. *Interval analysis* leads to overestimated results since the dependency amongst interval operands causes intractable interval expansion. In this case, the calculated response encloses the actual response space. *Vertex analysis* assumes that the response is monotonic to changes in any parameter and at any point in the parameter space. Therefore, the worst case parameter sets are located at the vertices of the parameter space and the response space can be calculated by taking the union of circuit simulation results at all possible vertices of the parameter space. In [15], a method is described which uses sensitivity bands to identify

the parameters that satisfy monotonicity over the parameter space and substitute them with their corner values, in order to generate an uncertainty-reduced simulation problem. It is shown that the derived bounds are enclosed in the respective bounds that result from Monte Carlo and Interval analysis.

Worst-case analysis is applied to find upper and lower bound for the gain $(\hat{y} - y)/u$. Hence, the resulting threshold is of the form $V_\delta = e_r(\omega)u$, that is, it should depend on both frequency and input stimulus magnitude. An analog checker for fully differential circuits that has the ability to adjust its threshold to the input stimuli magnitude is presented in [16]. The idea can be easily extended for the case of duplicate signals. The proposed checker implements a threshold $V_\delta = e_r|u|$. While its dependency on frequency is not consistent, this particular checker is the best known solution to date. The constant e_r can be chosen to achieve the highest possible accuracy in the bandwidth of interest. Accurate threshold assignment is a common problem among existing analog test solutions [17]. Essentially, the probability of false alarms can be minimized but cannot be eliminated.

5. Examples

In this section, we demonstrate the various points of the developed concurrent error detection methodology on two example filters. In both cases, the derived estimation circuit is very simple in comparison to a duplicate of the circuit under consideration. In error-free operation, the estimate converges exponentially to the output value for all input signals and for all frequencies. Since the input code space is infinite, we illustrate this convergence through simulation for an arbitrarily chosen input. In the first example, the output of the circuit is not a state variable, while in the second example it is. In either case, a single state variable needs to be estimated. Hence, H reduces to a scalar. In order to illustrate the impact of H on the rate of convergence, we use two largely different values for the two circuits.

We note that in case the initial condition of the error differential equation (10) is zero, i.e. $e_b(t = 0) = 0$, the estimate converges to the output value immediately. In order to simulate a non-zero initial condition of the differential equation and demonstrate the exponential reduction of $\hat{V}_{out} - V_{out}$, we insert switches at the input of the estimation circuit, which we turn on at an arbitrarily chosen time.

We also demonstrate that the estimate fails to converge to the output value in the presence of faults in the circuit components. While we only present simulations for a single fault for each circuit, we note that we simulated all possible catastrophic faults and a large subset of the infinite possible parametric faults and convergence failed in all cases where the output was erroneous. Qualitatively, this observation can be explained as follows. If the output is erroneous, then at least one of the monitored state variables will be erroneous as well. Since the estimator is inherently different than the

circuit under test, we expect that this voltage inconsistency will affect the two responses differently. Therefore, there is a reasonably accurate mapping between voltage specifications and the test output.

We emphasize that this method does not rely on a prescribed fault model but rather detects any malfunction that leads to an unacceptably large discrepancy between the estimate and the output of the circuit. The threshold of the analog checker that detects this discrepancy depends on the specific application and data collected from the manufacturing process. Issues regarding analog checker design as well as the selection of an appropriate threshold were analyzed in section 4. This topic is not discussed further herein. The interested reader may consult the existing literature [2, 16, 18, 19].

5.1. Highpass filter

A highpass filter implemented by a summing four-amplifier biquad is shown in Fig. 1. This circuit has three state-variables. The output itself is not a state-variable and can be expressed as $V_{out} = -x_1 + 10x_2 - V_{in}$. All state-variables are observable. The problem reduces to estimation of x_1 by monitoring x_2 , x_3 and V_{in} . Equivalently, we could have chosen to estimate x_2 by monitoring x_1 and noting that $x_2 = -x_3$. The state-variable equation for the circuit can be formulated as

$$\dot{x} = \begin{bmatrix} -100 & 1000 \\ -1000 & 0 \end{bmatrix} x + \begin{bmatrix} -100 \\ 0 \end{bmatrix} V_{in}$$

where $x = [x_1 \ x_2]^T$. The reduced-size state variable vector is $y = [0 \ 1] x$. Letting $P_b = [1 \ 0]$,

$$F = \begin{bmatrix} 0 & -1000 \\ 1000 & -100 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

