The long-term distribution of differential group delay in a recirculating loop

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I. Introduction

It is well known that the distribution of differential group delay (DGD) in a straight-line optical fiber transmission system is a Maxwellian when the fiber realization drifts ergodically and the fiber is statistically homogeneous [1]. However, in a recirculating loop system, a common test-bed in laboratories to study long-haul optical transmission, the periodicity of the fiber path induces different polarization-dependent behavior from that of a straight-line system [2, 3]. To break this periodicity, the loop-synchronous polarization scrambling was developed, whereby the signal undergoes a random polarization rotation each round trip [3]. If the DGD per round trip is constant, the DGD distribution in a loop with such scrambling will be a Maxwellian, similar to the DGD distribution in a uniform straight-line system. However, because of the inevitable drift of the loop fiber birefringence, the DGD per round trip varies over time. Consequently, the DGD distributions in recirculating loops are not Maxwellian, as has been shown both experimentally and theoretically [4, 5]. To our knowledge, there has been no published measurement result to date for the DGD distributions in a loop system from non-ergodic drift to ergodic drift.

In this work, we provide an analytical formula for the DGD distribution after multiple round trips in a loop when the fiber drifts ergodically. Experimentally, we measured the DGD distribution in a 107 km loop system for 6.5 days, during which the fiber drift was nearly ergodic. After nearly ergodic fiber drift, the DGD distribution is very close to the analytical formula. However, over shorter time intervals, the fiber does not drift ergodically, and the DGD distribution strongly depends on the choice of time interval over which the measurements were taken. In comparison to our previous experimental work [4], we shortened the time for one DGD measurement from 16 seconds to 180 ms thus making the measurement uncertainty induced by the fiber drift negligible. As a result, the overall measurement uncertainties were greatly reduced.

II. Theoretical model of ergodic drift in a loop

For a recirculating loop with loop-synchronous polarization scrambling with ergodic fiber drift, the DGD distribution, $F_N(x)$, after N round trips was given and numerically evaluated in [5] by

$$F_N(x) = \int_0^\infty f_N(x;\tau) f_1(\tau)\,\mathrm{d}\,\tau \tag{1}$$

Here, $f_1(\tau)$ is the distribution of the DGD in one round trip, which is Maxwellian with mean $\langle\tau\rangle$ if the fiber drift is ergodic. Also, $f_N(x;\tau)$ is the distribution of the DGD, x, after N round trips, given that the round trip DGD is τ. Since f_N is close to Maxwellian when $N >> 1$, in this case we can evaluate the integral to obtain the closed-form analytical expression

$$F_N(x) = \left(1024x^2/\pi^4\langle\tau_N\rangle^3\right) K_0\left(8x/\pi\langle\tau_N\rangle\right), \tag{2}$$

where K_0 is the zeroth order modified Bessel function of the second kind, and $\langle \tau_N \rangle = \sqrt{8N/3\pi}\langle \tau \rangle$ is the mean DGD after N round trips.

III. Experimental setup

As shown in Fig. 1, our recirculating loop consists of four spans of 25-km-dispersion-shifted fiber and two spans of 3.5-km-single-mode fiber. The details of the loop were described in [4]. We used a loop-synchronous lithium-niobate ($LiNbO_3$) polarization scrambler (PS) to randomly rotate the polarization state of the signal once per round trip. We then measured the DGD after each round trip of the loop using the Jones matrix eigenanalysis (JME) technique [6].

