Reduction of intra-channel four-wave mixing using subcarrier multiplexing

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Abstract: We propose to reduce intra-channel four-wave mixing by using subcarrier multiplexing for each WDM channel. There is a trade-off between propagation distance and spectral efficiency. The method performs best with strong dispersion management.

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1. Introduction Intra-channel four-wave mixing is a major limitation to long-haul propagation at data rates of 40 Gb/s and above [1]. The intra-channel four-wave mixing process transfers energy from ONEs to ZEROs and among the ONEs themselves, creating ghost pulses in the bit slots of the logical ZEROs and inducing amplitude jitter in the pulses that correspond to logical ONEs [2]. Recently, several methods have been proposed for reducing the effects of intra-channel four-wave mixing, such as optimizing the design of the dispersion map, choosing appropriate amounts of pre- and post-dispersion compensation, and using Raman amplification [3]–[5]. The effects of intra-channel nonlinear interactions between pulses have also been reduced in new high data-rate systems by using specialty fibers with low nonlinearity and an ultra-low average dispersion slope [6]. However, when installed systems are upgraded to higher per-channel data rates it is more cost effective to modify the transmitter and receiver subsystems than to install new fiber or to redesign the dispersion map. In this paper we propose to reduce the effect of intra-channel four-wave mixing by using two subcarriers for each wavelength-division multiplexed (WDM) channel. This method works best for systems in which there is a substantial pulse overlap, such as one would encounter when upgrading an installed system from 10 to 40 Gb/s. The method relies on trading off decreased spectral efficiency for increased propagation distance. For example, in a prototypical system, when two subcarriers are used instead of a single carrier, single channel simulations show that by choosing the combined bandwidth of the two subchannels to be 50% larger than that of the single carrier channel, we can propagate about three times as far.

2. Theory We are proposing to replace each channel by a pair of subchannels that are created by shifting the central frequencies of the pulses in the even numbered bit slots by $+\Omega$ and those in the odd bit slots by $-\Omega$. For a 40 Gb/s signal this procedure produces two 20 Gb/s signals whose pulse widths are appropriate for a 40 Gb/s signal and that are spaced 2Ω apart in frequency with a time offset of half a 20 Gb/s time slot. At the receiver the two subchannels are treated as though they were a single 40 Gb/s channel. To ensure that the pulses return to the correct temporal positions at the receiver, we assume throughout this paper that the total accumulated dispersion and dispersion slope are zero.

To analyze the method, we use a standard perturbative analysis of quasilinear solutions of the nonlinear Schrödinger equation in which solutions are represented in the form $u+q$, where u is a solution of the linear dispersive equation and q represents the perturbation due to nonlinearity [7], [8]. In this approximation, the perturbation q satisfies the equation

$$i\frac{\partial q}{\partial z} - \tfrac{1}{2}\beta''(z)\frac{\partial^2 q}{\partial t^2} + i\Gamma q = \gamma F(u), \tag{1}$$

where z is propagation distance, t is retarded time, $\beta''(z)$ is the group velocity dispersion, Γ is the fiber attenuation parameter, γ is the nonlinear coefficient, and $F(u) = |u|^2 u$ is the forcing function. Suppose three pump pulses, u_l, u_m, and u_n, whose relative central frequencies are all zero, are centered at times $t = lT$, $t = mT$, and $t = nT$, where T denotes the bit period. In intra-channel four-wave mixing, the nonlinear perturbation q due to the forcing function $F = u_l^* u_m u_n$ grows rapidly provided that the forcing function has a large amplitude at the time, $t = (m+n-l)T$, at which the resonance condition of phase matching is met. The phase matching condition is met when the time derivative of the phase of the forcing function equals zero.