Hence, the necessary conditions for the output of the observer (7) to exponentially converge to x_1 are

$$H = -100 + 1000N < 0$$

$$K = 1000 + HN$$

$$\Gamma = -100$$

Choosing $N = -0.1$, yields $H = -200$ and $K = 1020$. Taking the Laplace transform of (7), solving for \hat{x}_1 and substituting in the expression of the filter output, we obtain the following estimate

$$\hat{V}_{out} = -\frac{1020}{s+200}x_2 - 10.5x_3 + \left[\frac{100}{s+200} - 1 \right] V_{in}$$

This function can be implemented with the active circuit shown in Fig. 2. In Fig. 3 we illustrate the exponential convergence for the fault-free case. The estimation circuit is

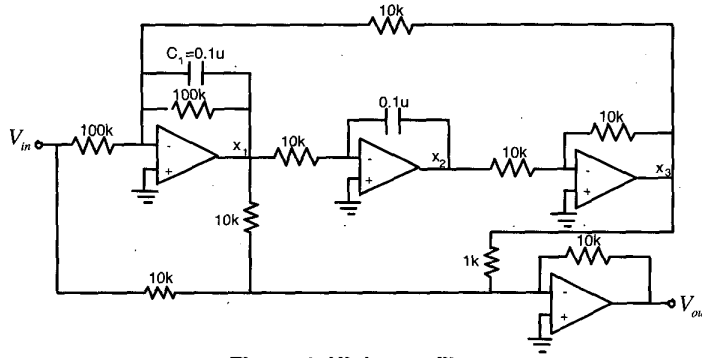


Figure 1. Highpass filter.

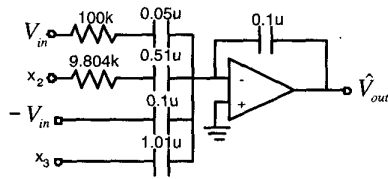


Figure 2. Estimation circuit for the highpass filter.

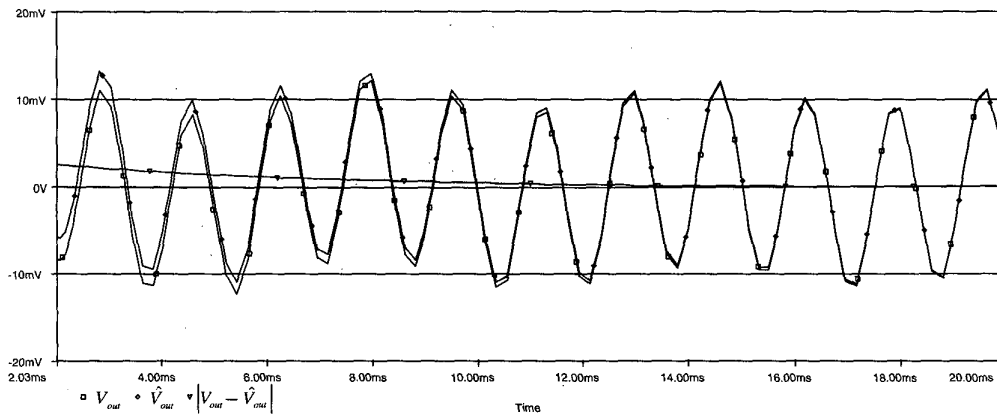


Figure 3. Estimation converges in the fault-free case.

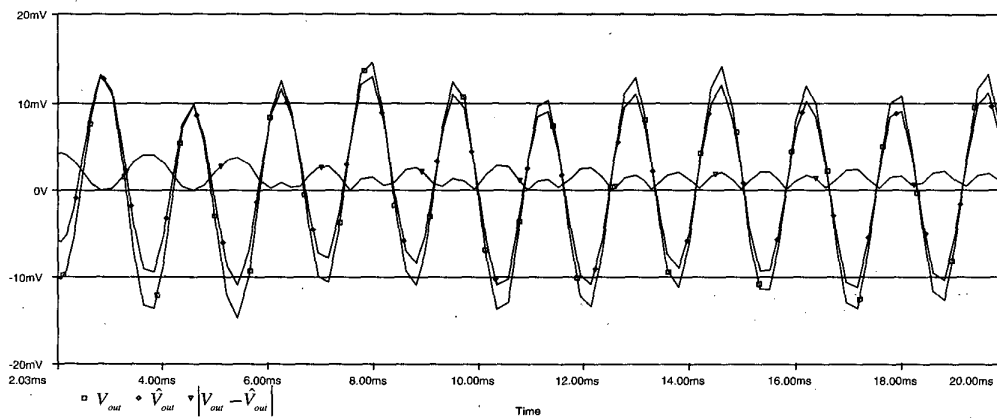


Figure 4. Estimation fails to converge in the case of a faulty value of 0.12μ for the capacitor C_1 .

activated at $t = 2ms$ so that $e_b(t = 0) \neq 0$. The simulation is repeated in the presence of a fault that increases the value of capacitor C_1 by 20%. The result is shown in Fig. 4. Due to this fault, the estimate never converges to the output.

