Probability Density Functions of Rotations in Loop-Synchronous Polarization Scrambling for Recirculating Loop Experiments

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Abstract: The performance in a recirculating loop with a loop-synchronous polarization scrambler is independent of the choice of probability density function (pdf) for the rotations in the polarization scrambler, unless the pdf is strongly biased.

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1. Introduction Polarization effects can be quite different in recirculating loops than in the straight-line transmission systems they are designed to emulate [1], [2]. Recently however, theoretical and experimental studies have shown that when loop-synchronous polarization scrambling is used, the distributions of the bit-error ratio, the total polarization-dependent loss (PDL), and the total differential group delay (DGD) in a recirculating loop are similar to those in a straight-line system [3]–[6]. A polarization scrambler (PS) is used within the loop to break the periodicity of the optical path by imparting a different random rotation to the light each round trip of the loop for each statistical sample. If the rotations of the PS are uniformly distributed, one would expect the performance in the loop to closely approximate that in a straight-line system [3]. In experiments, loop-synchronous rotations are typically performed using an electro-optic lithium niobate polarization scrambler [7]. Although uniformly distributed rotations have been generated experimentally using fiber-squeezer-based scramblers [8], because of manufacturing imperfections we have found it difficult to generate them with lithium niobate scramblers.

It is commonly assumed that even if the pdf of the rotations is not uniform, the system performance will be close to that obtained using uniform rotations, provided that for each input polarization state the output polarization states cover the Poincaré sphere, and that the signal is scrambled enough times during transmission. In this paper, we study the validity of this assumption by quantifying the effect that the pdf used for the rotations has on the system performance, as measured by the pdf of the total PDL, especially in the low-probability tails which are important for the determination of outage probabilities. We use theory and simulations to show that even very non-uniform pdfs will result in essentially the same performance statistics as in the uniform case, unless there is an input polarization state to the scrambler for which the output polarization states are biased towards a particular Stokes vector on the Poincaré sphere.

2. Theoretical model We use the reduced Stokes model [9] and standard Monte Carlo simulations to propagate the average Stokes parameters of the signal through the loop and compute the pdf of the total PDL at the receiver. We model a single round trip of the loop as a rotation of a loop-synchronous PS followed by a fiber rotation, a PDL element with a PDL of 0.1 dB, and finally a second fiber rotation. We assume that the low and high loss axes of the PDL are $(1, 0, 0)$ and $(-1, 0, 0)$, respectively, and that the two rotations due to the fibers in the loop do not change during the experiment. We use three different pdfs for the rotations of the PS: a uniformly distributed pdf, an unbiased non-uniform pdf, and a biased pdf. We model an ideal lithium niobate PS as the concatenation of a quarter, a half, and a quarter waveplate, so that the rotation of Stokes vectors on the Poincaré sphere is given by $R_{\mathrm{PS}}(\psi, \theta, \phi) = W_{\pi/2}(\psi)W_{\pi}(\theta)W_{\pi/2}(\phi)$, where $W_{\alpha}(\theta) = R_z(-\theta)R_x(-\alpha)R_z(\theta)$ [7]. Here $R_z(\theta)$ is a rotation about the z-axis through θ. To generate uniform rotations, we observe that R_{PS} can be represented using Euler angle rotations as $R_{\mathrm{PS}}(\psi, \theta, \phi) = R_z(-\psi)R_y(\overline{\theta})R_z(\phi)$, where $\overline{\theta} = 2\theta - \psi - \phi$. Consequently, for *uniform rotations* we choose ψ, ϕ uniformly from $[0, 2\pi]$, $\gamma = \cos\overline{\theta}$ uniformly from $[-1, 1]$ and then set $\theta = (\arccos(\gamma) + \psi + \phi)/2$. We define an *unbiased non-uniform pdf* by choosing the waveplate angles ψ, θ, and ϕ uniformly from $[0, 2\pi]$. This pdf is the one we used to set the voltages in the PS for the experimental results reported in [4]–[6]. However, the rotations performed by the PS did not exactly correspond to those we set due to imperfections in the device. We define a *biased pdf* in terms of a biasing parameter, $\beta > 0$, and a sign, $\epsilon = \pm 1$. We choose ψ, ϕ uniformly in $[0, 2\pi]$, and we introduce a bias, by selecting $\gamma = \cos\overline{\theta}$ using the pdf [10]

$$f_{\beta,\epsilon}(\gamma) = \frac{\beta}{1 - \exp(-2\beta)} \exp(-\beta(1 - \epsilon\gamma)) \qquad \text{for } |\gamma| \leq 1, \qquad \text{and } f_{\beta,\epsilon}(\gamma) = 0 \text{ otherwise.}$$