The motivation behind our approach is to decrease the effect of intra-channel four-wave mixing by decreasing the amplitude of the forcing function, particularly at the time that the phase matching condition is met. To analyze the forcing function in the case of two subcarriers, suppose that initially the three pulses, u_l, u_m, and u_n, are Gaussians with relative central frequencies, Ω_l, Ω_m, and Ω_n, so that $u_k(z=0,t) = \exp[-(t-kT)^2/2\tau_0^2]\ \exp(-i\Omega_k t)$. Then, the amplitude of the forcing function is given by

$$A(z,t) = [1+B_*^2(z)]^{-3/4} \exp\left\{ \frac{-(Q_l+Q_m+Q_n)}{2[1+B_*^2(z)]} \right\}, \tag{2}$$

where $Q_k(z,t) = 4\pi^2\Omega_{k,*}^2 B_*^2(z) - 4\pi\Omega_{k,*}B_*(z)(t_* - kT_*) + (t_* - kT_*)^2$. Here, we let $t_* = t/\tau_0$, $T_* = T/\tau_0$, and $\Omega_* = \Omega\tau_0$, while $B_*(z) = \int_0^z \beta''(w)/\tau_0^2 dw$ is the normalized accumulated dispersion. It follows that by using two subcarriers, the amplitude of the forcing function will be decreased, provided that the terms of the functions Q_k that are quadratic in Ω dominate the terms that are linear in Ω. We have verified numerically that, when it is large for a single carrier, the forcing function decreases provided that $B_*(z) \gg 1$, and that the relative central frequencies of the three pump pulses are not all equal. However, the forcing function can actually be increased by using subcarrier multiplexing when $B_*(z)$ is close to zero, especially when Ω is small. Consequently, our method will reduce the effects of intra-channel four-wave mixing when the accumulated dispersion is large throughout the dispersion map, but it is not as effective in systems in which the accumulated dispersion function passes through zero many times, and the spacing between the two subcarriers is small.

3. Results In this section we test the method by performing noiseless, single-channel simulations on prototypical systems based on D_+ and D_- fiber. We considered three dispersion maps with lengths $L = 48$, 72, and 96 km. The average dispersion and dispersion slope of the maps were zero. Each dispersion map consisted of D_+ fiber of length $2L/3$ followed by D_- fiber of length $L/3$ and an amplifier. The D_+ fiber had a dispersion of 20 ps/nm-km and a slope of 0.06 ps/nm^2-km at 1550 nm, with an effective area of 110 μm^2, and a loss of 0.19 dB/km. The D_- fiber had an effective area of 30 μm^2, and a loss of 0.25 dB/km. The input 40 Gb/s return-to-zero pulse train used a pseudo-random bit sequence of length 2^{10} and consisted of Gaussians with a central frequency of 1550 nm, a FWHM of 5 ps, and a peak power of 4 mW. The method performed very similarly for peak powers between 1 and 8 mW. Since previous studies and our own simulations show that the effect of intra-channel four-wave mixing is reduced by using an accumulated dispersion function that is approximately symmetric about the zero dispersion axis, we used an additional $-20L/3$ ps/nm of dispersion at the transmitter and $+20L/3$ ps/nm of dispersion at the receiver [4].

To examine the tradeoff between increasing propagation distance and decreasing spectral efficiency, we performed simulations in which the spacing between the two subcarriers was increased from 0 to 200 GHz in 25 GHz increments. We used the root mean square spectral width as a measure of the signal bandwidth. For our first simulation, we measured the mean energy of the ZEROs and the standard deviation of the energy in the ONEs in the optical domain, as a function of the subcarrier spacing, for a system with a single map period. Using the fact that the mean of the ZEROs grows quadratically and the standard deviation of the ONEs grows linearly with the number of maps [8], we used the measured data to calculate the gain in propagation distance that can be achieved by using two subcarriers instead of a single carrier. In Fig. 1(a) we plot the relative propagation distance versus the relative spectral bandwidth as computed from the mean and standard deviation of the energy in the ZEROs and ONEs, respectively, for a dispersion map of length $L = 96$ km. The circle at $(2, 3)$ means that when the bandwidth of the subcarrier multiplexed signal is twice that of the single carrier signal, the subcarrier multiplexed signal can propagate three times as far for the same mean energy of the ZEROs. The results in Fig. 1(a) show that, for this system, by only increasing the signal bandwidth by 20%, the propagation distance is increased by 50%. Furthermore, as the relative signal bandwidth increases out to 2.2, the propagation distance increases rapidly, after which it levels off. Once the spacing between the two subcarriers exceeds 160 GHz, implying that the bandwidth is more than twice the bandwidth of a single carrier, the two subchannels have separated into two distinct 20 Gb/s WDM channels and the four-wave mixing interaction between pulses in these two channels is extremely small. However, pulses in a given 20 Gb/s channel still undergo some intra-channel four-wave mixing with each other. Therefore, we would expect that as the signal bandwidth increases, the propagation distance would level off to that of a 20 Gb/s single subcarrier signal with 5 ps pulses. Indeed, simulations show that such a 20 Gb/s signal propagates 4.5 times as far as a single carrier 40 Gb/s signal. The graphs in Fig. 1(a) are not smooth because with only 1024 bits it is not possible to correctly average the energy of the ZEROs and ONEs over all possible bit patterns. However, we verified that when the number of bits is doubled to 2048, the relative propagation distances change by less than 10%.