5.2. Leapfrog lowpass filter

The second example is a leapfrog lowpass filter [12], shown in Fig. 5. There are six state-variables one of which is the output itself, $V_{out} = x_6$. Moreover, note that $x_2 = -x_1 - x_4$ and $x_5 = -x_4$. Hence, if we choose $x = [x_1 \ x_3 \ x_4 \ x_6]^T$, the state-variable equation is

$$\dot{x} = \begin{bmatrix} -\frac{1}{RC_1} & -\frac{1}{RC_1} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & \frac{1}{RC_2} & 0 \\ 0 & -\frac{1}{RC_2} & 0 & -\frac{1}{RC_2} \\ 0 & 0 & \frac{1}{RC_1} & \frac{1}{RC_1} \end{bmatrix} x + \begin{bmatrix} -\frac{1}{RC_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

where $R = 10k$, $C_1 = 7.654n$ and $C_2 = 18.48n$. We will estimate V_{out} by monitoring x_1, x_3, x_4 and V_{in} . Hence, the reduced-size state variable vector is specified as

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x$$

Letting $P_b = [0 \ 0 \ 0 \ 1]$, it turns out that $P = I_{4 \times 4}$ and

$$F = \begin{bmatrix} -\frac{1}{RC_1} & -\frac{1}{RC_1} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & \frac{1}{RC_2} & 0 \\ 0 & -\frac{1}{RC_2} & 0 & -\frac{1}{RC_2} \\ 0 & 0 & \frac{1}{RC_1} & \frac{1}{RC_1} \end{bmatrix}, \quad G = \begin{bmatrix} -\frac{1}{RC_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If we denote by $N_i, i = 1, 2, 3$, the elements of the 1×3 matrix N , the conditions for exponential convergence become

$$H = -\frac{1}{RC_1} + \frac{N_3}{RC_2} < 0$$

$$K = \begin{bmatrix} \frac{N_3 N_1 - N_2}{RC_2} & \frac{N_1 - N_2}{RC_1} + \frac{N_3(N_2 + 1)}{RC_2} & \frac{1 - N_3}{RC_1} + \frac{N_3^2 - N_2}{RC_2} \end{bmatrix}$$

$$\Gamma = \frac{N_1}{RC_1}$$

Hence, choosing $N_3 = 1, N_1 = N_2 = -1$, we obtain

$$H = -7.65 \cdot 10^3$$

$$K = [0 \ 0 \ 10.822 \cdot 10^3]$$

$$\Gamma = -13.065 \cdot 10^3$$

The estimate for the output is

$$\hat{V}_{out} = \hat{x}_6 = -\frac{10.822 \cdot 10^3}{s + 7.653 \cdot 10^3} x_5 - \frac{13.065 \cdot 10^3}{s + 7.653 \cdot 10^3} V_{in} - x_1 - x_3 - x_5$$

The estimation circuit is shown in Fig. 6. Fig. 7 illustrates the convergence when all components have their nominal values. Once again, we utilize switches which we turn on at $t = 2ms$ to impose random initial conditions. Since the value of H is smaller in this example than in the previous one, convergence is faster. Fig. 8 shows that the estimate fails to converge when resistor R_1 takes the faulty value of $12k$. Finally, Fig. 9 illustrates the respective responses in the occurrence of a transient error. A short pulse is added to the signal of a randomly selected node at around $4ms$. The invariant difference $|\hat{y} - y|$ becomes temporarily nonzero and decreases exponentially to zero again.

6. Conclusion

We presented a new approach for concurrent error detection in linear analog circuits. In general, the proposed approach results in an error detection circuit that is smaller than a duplicate of the original circuit. The size of the derived error detection circuit is predictable by simple observation of the circuit under consideration and, furthermore, decreases with the number of observable nodes. The invariant property used for concurrent error detection reflects the entire parameter space of the circuit and is monitored continuously. Additionally, response correctness information is obtained at the operational speed for any input value and for the entire frequency spectrum. Hence, concurrent detection of both transient errors and permanent faults is ensured. The various points of the developed scheme were demonstrated through representative simulations.

