

# Electrical Estimation of Conditional Probability for Maximum-likelihood Based PMD Mitigation

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## Abstract

Accurate probability density functions in the presence of both all-order PMD and ASE noise are estimated electronically. Maximum likelihood sequence estimation and maximum a posteriori detection are comparatively studied for PMD mitigation.

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## 1 Introduction

Electrical-domain equalization techniques, such as feedforward and feedback adaptive filters, have been demonstrated to be effective in mitigating polarization mode dispersion (PMD) for optical communications systems [1], [2]. These equalizers are typically implemented by tap delay lines, and their coefficients are updated such that the mean square error (MSE) or another error statistics is minimized. Maximum-likelihood detection based techniques, such as maximum likelihood sequence estimation (MLSE) or maximum a posteriori (MAP) detection, are recently proposed for PMD mitigation [3]–[7]. Maximum likelihood sequence (MLS) estimator bases its decision on the observation of a sequence of received signals, and searches for the best path through a trellis that maximizes the joint probability of received signals. MAP detector, on the other hand, makes decisions on a symbol-by-symbol basis and is optimum in the sense that it minimizes the probability of bit errors. Both MAP detector and MLS estimator are superior to equalizers that rely on error metrics such as the MSE, as they directly minimize the errors in a symbol or sequence. However,

they require knowledge of channel characteristics as well as of noise statistics. Previous implementations of maximum-likelihood based equalization techniques for optical communication systems have important limitation that they rely on generating look-up tables through histograms for noise statistics, a very challenging task at the very low bit error rates (BER) of  $10^{-8}$  or less at which optical communications systems operate.

In this paper, we develop a practical method to accurately estimate the entire probability density functions (pdfs) of the filtered electrical current, including its low-probability tails, from measurements of the averaged electrical current as a function of time. The estimation takes both the all-order PMD and the amplified spontaneous emission (ASE) noise into account for a realistic receiver. The importance of this method is that complicated optical domain measurements of the signal can be avoided by using electrical signal processing techniques, greatly simplifying the MLS estimator and MAP detector implementation. Using the estimated pdf, we calculate the conditional pdf of the current in a bit given a transmitted bit sequence. We implement both an MLS estimator and an MAP detector [7] using the estimated conditional pdf. Although the BER performance of the MLS estimator is slightly better than symbol-by-symbol based MAP detector, MAP detector has a structure that is much simpler than the MLS estimator. Moreover, considering the significant amount of time delay MLS estimator introduces, MAP detector presents a realistic maximum-likelihood based detection approach for PMD mitigation in the optical communications systems.

## 2 Estimation of the Electronic Conditional Pdf

It is a very difficult task to directly measure the pdf in an experimental or real system, especially in the low probability tails because of the time that would be required to accumulate the required statistics. Instead, we would like to calculate the pdf directly given an optical channel and a receiver model. Our channel model assumes that all-order PMD and ASE noise are the dominant sources of signal distortion. We assume that the receiver consists of an optical filter, an ideal square-law photodetector that converts the optical signal into an electrical current, and a low-pass electrical filter. If both the noise-free optical signal  $S_x(t)$ ,  $S_y(t)$ , and the ASE noise  $N_x(t)$ ,  $N_y(t)$  in the two orthogonal states of polarization  $x$  and  $y$  are known, the characteristic function of the output electrical current  $y(t)$  after the low-pass electrical filter can be written as [6], [7]:

$$\Phi_y(\xi) = \prod_{k=1}^{2N} \frac{1}{1 - 2i\lambda_k\xi} \exp \left[ i\xi \sum_{k=1}^{2N} \frac{\lambda_k(u_x^2(k) + u_y^2(k))}{1 - 2i\lambda_k\xi} \right]. \quad (1)$$

where  $\lambda_k$  are defined by the Fourier coefficients of the optical filter, the electrical filter, and the covariance matrix of the noise [7]. Here,  $u_x(k)$  and  $u_y(k)$  are the Fourier coefficients of the optical signal  $S_x(t)$  and  $S_y(t)$  up to a linear transformation, and  $N$  is the number of Fourier coefficients in each expansion [7]. Inversion of the characteristic function  $\Phi_y(\xi)$  yields the pdf of the received

electrical current  $y(t)$

$$f_y(y(t)) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi_y(\xi) \exp[-iy(t)\xi] d\xi. \quad (2)$$