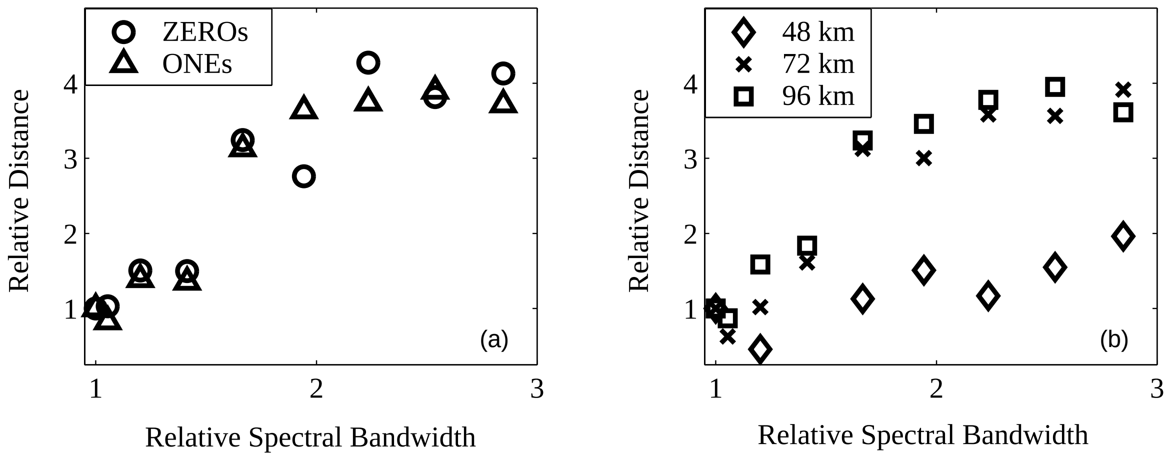

Fig. 1. Relative propagation distance versus relative signal bandwidth as calculated (a) from the mean and standard deviation of the energy in the ZEROs and ONEs, respectively, for the 96 km dispersion map, and (b) from the power margin for all three dispersion maps.

For our second series of simulations, we computed the decrease in the power margin as a function of propagation distance. In these simulations we used a 34 GHz fourth-order electrical Bessel filter in the receiver. In Fig. 1(b) we plot the propagation distance required for the power margin to decrease to 75% of the back-to-back power margin in the subcarrier multiplexed signal, relative to the corresponding propagation distance for the single carrier signal, as a function of the relative signal bandwidth. The results for the system with a 96 km dispersion map that we show using squares are very similar to the results that we show in Fig. 1(a). The performance of the method for the 72 km dispersion map is very similar to that of the 96 km dispersion map, except when the relative signal bandwidth is less than 1.4. However, the performance for the 48 km dispersion map is very poor. From the theoretical analysis that we briefly summarized earlier, we find that this poor performance occurs because the accumulated dispersion function is close to zero for a larger percentage of the dispersion map than is the case for the other two maps. The performance of the method is insensitive to the particular dispersion map, provided that the accumulated dispersion is sufficiently large. In particular it does not depend significantly on the fiber types nor does it require a zero average dispersion in each period of the dispersion map.

4. Conclusions We have proposed to decrease the effects of intra-channel four-wave mixing in high data-rate systems by replacing each channel with two subcarrier multiplexed channels. This method trades off spectral efficiency for propagation distance, and it performs best for systems in which there is a large amount of pulse overlap. Consequently, the method may be useful when installed systems are upgraded to higher per-channel data rates.

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