Performance evaluation of single-section and three-section PMD compensators using extended Monte Carlo methods

A. O. Lima,[1] I. T. Lima, Jr.,[3] C. R. Menyuk,[1] and J. Zweck[2]

[1] *Department of Computer Science and Electrical Engineering,*
[2] *Department of Mathematics and Statistics,*
University of Maryland Baltimore County, Baltimore, MD, 21250, e-mail: alima1@umbc.edu
[3] *Department of Electrical and Computer Engineering,*
North Dakota State University, ECE 101E, Fargo, ND, 58105-5285.

Abstract: We evaluate the performance of single-section and three-section PMD compensators. Three-section compensators offer less than twice the advantage of single-section compensators (in dB) due to higher-order PMD correlations.

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1 Introduction

Polarization-mode dispersion (PMD) is one of the barriers to upgrading the current per-channel data rates to 10 Gbits/s and beyond in a large number of terrestrial optical fiber systems. One approach to mitigating the effects of PMD is to build optical PMD compensators with a number of high-birefringence sections. Since PMD is a random process, the polarization rotation produced by the polarization controllers of the compensator must be actively controlled through an electronic feedback process that monitors the system performance. Sometimes, the magnitude of the birefringence in one or more of the sections is controlled as well. Better compensation as a function of frequency can be provided by using multiple sections. As a consequence, there has long been a view that the use of multi-section compensators should yield a large improvement relative to single-section compensators. Compensators with as many as 15 sections have been built and studied [1]. On the other hand, recent work has shown that higher orders of PMD are highly correlated with the lower orders and that PMD compensation can induce additional higher-order PMD correlations, calling into question the efficacy of higher-order PMD compensation [2], [3].

In this work, we investigate the effectiveness of higher-order PMD compensators by comparing a single-section, variable-DGD compensator to a three-section compensator with two fixed-DGD elements followed by one variable-DGD element. Our reason for focusing on the three-section compensator is that it is the simplest compensator that allows one in principle to compensate both first- and second-order PMD [4]. We find that the use of three sections does significantly improve the compensation. However, the improvement is less than a factor of two (in dB), despite the large increase in complexity (7 feedback quantities instead of 3). We show that the residual PMD is highly correlated with the first two compensated orders in the three-section compensator, to which we attribute the diminished returns with increased complexity.

To study the performance of the compensators, we used two extended Monte Carlo methods. The first is multiple importance sampling (IS) [5], in which one biases the first- and second-order PMD, and the second is the multicanonical Monte Carlo (MMC) method [6]. The first method requires *a priori* knowledge of how to bias the simulation; the second is an adaptive method that requires no *a priori* knowledge. Both methods work well with both compensators. They yield the same results within the limit of their statistical errors, and the multiple importance sampling is more efficient. It may appear surprising at first that multiple importance sampling works well with the three-section compensator, in which both first- and second-order PMD are compensated by the feedback process. Its success indicates that there exists a large correlation between first- and second-order PMD and higher orders, so that the first two orders, even when compensated, remain a good predictor for the residual penalty.

2 Theory

In order to compensate for PMD, we use a single-section PMD compensator [1], which is a variable-DGD compensator that was programmed to eliminate the residual DGD at the central frequency of the channel after compensation, and a three-section PMD compensator based on the compensator proposed in [4]. The three-section compensator consists of two fixed-DGD elements that compensate for the second-order PMD, and one variable-DGD element that eliminates the residual DGD at the central frequency of the channel after compensation. Second-order PMD has two components: Polarization chromatic dispersion (PCD) and the principal states of polarization rotation rate (PSPRR) [4]. Let $\boldsymbol{\tau}_1$ be the polarization dispersion vector (PDV) of the transmission line, and $\boldsymbol{\tau}_2$ and $\boldsymbol{\tau}_3$ the PDVs of the two fixed-DGD elements of the three-section compensator. The first- and second-order PMD vector of these three concatenated fibers are given by

$$\boldsymbol{\tau}_{\rm tot} \,=\, \mathsf{R}_3\,\mathsf{R}_2\,\boldsymbol{\tau}_1 \,+\, \mathsf{R}_3\boldsymbol{\tau}_2 \,+\, \boldsymbol{\tau}_3, \tag{1}$$