To compute  $f_y(y(t))$  using Eqn. (2), we need to know the noise-free optical signal  $S_x(t)$ ,  $S_y(t)$ , and the total noise spectral density  $2\sigma^2$ . The total noise spectral density can be easily measured using an optical spectrum analyzer. However, in practice, it is difficult to directly measure the noise-free optical signal. Nevertheless, one can fairly readily measure the first order moment  $\langle y(t) \rangle$  of the electrical current as a function of time  $t$ . The expectation of  $y(t)$  can be calculated using the first order moment-generating property of the characteristic function given in Eqn. (1), which can be written as

$$\langle y(t) \rangle = -i \frac{\partial \Phi_{y(t)}}{\partial \xi} \Big|_{\xi=0} \equiv \bar{y}_n + \bar{y}_s = 2 \sum_{k=1}^{2N} \lambda_k + \sum_{k=1}^{2N} \lambda_k [u_x^2(k) + u_y^2(k)] \quad (3)$$

where  $\langle \cdot \rangle$  denotes expectation. The first term  $\bar{y}_n$  in Eqn. (3) is the mean filtered electrical current due to noise and the second term  $\bar{y}_s$  is the filtered noise-free electrical current. An estimate of the first term  $\bar{y}_n$  can be obtained by transmitting a sequence of all zeros and taking its time average. Similarly, after repeated transmission of a known sequence, observation of the ensemble average for the received electrical current yields an estimate  $\langle y(t) \rangle$ . Subtracting  $\bar{y}_n$  from the estimate of  $\langle y(t) \rangle$  yields an estimate of the mean noise-free electrical current  $\bar{y}_s$ .

In the optical receiver, the photodiode converts optical power into electrical current and the optical phase and polarization information is lost during the conversion. Due to optical phase loss in the conversion, we obtain multiple solutions for noise-free optical signal  $S_x(t)$  and  $S_y(t)$  using only the estimated filtered noise-free electrical current. A key observation in our development is that the calculation of the pdf of the electrical current is insensitive to the optical phase provided that the optical filter bandwidth is wide compared to the signal bandwidth. Hence the choice for splitting the power between  $S_x(t)$  and  $S_y(t)$  does not have much effect on the calculation of  $f_y(y(t))$  [7]. In Fig. 1, we compare the true pdfs and the estimated pdfs for different values of optical filter bandwidth. We use  $y_n$  to denote the sampled electrical current  $y(t)$  in the  $n$ -th bit after clock recovery, and  $x_n$  the corresponding transmitted information bit. Simulations show that the two pdfs agree very well.

### 3 MLSE and MAP Detection

The conditional pdfs we derived can be used to implement a symbol-by-symbol MAP detector or an MLS detector to compensate for PMD-induced pulse spreading and distortion in the received signal. We assume that the PMD-induced pulse spreading is contained within a window of length  $2k - 1$  bits with  $k$  an integer.