Appendix

The proof is based on three well known results in linear system theory [10, 20], which are presented in theorems A.1 through A.3. First, controllability is defined.

A n -dimensional time invariant linear system (1) is controllable if and only if

$$\text{rank} [B \ AB \ \dots \ A^{n-1}B] = n$$

Theorem A.1: For every n -dimensional controllable pair (A, B) and each symmetric set of complex numbers Λ , there exist a matrix L such that

$$\text{spectrum}(A + BL) = \Lambda$$

Theorem A.2: A pair (C, A) is observable if and only if for every complex scalar λ the only complex $n \times 1$ vector p that satisfies

$$Ap = \lambda p, \quad Cp = 0$$

is $p = 0$.

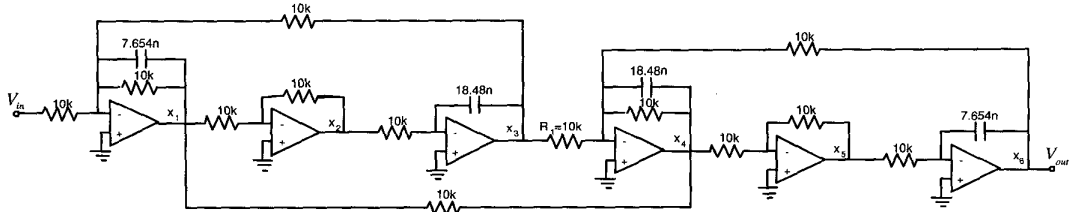


Figure 5. Leapfrog lowpass filter.

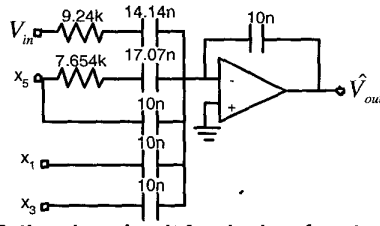


Figure 6. Estimation circuit for the leapfrog lowpass filter.

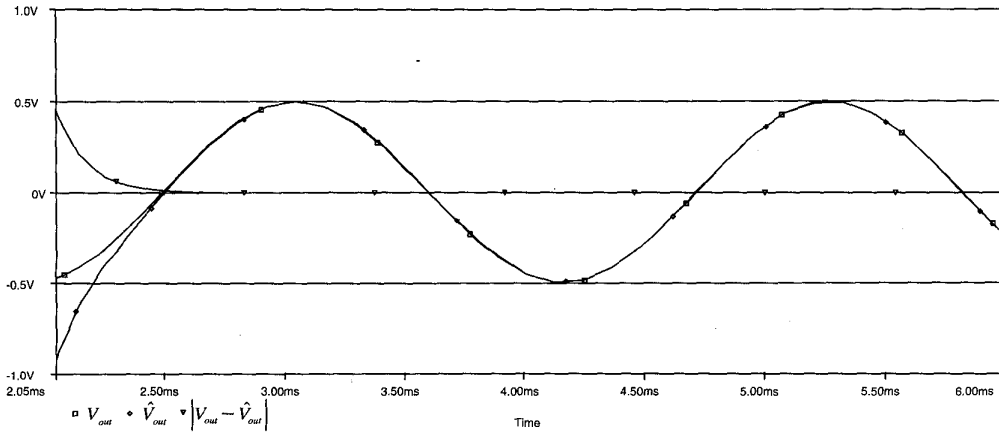


Figure 7. Estimation converges in the fault-free case.

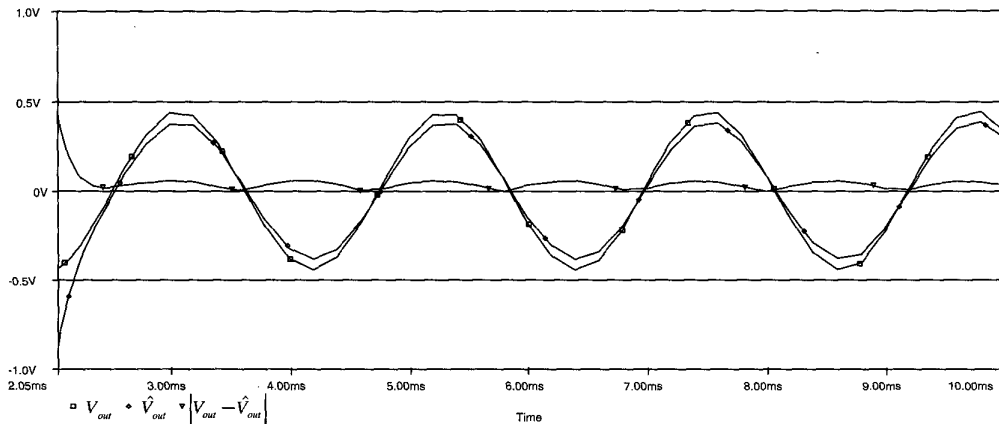


Figure 8. Estimation fails to converge in the case of a faulty value of 12k for the resistor R_1 .