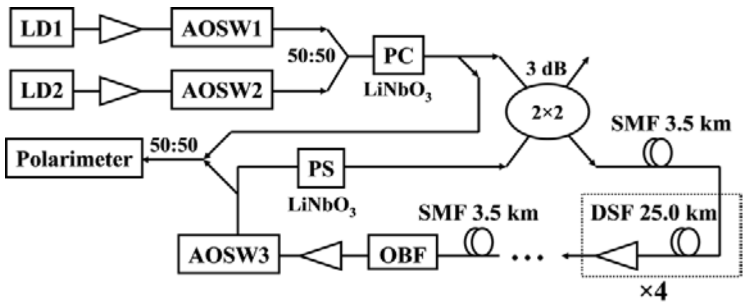

Fig. 1. Experimental setup

We require the DGD measurements to be as fast as possible so that the loop does not drift significantly during a single set of measurements. Therefore, in comparison to [4], we modified the transmitter to shorten the DGD measurement time. Two continuous wave signals at 1551.0 ± 0.04 nm were generated from two laser diodes. These two signals were sent into the loop one at a time, using two acoustic-optic switches. Each wavelength was transmitted for 30 ms — longer than the propagation time over 50 round trips. Every 60 ms, we varied the input polarization states using another $LiNbO_3$ polarization controller PC. Different input polarization states were widely separated on the Poincaré sphere. Consequently, over each 180 ms, we launched 2 wavelengths each at 3 different polarization states. From the input and output polarization states in these 6 combinations, we calculated the DGD using the JME for all 50 round trips in 180 ms.

Both the input and output polarization states were measured by a real-time polarimeter (Adaptif A1000), with a sampling rate of 100 kHz, corresponding to 50 samples per round trip. We averaged these 50 samples to obtain the Stokes vector for one round trip. The sampling window of the polarimeter was 240 ms. In the first 180 ms, the DGD values after all 50 round trips were measured. In the remaining 60 ms, the wavelengths and input polarization states were the same as those in the first 60 ms. We computed the difference between the Stokes vectors measured in the last 60 ms and those in the first 60 ms, and we call the angle between these two vectors the *noise angle* [4], which is a measure of the uncertainty of the Stokes vector.

Every 10 seconds, the loop-synchronous polarization scrambler PS was set to randomly choose a new set of rotations and one sample of DGD was measured for each round trip. For 6.5 days we repeated this process continuously and collected 55 000 samples of DGD.

IV. Experimental results and discussion

Fig. 2a shows the distribution of the noise angle at the first and the 50th round trip, collected from 55 000 DGD measurements in 6.5 days. For each DGD measurement, we used the corresponding measured noise angle to simulate the uncertainty in that DGD value as follows. We perturbed the measured Stokes vector to a random point on a circle, the center and radius of which are the measured Stokes vector and noise angle. Then we re-calculated the DGD using the perturbed Stokes vector, and the difference from the measured DGD is called the *DGD deviation*. For each DGD measurement, we obtained 50 values of the DGD deviation by repeating this perturbation. In Fig. 2b we show the probability that the DGD deviation exceeds certain value at the first and the 50th round trip from 55 000×50 samples. Fig. 2b show the extent to which the DGD values are affected by the uncertainties of the Stokes vectors in our JME measurement.

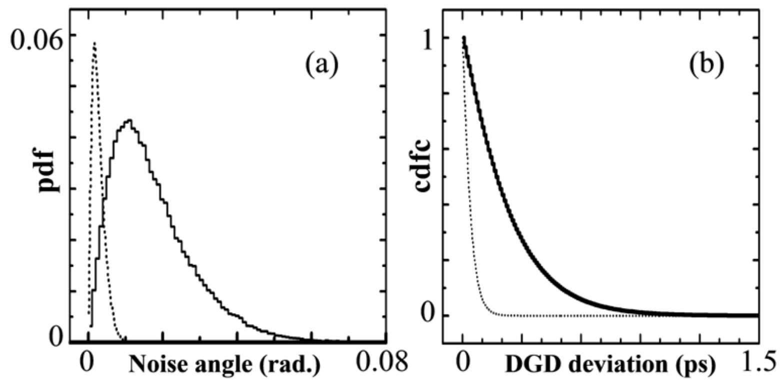

Fig. 2. (a) Probability density function (pdf) of noise angle after one (dotted line) and 50 round trips (solid line). (b) Probability of the DGD deviation larger than certain value for one (dotted line) and 50 round trips (solid line).

The mean and the standard deviation of the DGD deviation are 0.04 ps and 0.04 ps after one round trip. These values increase to 0.23 ps and 0.26 ps after 50 round trips due to a larger amount of Stokes noise at longer propagation distance. In comparison to our previous experiment [4], we reduced the measurement uncertainties by a factor of two by shortening the measurement time to 180 ms. In such a short measurement time, the influence of the fiber drift is insignificant, and the uncertainty is mostly caused by the optical noise in the system, particularly at longer distances. As shown in our previous simulation study [7], for the JME method the effects of high-order polarization-mode dispersion and polarization dependent loss on the DGD measurement are much less than the Stokes noise.