As $\beta \to 0$, $f_{\beta,\epsilon}$ converges to a uniform pdf on $[-1, 1]$, and as β increases, it is biased towards $\gamma = \epsilon$, *i.e.*, towards $\theta = 0$ when $\epsilon = +1$ and $\theta = \pi$ when $\epsilon = -1$. Although the biased pdfs, $f_{\beta,\pm 1}$, are unlikely to correspond to the pdf of the rotations in a real PS, as we will show, they represent two extreme cases for the pdf of the total PDL.

3. Results We begin by analyzing the two non-uniform pdfs of the rotations for the PS. To study these pdfs, we computed the pdf of the output Stokes vectors, $S_{\rm out}$, to the PS for different choices of input Stokes vector, $S_{\rm in}$. In Fig. 1 (left), for the unbiased non-uniform pdf, we show 1000 output Stokes vectors, $S_{\rm out}$, on the Poincaré sphere when $S_{\rm in} = (0, 0, 1)$. The corresponding result for the biased pdf with $\beta = 0.6$ and $\epsilon = +1$ is shown in Fig. 1 (right). For both pdfs, although the output states cover the sphere, the coverage is not uniform. For a better comparison, we construct a one-dimensional (1D) representation of the histograms of $S_{\rm out}$. We let (Φ, Θ) be spherical coordinates, where Φ is the angle from the north pole. We divide the sphere into 10×10 equal area regions A_{ij} using circles of latitude, $\Phi = \Phi_i$, and longitude, $\Theta = \Theta_j$. We obtain the first 10 bins of the 1D histogram by going east around the 10 regions on the sphere nearest the north pole, $(0, 0, 1)$, and so on, with the last 10 bins corresponding to the regions nearest the south pole, $(0, 0, -1)$. For $S_{\rm in} = (0, 0, 1)$, we show the results for the unbiased non-uniform pdf and the biased pdf with thick dashed lines on the left and thin solid lines in the middle of Fig. 2, respectively. These results were obtained using Monte Carlo simulations with 10^6 samples. We chose a biasing parameter of $\beta = 0.6$ so that the ratio of the largest to the smallest probabilities in these two histograms is approximately the same. However, the biased and unbiased non-uniform pdfs have quite different symmetries.

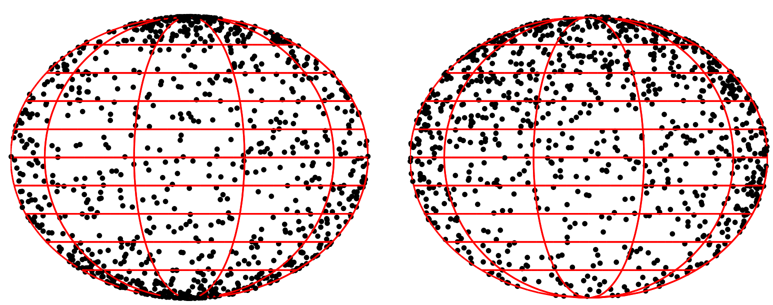

Fig. 1. Left: Output Stokes vectors for unbiased nonuniform rotations. Right: Output Stokes vectors for biased rotations with $\beta = 0.6$, $\epsilon = +1$.

From Figs. 1 (right) and 2 (middle), we see that for the biased pdf, when $S_{\rm in} = (0, 0, 1)$, the output Stokes vectors are biased towards $(0, 0, 1)$: The pdf is rotationally symmetric about the z-axis, and the probability per unit area on the sphere decreases from a maximum at $(0, 0, 1)$ to a minimum at $(0, 0, -1)$ along any circle of longitude. The corresponding pdf for $\beta = 0.2$ is shown with a thin dashed line in Fig. 2 (middle). Similarly, if $S_{\rm in} = (0, 0, -1)$, $S_{\rm out}$ is biased towards $(0, 0, -1)$, and if $\epsilon = -1$, $S_{\rm out}$ is biased towards $(0, 0, \mp 1)$ when $S_{\rm in} = (0, 0, \pm 1)$. However, when $S_{\rm in}$ is not close to $(0, 0, \pm 1)$ the pdf of $S_{\rm out}$ is not biased towards any point. On the other hand, for the unbiased non-uniform pdf, as we see in Figs. 1 (left) and 2 (left), when $S_{\rm in} = (0, 0, 1)$ the pdf is symmetric both about the z-axis and with respect to reflection over the equator, and decreases from maxima at the two poles to a minimum on the equator. The thin solid curve in Fig. 2 (left) is the histogram for $S_{\rm in} = (0, 0, \pm 1)$ obtained using the analytical formula for the probability per unit area on the sphere, $f_{\rm pole}(\Phi, \Theta) = \operatorname{cosec}(\Phi)/(2\pi^2)$, while the thin dashed curve is the histogram for $S_{\rm in} = (\cos\alpha, \sin\alpha, 0)$, obtained using the analytical formula $f_{\rm equator}(\Phi, \Theta) = K(\sin^2\Phi)/\pi^3$, where K is the complete elliptic integral of the first kind. More generally, as $S_{\rm in}$ varies from a pole to the equator, the high-probability bands in the pdf of $S_{\rm out}$ near the poles move symmetrically towards the equator, merging when $S_{\rm in}$ is on the equator. Therefore, although it is not uniform, for this pdf $S_{\rm out}$ is not biased for any $S_{\rm in}$.