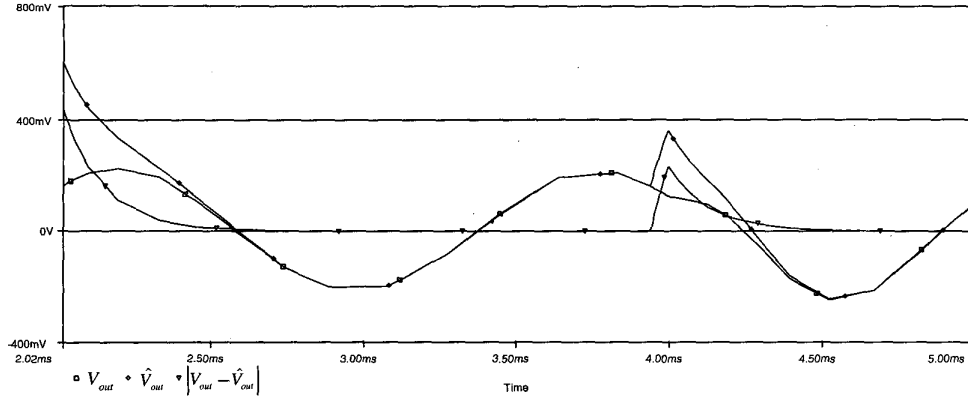


Figure 9. Estimation diverges temporarily in the occurrence of a transient error.

The next theorem follows directly from theorem A.2. and

Theorem A.3: A pair (C, A) is observable if and only if

$$\text{rank} \begin{bmatrix} C \\ sI - A \end{bmatrix} = n$$

for every complex scalar s .

It is clear that this observability test criterion need only be applied for those values of s that are eigenvalues of A .

Since $H = F_{22} - NF_{12}$, we need to prove that

$$\det[\lambda I - (F_{22} - NF_{12})] = q(\lambda) \quad (16)$$

Equivalently, noticing that $(F_{22} - NF_{12})^T = F_{22}^T - F_{12}^T N^T$, (16) becomes

$$\det[\lambda I - (F_{22}^T - F_{12}^T N^T)] = q(\lambda) \quad (17)$$

According to theorem A.1, it is adequate to show that the pair (F_{22}^T, F_{12}^T) is controllable. However, we know that controllability of (F_{22}^T, F_{12}^T) is equivalent to observability of (F_{12}, F_{22}) . Hence, it is adequate to show that the $(n - p)$ -dimensional linear state equation

$$\begin{aligned} \dot{x} &= F_{22}x \\ y &= F_{12}x \end{aligned} \quad (18)$$

is observable. Supposing the contrary, a contradiction is obtain as follows. If (18) is not observable, then by theorem A.2 there exists a nonzero $(n - p) \times 1$ vector, ζ , and a scalar, η , such that

$$F_{22}\zeta = \eta\zeta, F_{12}\zeta = 0 \quad (19)$$

Therefore,

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} 0_{p \times 1} \\ \zeta \end{bmatrix} = \begin{bmatrix} F_{12}\zeta \\ F_{22}\zeta \end{bmatrix} = \eta \begin{bmatrix} 0_{p \times 1} \\ \zeta \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} I_p & 0 \end{bmatrix} \begin{bmatrix} 0_{p \times 1} \\ \zeta \end{bmatrix} = 0 \quad (21)$$

The above two equations can be written in matrix form

$$\begin{bmatrix} I_p & 0 \\ \eta I - F \end{bmatrix} \begin{bmatrix} 0_{p \times 1} \\ \zeta \end{bmatrix} = 0 \quad (22)$$

Hence,

$$\text{rank} \begin{bmatrix} I_p & 0 \\ \eta I - F \end{bmatrix} < n \quad (23)$$

and by theorem A.3, the linear state equation (6) is not observable. But (6) is related to (1) by a state variable change and thus a contradiction to the observability hypothesis for (1) is obtained. This completes the proof.

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