$$\boldsymbol{\tau}_{{\rm tot},w} \,=\, (\boldsymbol{\tau}_3 \,+\, \mathsf{R}_3\boldsymbol{\tau}_2)\times \mathsf{R}_3\,\mathsf{R}_2\,\tau_1\hat{\mathbf{q}}_1 \,+\, \boldsymbol{\tau}_3\times \mathsf{R}_3\boldsymbol{\tau}_2 \,+\, \mathsf{R}_3\,\mathsf{R}_2\,\tau_{1w}\hat{\mathbf{q}}_1 \,+\, \mathsf{R}_3\,\mathsf{R}_2\,\tau_1\hat{\mathbf{q}}_{1w}, \tag{2}$$

where R_2 and R_3 are the rotation matrices of the polarization controllers before the first and the second fixed-DGD elements of the compensator, respectively. In (2), $\tau_{1w}\hat{\mathbf{q}}_1$ and $\tau_1\hat{\mathbf{q}}_{1w}$ are the transmission line PCD and the PSPRR components, respectively, where we express the PDV of the transmission fiber as $\boldsymbol{\tau}_1 \,=\, \tau_1\hat{\mathbf{q}}_1$. Here τ is the DGD and $\hat{\mathbf{q}} = \boldsymbol{\tau}/\left|\boldsymbol{\tau}\right|$ is the Stokes vector of one of the two orthogonal principal states of polarization.

The second-order PMD compensator has two operating points [4]. For the first operating point, the term $\boldsymbol{\tau}_3 \times \mathsf{R}_3\boldsymbol{\tau}_2$ in (2) is used to cancel the PSPRR component $\mathsf{R}_3\,\mathsf{R}_2\,\tau_1\hat{\mathbf{q}}_{1w}$, provided that we choose R_3 and R_2 so that $\mathsf{R}_3^{\dagger}\boldsymbol{\tau}_3 \times \boldsymbol{\tau}_2$ and $\mathsf{R}_2\tau_1\hat{\mathbf{q}}_{1w}$ are antiparallel, where $\mathsf{R}_3^{\dagger}$ is the Hermitian conjugate of R_3. Note that with this configuration one cannot compensate for PCD. For the second operating point, $\boldsymbol{\tau}_3 \times \mathsf{R}_3\boldsymbol{\tau}_2$ in (2) is used to compensate for PCD by choosing $\mathsf{R}_3^{\dagger}\boldsymbol{\tau}_3 \times \boldsymbol{\tau}_2$ and $\mathsf{R}_2\tau_{1w}\hat{\mathbf{q}}_1$ to be antiparallel. Moreover, we can add an extra rotation to R_2 so that $\left[\left(\mathsf{R}_3^{\dagger}\boldsymbol{\tau}_3 + \boldsymbol{\tau}_2\right) \times \mathsf{R}_2\,\tau_1\hat{\mathbf{q}}_1\right]$ and $\mathsf{R}_2\,\tau_1\hat{\mathbf{q}}_{1w}$ are also antiparallel. In this way, the compensator can also reduce the PSPRR term. In our simulations, we compute the reduction of the PCD and PSPRR components for the two operating points and we select the one that presents the largest reduction of the second-order PMD. Finally, the third, variable-DGD, section of the compensator cancels the residual DGD $\boldsymbol{\tau}_{\rm tot}$ after the first two sections.

3 Simulation Results and Discussions

We evaluate the performance of a single-section and a three-section PMD compensator in a 10 Gbits/s nonreturn-to-zero system with a mean differential group delay (DGD) of 30 ps. The three-section compensator has two fixed-DGD elements of 45 ps and one variable-DGD element. The results that we present here were obtained using 30 MMC iterations with $8,000$ samples each and using importance sampling with a total of 2.4×10^5 samples. We estimate the errors in MMC using a transition matrix method that will be described in detail elsewhere, while we estimate the errors in IS as in [7].

In Fig. 1, we plot the outage probability [7] ($\hat{P}_{\rm op}$) as a function of the eye-opening penalty for the two compensators that we study. The maximum relative error ($\hat{\sigma}_{\hat{P}_{\rm op}}/\hat{P}_{\rm op}$) for the curves computed with MMC shown in this plot equals 0.2, but is smaller in almost all bins, typically around 0.1. The relative error for the curves computed with IS is smaller than with MMC, and is not shown in the plot. The maximum relative error for the curves computed with IS equals 0.12. The results obtained using MMC simulation (solid lines) are in agreement with the ones obtained using IS (symbols). The agreement between the MMC and IS results was expected for the case that we use a single-section compensator, since this type of compensator can only compensate for first-order PMD [8], so that the dominant source of penalty after compensation is essentially the second-order PMD of the transmission line that is properly biased by importance sampling. As a consequence, MMC and IS give similar results.