To study the effect that the choice of pdf for the rotations in the PS has on the pdf of the total PDL, we first choose the two fixed fiber rotations to align the vectors $(0, 0, \pm 1)$ in the PS with the high and low loss axes $(\pm 1, 0, 0)$ in the PDL element. In this case, when β is large, the biased pdfs, $f_{\beta,\pm 1}$, represent two extreme cases for the pdf of the total PDL. For example, if $\epsilon = +1$ and the input state to the PS on the first round trip of the loop is $S_{\rm in,1} = (0, 0, -1)$, then with high probability the Stokes vector of the signal will be near the high loss axis each round trip, and when $S_{\rm in,1} = (0, 0, +1)$, it will be near the low loss axis. Therefore, there will be a greater probability of large total PDL values than in the case of uniform rotations. Similarly, if $\epsilon = -1$, for either of the input states $(0, 0, \pm 1)$ to the PS on the first round trip, with high probability, the Stokes vector will oscillate between the high and low loss axes from round trip to round trip. Consequently, there will be a lower probability of large total PDL values than in uniform case. In Fig. 2 (right), we show the pdf of the total PDL for 15 round trips. The result for the unbiased non-uniform case, which we show with a partially obscured thick dashed curve, is almost identical to that for the uniform case, which we show with a thick solid curve. The results for the two biased rotations with $\beta = 0.6$ and $\epsilon = \pm 1$ are shown with two thin solid curves, while those for $\beta = 0.2$ are shown with thin dashed curves. Even for $\beta = 0.2$, the 1 dB outage probability is within a factor of 3 of that for the uniform case, which is a good agreement.

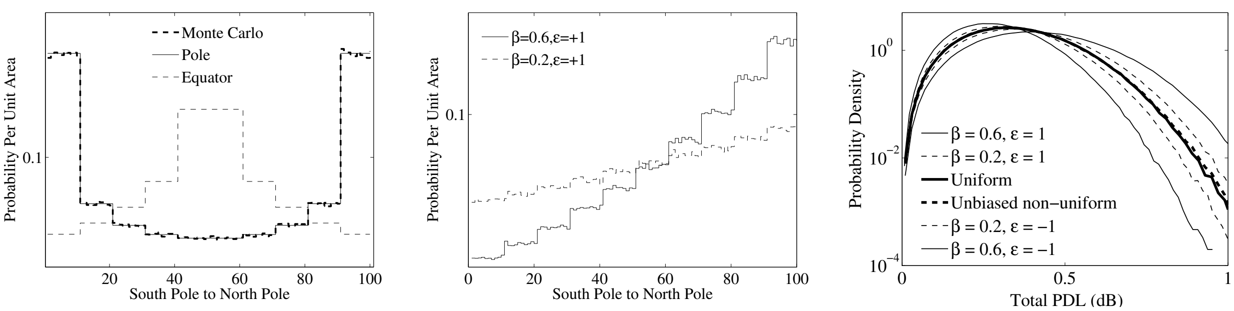

Fig. 2. Left: 1D histogram of output Stokes vectors for unbiased nonuniform rotations. Middle: 1D histogram of output Stokes vectors for biased rotations. Right: Probability density function of total PDL.

4. Conclusions We studied the effect that the pdf of the rotations in a loop-synchronous PS has on the performance of a loop. Provided the pdf of the rotations is not strongly biased, the pdf of the total PDL is very close to that obtained with uniform rotations.

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