A Chromatic Dispersion Estimation Method for Arbitrary Modulation Formats

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Abstract: Simulations show that a modulation-format-independent method for estimating chromatic dispersion from the phase of a coherently-received signal at four frequencies can estimate 3000 ps/nm of dispersion to within 2% at an OSNR of 10 dB.

1.   Introduction

Recent advances in coherent receivers now allow for optical noise and chromatic dispersion (CD) to be monitored and mitigated using digital signal processing techniques [1, 2]. Typically, compensation of CD is performed using electrical equalization filters that are continuously adapted using optimization algorithms such as minimum mean square estimation (MMSE) [1]. All such optimization methods use *a priori* knowledge of the optical data modulation format to define the cost function to be minimized. For example, with the constant modulus algorithm (CMA) [3], the data symbols are assumed to lie on a circle in the constellation diagram. Consequently, such methods often need to be individually designed for each format (see, for example, [4]), which is challenging—particularly in light of the recent proliferation of novel optical modulation formats [5].

In this paper, we introduce a method to estimate CD for arbitrary modulation formats impaired by arbitrary amounts of CD and noise. The method is based on an analytical formula for the CD in terms of the phase of the optical signal at four frequencies, and does not use optimization. We describe a preliminary version of the method and present initial simulation results demonstrating that the method is effective for a wide variety of formats even when the OSNR approaches the FEC limit. The method could be used to monitor performance in reconfigurable heterogeneous optical networks and in the design of general purpose coherent receiver systems.

2.   Theory

If the signal is impaired only by CD, then the Fourier transform, $\widetilde{u}$, of the coherently-received signal, u, is given by

$$\widetilde{u}(\omega) = \widetilde{v}(\omega) \left(\sum_{n=0}^{N-1} b_n \exp(i\omega nT) \right) \exp(i\beta \omega^2/2) \exp(i\theta_0 + i\omega t_0), \tag{1}$$

where ω is angular frequency, $v = v(t)$ is the pulse shape at the transmitter, b_n is the n-th data symbol, T is the symbol period, β is the total dispersion, and t_0 and θ_0 are unknown time and phase offsets. To determine the chromatic dispersion, β, we choose two frequencies, $\omega_j = 2\pi f_j$ for $j = 1, 2$, and define

$$\Theta(f_1, f_2) \;=\; \arg[\widetilde{u}(f_1)] + \arg[\widetilde{u}(-f_1)] - \arg[\widetilde{u}(f_2)] - \arg[\widetilde{u}(-f_2)]. \tag{2}$$

We assume that $\arg[\widetilde{v}(f)]$ is an even function of f, which, for example, holds if v is real and even. Then, we find that

$$\Theta(f_1, f_2) \;=\; 4\pi^2\beta(f_1^2 - f_2^2) \;+\; \varphi_b(f_1) \;+\; \varphi_b(-f_1) \;-\; \varphi_b(f_2) \;-\; \varphi_b(-f_2) + 2k\pi, \qquad k = \pm1, \pm2, ..., \tag{3}$$

where

$$\varphi_b(f) \;=\; \arg\left[\sum_{n=0}^{N-1} b_n \exp(2\pi i f T n) \right]. \tag{4}$$

If the data symbols, b_n, are real, then φ_b is an odd function and so $\Theta(f_1, f_2) = 4\pi^2\beta(f_1^2 - f_2^2) + 2k\pi$. On the other hand, for complex-valued symbols, b_n, we may choose $f_1 = (1/2T) + f$ and $f_2 = (1/2T) - f$, for some frequency, f. We then find that $\varphi_b(f_2) = \varphi_b(-f_1)$ and $\varphi_b(f_1) = \varphi_b(-f_2)$, and so $\Theta(f) = (8\pi^2 f/T)\beta + 2k\pi$. For the results in this paper, we suppose that the absolute value of the dispersion does not exceed a maximum value given by

$$|\beta| < \beta_{\max} = \frac{1}{4\pi(f_1^2 - f_2^2)} = \frac{T}{8\pi f}. \qquad (5)$$

Then the dispersion is unambiguously given by

$$\beta = \frac{\Theta(f_1, f_2)}{4\pi^2(f_1^2 - f_2^2)} = \frac{T\Theta(f)}{8\pi^2 f}, \qquad \text{where } -\pi \leq \Theta < \pi. \qquad (6)$$

To mitigate for errors due to noise, we average the angle Θ in Eqn. (2) over many noise (and data) realizations. For the preliminary results presented here, we arranged for each realization of the signal to be periodic in time. The restrictions on the periodicity of the signal and the maximum allowed value of the disperison will be removed in a future paper.

3. Simulation Results

We tested the method on six formats generated using a Mach-Zehnder modulator (NRZ, RZ33 (33% duty cycle), RZ50, CSRZ, BPSKMZ, and QPSKMZ) and on two formats generated using a phase modulator (BPSKPM and QPSKPM). We generated signals at a baud rate of 10 Gb/s, with OSNR values of of 5, 10, and 15 dB, and with dispersions of 500 ps/nm and 3000 ps/nm. For each format, OSNR, and dispersion value we generated 2048 periodic signals each with 64 symbols. For each signal, we used different realizations of the data symbols and noise. We used $f_1 = 6$ GHz and $f_2 = 4$ GHz. The maximum dispersion was $\beta_{\max} = 3120$ ps/nm. On the left in Table 1, we show the errors in the estimated dispersion (in ps/nm) when the dispersion was 500 ps/nm, and on the right when the dispersion was 3000 ps/nm. The results suggest that the relative error in the overall estimate could be reduced to less than 2% for signals with 3000 ps/nm dispersion and an OSNR of at least 10 dB if we were to first compensate for the dispersion using the estimated dispersion values and then estimate the dispersion of these compensated signals.

Table 1. Error in the estimated dispersion (in ps/nm) displayed as a function of the modulation format (rows) and OSNR (columns). The total dispersion was 500 ps/nm (left) and 3000 ps/nm (right).

Format	5 dB	10 dB	15 dB
NRZ	162	59	1
RZ33	14	17	27
RZ50	55	37	27
CSRZ	6	20	42
BPSKMZ	7	36	44
QPSKMZ	30	25	41
BPSKPM	107	29	3
QPSKPM	109	20	18

Format	5 dB	10 dB	15 dB
NRZ	148	164	189
RZ33	183	186	175
RZ50	236	168	195
CSRZ	226	212	187
BPSKMZ	155	191	186
QPSKMZ	191	205	179
BPSKPM	204	154	198
QPSKPM	75	160	191

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