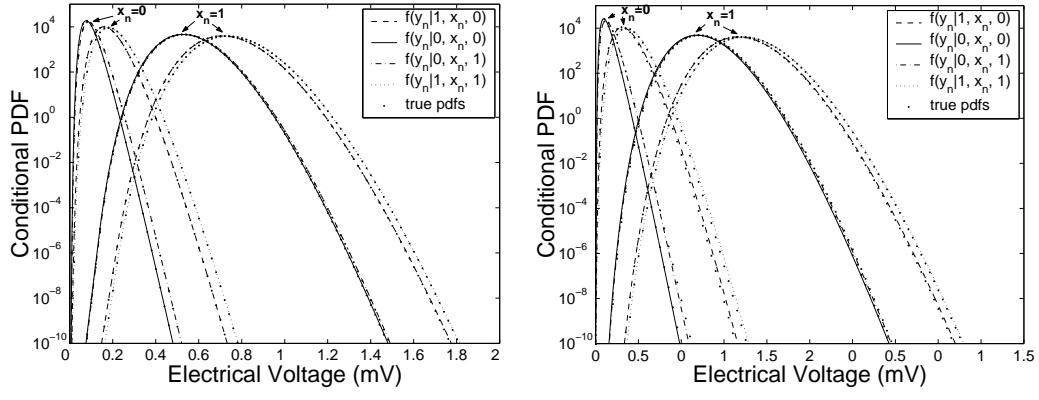


Figure 1: Comparison between the estimated electrical conditional pdfs and the true pdfs denoted by solid dots, the optical filter bandwidth is 80 GHz for the left figure and 40 GHz for the right figure.

For the MAP detector to detect the  $n^{\text{th}}$  symbol such that  $n > k$ , the decision window  $[m, m + 2k - 2]$  of length  $2k - 1$  is shifted over the received sequence where  $m > 1$ . The decision is made by the evaluation of [7]

$$\hat{x}_n = \arg \left\{ \max_{x_n} \sum_{x_{n+1}, \dots, x_{m+2k-2}} P(y_n | x_m, x_{m+1}, \dots, x_{m+2k-2}) \right. \\ \left. \sum_{m+2k-1} P(y_{n+1} | x_{m+1}, x_{m+2}, \dots, x_{m+2k-1}) \cdots \sum_{m+3k-2} P(y_{m+2k-1} | x_n, \dots, x_{m+3k-2}) \right\} \quad (4)$$

The MLS detector in the presence of both PMD-induced ISI and ASE noise is given by

$$\hat{\mathbf{x}} = \arg \left\{ \max_{\mathbf{x}} \prod_{n=m+k}^N P(y_n | x_m, x_{m+1}, \dots, x_{m+2k-2}) \right\} \quad (5)$$

where  $\mathbf{x} = (x_n, x_{n+1}, \dots, x_N)$ . The Viterbi algorithm is used to determine the most likely sequence [8].

## 4 Simulation Results

Our numerical simulations are for a 10 Gb/s return-to-zero (RZ) transmission system using Gaussian pulses with full width at half maximum (FWHM) of 50 ps and peak power of 1 mW. To include the effects of inter-symbol interference (ISI) due to all-order PMD, a 1000 km fiber is used, which is modeled by 800 sections of polarization maintaining fiber with polarization transformation between

each section [9]. ASE noise is added in the optical domain. Fiber nonlinearity and chromatic dispersion are considered small so that they can be neglected. After fiber propagation and optical amplification, the distorted optical signal—in two polarization states—is filtered by a Gaussian optical filter with a bandwidth of 80 GHz, and goes through a photodetector and a 5th order electrical Bessel filter with a 3 dB bandwidth of 8 GHz. The electrical current is sampled after clock recovery and the conditional pdfs shown in Fig. 1 can be used to generate the conditional cumulative density functions (cdfs) stored in a look-up table to be used in both the MAP detector or the MLS estimator.

For the simulation results we present, we consider a 3-bit MAP detector and an eight-state MLS estimator. We assume that the PMD-induced pulse spreading does not extend beyond the immediate neighboring bit slots, i.e., the pulse spreading of  $x_n$  due to PMD is well contained inside the received three-bit sequence  $(y_{n-1}, y_n, y_{n+1})$ .