In Fig. 3, we show the distribution of DGD over different time intervals for one (left column) and 50 round trips (right column). The three distributions in Figs. 3a and 3a' are from DGDs measured over three hours: hours 11−14, 48−51, and 91−94. The three distributions in Figs. 3b and 3b' are measured over two days: days 0−2, 2−4, and 4−6. The distributions in Figs. 3c and 3c' are from the entire 6.5 days, together with the Maxwellian fits. In addition, in Fig. 3c', the distribution obtained using the analytical formula in Eq. (2) is shown with a dotted line. These fits have the same mean DGD values as the measured results. There are 1 060, 16 940, and 55 000 DGD samples in the three-hour, two-day, and 6.5-day distributions, respectively.

As shown in Fig. 3a, over three-hour time intervals, the DGD per round trip cannot be treated as a constant value because the fiber can drift significantly. Consequently the DGD distribution at the 50th round trip can be very different from a Maxwellian, as shown in Fig. 3a'. On the other hand, a three-hour drift is far from ergodic. As a result, the DGD distribution depends on the time interval over which the measurements were taken. The fiber drift over two-day intervals is still not ergodic, and the DGD distribution varies for different two-day intervals, as shown in Figs. 3b and 3b'. However, the variation is less than in the three-hour distributions. As the measurement time interval increases, at the first round trip the distribution becomes closer to a Maxwellian, and at the 50th round trip, the distribution becomes closer to the analytical expression of Eq. (2). After 6.5 days, as shown in Figs. 3c and 3c', the DGD distribution in one round trip is nearly Maxwellian, indicating a nearly ergodic drift. The DGD distribution at the 50th round trip agrees well with the analytical expression of Eq. (2). For a loop system with ergodic fiber drift, the Maxwellian distribution underestimates the probability of both small and large DGD.

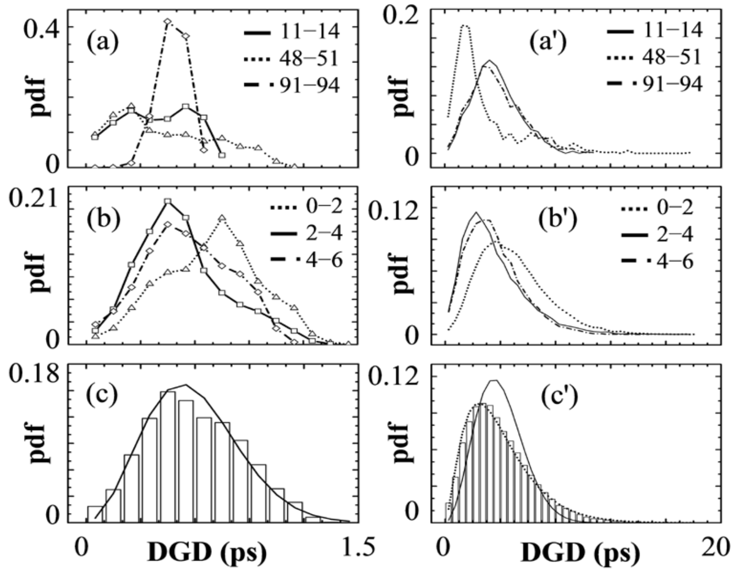

Fig. 3. (a, a') The DGD distribution measured at the first (a) and 50th round trip (a') over 3 hours, with the starting and ending time shown in the insets with units of hours; (b, b') The DGD distribution measured at the first (b) and 50th round trip (b') over 2 days, with the starting and ending time shown in the insets with units of days; (c, c') bar: The DGD distribution measured at the first (c) and 50th round trip (c') in 6.5 days, solid line: Maxwellian fit, dotted line: analytical expression in Eq. (2). In (a) and (b), the center of each histogram bin is marked.

V. Conclusion

We present an analytical formula for the DGD distribution after many round trips in a recirculating loop with loop-synchronous scrambling when the loop fiber drifts ergodically. Our DGD measurements over 6.5 days agree with this formula. When the loop fiber drifts only partially, the DGD distribution varies from measurement window to measurement window.

References:

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