Computation of the outage probability due to the polarization effects using importance sampling

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Abstract: We compute the probability density function of ΔQ, the reduction of the Q-factor due to polarization effects, and hence the outage probability as a function of the allowed margin in WDM systems. We use a reduced Stokes parameter model and importance sampling.

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A fundamental problem in the design of optical communication systems is to minimize channel outages due to the polarization effects. System designers commonly allocate a prescribed margin to polarization effects, such as 2 dB, with a certain probability that the margin will be exceeded, such as 10^{-6}. When this margin is exceeded an outage is said to occur. Because outages are so rare, it has been difficult to obtain them from experiments or from standard Monte Carlo simulations.

There are three polarization effects that lead to impairments in long-haul optical fiber transmission systems: polarization-mode dispersion (PMD), polarization-dependent loss (PDL), and polarization-dependent gain (PDG) [1, 2]. Since PMD, PDL, and PDG are slow time effects, it is reasonable to assume that they can be separated from the fast effects of nonlinearity and chromatic dispersion [3]. Wang and Menyuk validated this assumption and proposed the reduced Stokes model as a tool for the computation of the penalty induced by the polarization effects in long-haul transmission systems [4]. The reduced model only follows the evolution of the Stokes parameters and the average power of the signal and of the noise in each channel due to the combined effects of PMD, PDL, PDG, amplifier spontaneous emission noise, and the gain saturation of optical amplifiers. Thus, the reduced model applies when the PMD is not so large that it distorts the pulses within a single channel. We calculate the Q-factor from the signal-to-noise ratio [5] using a single fiber realization at a fixed level of PMD when PDL and PDG are present and when they are absent. From that, we may determine ΔQ (in dB) due to these effects. We note that only ΔQ is meaningful since the Q-factor does not contain the effects of chromatic dispersion and nonlinearity. We define the outage probability as the probability that ΔQ exceeds an allowed margin.

The reduced model decreases the computational time of simulations of the polarization effects by orders of magnitude when compared to full time domain simulations. Even so, until now efficient computation of outage probabilities as small as 10^{-6} has only been carried out using numerical extrapolation with a Gaussian function [4] to estimate the tails of the probability density function (pdf) of ΔQ obtained using Monte Carlo techniques in combination with reduced model simulations. In this contribution, we apply the technique of importance sampling [6] to resolve the tails of the pdf of ΔQ and thereby obtain a more accurate computation of the outage probability due to PMD and PDL. In addition, we have been able to determine the accuracy of the Gaussian extrapolation of the pdf of ΔQ. Importance sampling has been recently applied to the study of PMD emulators [7] and intra-channel PMD-induced distortions [8, 9] in optical transmission systems.

To apply importance sampling, we first recall that P_I, the probability of an event defined by the indicator function $I(\mathbf{x})$, may be written as

$$P_I = \frac{1}{N}\sum_{i=1}^{N} I(\mathbf{x}_i)L(\mathbf{x}_i), \tag{1}$$

where $L(\mathbf{x}) = p(\mathbf{x})/p^*(\mathbf{x})$ is the likelihood ratio, and $p(\mathbf{x})$ and $p^*(\mathbf{x})$ are the unbiased and biased density functions of the random vector $\mathbf{x}$. The key difficulty in applying importance sampling is to properly choose

$p^*(\mathbf{x})$. For a given channel, we have found that in order to bias towards large ΔQ values, the appropriate parameters to bias are the angles θ_n between the polarization state of the channel and the polarization state that undergoes the highest loss due to PDL in the n-th optical amplifier. The optical amplifiers are the main source of PDL in optical transmission systems. By biasing $\cos\theta_n$ towards one, we increase the likelihood that the ΔQ of the channel will be large. The angles θ_n are directly determined by the realization of the random mode coupling of the last birefringent section of the fiber that precedes the optical amplifiers. Thus, the values of $\cos\theta_n$ play the role of the components of the random vector $\mathbf{x}$ in Eq. (1). The indicator function I in Eq. (1) is chosen to compute the probability of having the value of ΔQ within a given range, such as a bin in a histogram. Thus, I is defined to be 1 inside the desired ΔQ range and 0 otherwise. Specifically, we select $\cos\theta_n$ using the same pdf used in [9]: $f(\cos\theta_n) = (\alpha/2)\left[(\cos\theta_n + 1)/2\right]^{\alpha-1}$, which corresponds to the unbiased case when $\alpha = 1$. With this pdf the likelihood ratio for each biased angle is given by $L(\cos\theta_n) = \alpha^{-1}\left[(\cos\theta_n + 1)/2\right]^{1-\alpha}$. Since the unbiased $\cos\theta_n$ are independent, the likelihood ratio of each realization of the system is equal to the product of the likelihood ratios of each biased angle. By varying α we can statistically resolve the pdf of the Q-factor in any desired range.