To evaluate the degree to which the three-bit MAP detector and eight-state MLS estimator, assuming 3-bit interactions, which can be reduced to four states, compensate for the all-order PMD distortion in the optical fiber. In Fig. 2 we compare the BER performance of different equalization techniques for different differential group delays (DGD) and optical signal-to-noise ratios (OSNRs). We choose the fiber parameters such that the DGD at center frequency of the optical channel is approximately equal to the mean DGD of the fiber realization. Fig. 2a is for a fixed fiber realization with a DGD 57 ps as the OSNR varies from 0 to 14 dB. Fig. 2b is for a fixed OSNR around 10 dB as DGD varies from 0 to 100 ps. We performed Monte Carlo simulations using a 32 bit or 512 bit pseudo-random sequence with  $10^7$  different noise realizations.

As shown in Fig. 2a, MLS estimator provides a slight improvement over the MAP detector and only as OSNR increases. However, both techniques provide more than two orders of magnitude gain with respect to adaptive thresholding and more for both feedforward equalizer (FFE) and decision feedback equalizer (DFE) (when the decision threshold is set in the center of mean values of marks and spaces). The slopes of the two BER curves suggest that the improvement in the BER will be even greater for larger values of the OSNR. In Fig. 2b, both the MLS estimator and MAP detector tend to have the lowest BER performance, and again, MLS estimator outperforming the MAP detector especially for higher DGD values. It is important to note that this slight performance gain for the MLS estimator comes at a significant cost. The Viterbi algorithm is computationally complex and there is a significant time delay introduced in the MLS estimator for decisions, 512 bits in our implementation.

We also note that MAP detector provides little gain with respect to adaptive thresholding in a fiber without PMD or when the DGD is small. This is to be expected since the MAP detector is designed for PMD mitigation by taking the ISI into account. Fiber realizations with no PMD or low DGD result in similar conditional pdfs for all the marks and the spaces in which case the MAP detector has almost the same effect as a normal threshold detector. However, as the DGD increases,

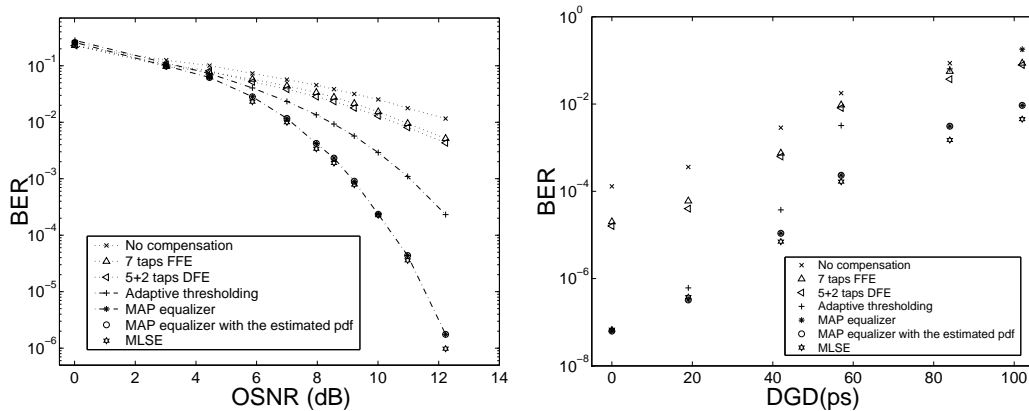


Figure 2: Equalizer performance comparison among FFE, DFE, adaptive thresholding, MAP detector, and MLSE: a) BER as a function of OSNR; b) BER as a function of DGD.

the MAP detector provides significant improvement as observed for mean DGDs of 42, 57, 84, and 102 ps. When the DGD is extremely large, for example more than 100 ps, the ISI produced by PMD will spread beyond the immediate neighboring bits, hence violating the assumption that the ISI is well contained in a three bit pattern. To compensate for large DGD, the memory length of MAP detector needs to be expanded.

## 5 Conclusion

We demonstrate an effective method for estimating the conditional pdf of the filtered electrical current. The MAP detector offers near MLS estimator performance with a much simpler structure and without introducing a significant delay for the decisions as does the MLS estimator.

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