In Fig. 1, we also observe good agreement between the MMC and IS results for the three-section compensator. This level of agreement indicates that three-section compensators that compensate for the first two orders of PMD of the transmission line in the Taylor expansion produce residual third and higher orders of PMD that are significantly correlated with the first- and second-order PMD of the transmission line. Therefore, the use of IS to bias first- and second-order PMD is sufficient to accurately compute the outage probability in systems where the first two orders of PMD of the transmission line are compensated. Significantly, the performance improvement with the addition of two sections, from the single-section compensator to the three-section compensator, is not as large as the improvement in the performance when one section is added, from the uncompensated to the single-section compensator. The diminishing returns that we observe for increased compensator complexity is consistent with the existence of correlations between the residual higher orders of PMD after compensation and the first two orders of PMD of the transmission line that are compensated by the three-section compensator. In Fig. 2, we provide partial evidence for this correlation by plotting the conditional expectation of the magnitude of third-order PMD as a function of the DGD of the transmission line, $\langle|\boldsymbol{\tau}_{\omega\omega}| \;|\; |\boldsymbol{\tau}|\rangle$, before and after the three-section compensator. We normalize the DGD $|\boldsymbol{\tau}|$ by the mean

DGD $\langle|\boldsymbol{\tau}|\rangle$, and $|\boldsymbol{\tau}_{\omega\omega}|$ by $\langle|\boldsymbol{\tau}_{\omega\omega}|\rangle$ to obtain results that are independent of the mean DGD and of the mean of the magnitude of third-order PMD. As one can observe, the conditional expectation is actually larger after compensation demonstrating the strong correlation between the residual third-order after the compensator and the first-order PMD of the transmission line. The increase in the third-order PMD after the compensator is on the order of 30% for DGD values larger than three times the mean DGD.

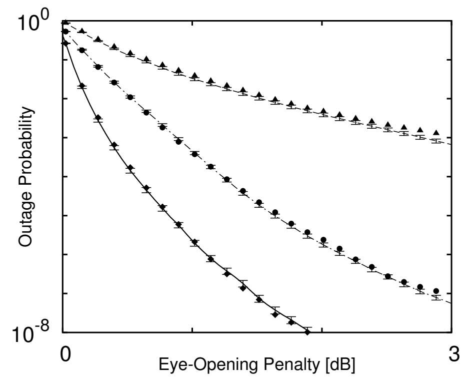

Fig. 1. Outage probability as a function of the eye-opening penalty. Curves from top to bottom of the plot: (i) dashed line (MMC) and triangles (IS): uncompensated system. (ii) dot-dashed line (MMC) and circles (IS): system with a single-section compensator. (iii) solid line (MMC) and diamonds (IS): system with a three-section compensator.

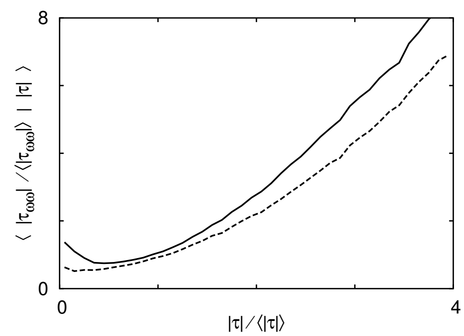

Fig. 2. Conditional expectation of the magnitude of the third-order PMD, $|\boldsymbol{\tau}_{\omega\omega}|$, given a value of the DGD of the transmission line, $|\boldsymbol{\tau}|$. (i) solid line: conditional expectation after the three-section compensator. (ii) dashed line: conditional expectation before the three-section compensator.

4 Conclusions

We showed that the three-section compensator offers less than twice the advantage (in dB) of single-section compensators due to the correlation between the residual PMD after compensation and the first two orders of PMD of the transmission line. We attribute the diminishing returns with increased complexity to the existence of correlations between the first two orders of PMD prior to compensation and higher orders of PMD after compensation.

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