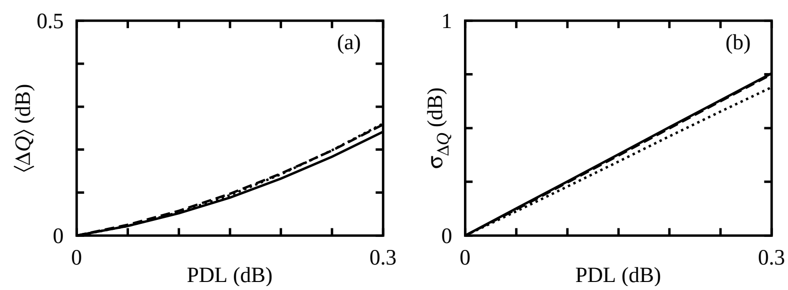

Fig. 1. Validation of the reduced Stokes model. (a) Mean of ΔQ in dB as a function of the PDL for each optical amplifier. (b) Standard deviation of ΔQ. The dotted lines are results of full model simulations with 20 samples of fiber realizations. The dashed lines are results of reduced model simulations with 20 samples. The solid lines are results of reduced model simulations with 1000 samples. The curves for the full model and the reduced model with 20 samples lie on top of each other in (a). The curves for the reduced model with 20 samples and with 1000 samples lie on top of each other in (b).

In order to compute outage probabilities using the reduced model we must first validate our implementation of the reduced model by comparison to a full time and frequency domain model using the Manakov-PMD equation [10]. We note that the importance sampling technique proposed here can also be applied to the full model. Figures 1.a and 1.b show numerical results of the mean of ΔQ in dB and its standard deviation, respectively, as a function of the PDL for each optical amplifier for the full and the reduced models. These results are for a trans-oceanic wavelength-division multiplexed (WDM) system with eight 10 Gbit/s return-to-zero channels spaced 1 nm apart. The total propagation distance is 8,910 km, with an amplifier spacing of 33 km, and 0.1 ps/km$^{1/2}$ of PMD. There is no PDG in this example. For the full model the nonlinear coefficient n_2 is 2.6×10^{-20} m^2/W and the effective area is 80 μm^2. The periodic dispersion map consists of one section of dispersion shifted fiber whose dispersion is -2 ps/nm-km at 1550 nm and whose length is 264 km, followed by a section of single mode fiber whose dispersion is 16 ps/nm-km and whose length is 33 km. In both sections the dispersion slope is equal to 0.07 ps/nm^2-km. The residual dispersion in each of the channels whose central wavelength is not equal to the zero-dispersion wavelength is compensated for using symmetric pre- and post-dispersion compensation. Since the full simulations require a large amount of computer time, we are only comparing the mean and the standard deviation of ΔQ, and we only use twenty random system realizations with the same PMD. To compare the two models we use the same twenty samples in the reduced model simulation to avoid the statistical uncertainty in the computation of the mean and the standard deviation of ΔQ, and these same fiber realizations were used for the different PDL values. The agreement between the two models is very good. When we increase the number of realizations in the reduced model to 1000, the agreement is still very good.

We now apply importance sampling to resolve the tails of the pdf of ΔQ, in order to compute outage probabilities. Figures 2.a and 2.b show, respectively, the pdf of ΔQ and the outage probability versus the allowed ΔQ margin with PDL equal to 0.13 dB and 0.2 dB in each optical amplifier. The outage probability at a given ΔQ is the complement of the cumulative density function. To validate the importance sampling

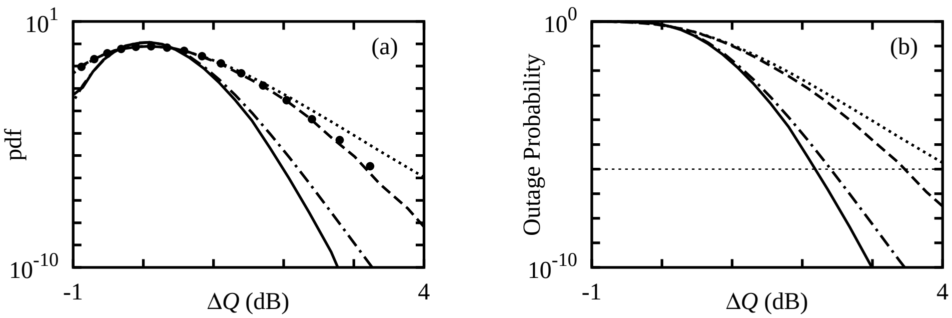

Fig. 2. (a) The pdf of ΔQ and (b) the outage probability with different values of PDL. The solid lines are results for PDL = 0.13 dB, and the dot-dashed lines are their Gaussian fit. The dashed lines are results for PDL= 0.20 dB, the dotted lines are their Gaussian fit, and the filled circles are results from standard Monte Carlo simulations with 2.6×10^6 samples. The horizontal line in (b) is the 10^{-6} outage probability level.

algorithm we compared it to standard Monte Carlo simulations. When the PDL is 0.2 dB, the agreement in Fig. 2.a between the importance sampling method (dashed lines) and the standard Monte Carlo method (filled circles) is excellent. In Figures 2.a and 2.b we observe that the actual pdf of ΔQ substantially deviates in the tail from the Gaussian pdf with the same mean and standard deviation for large values of ΔQ margin, such as 2 dB and 3 dB. In these cases, the Gaussian extrapolation of the pdf of ΔQ overestimates the margin at outage probability of 10^{-6} by up to 0.6 dB. In Fig. 3 we show that the deviation between the ΔQ margins of the true pdf and the Gaussian fit increases as the PDL increases. The Monte Carlo simulations using importance sampling were carried out with only 3×10^4 samples, which is a tiny fraction of the number of samples necessary to obtain an equivalent statistical resolution using standard Monte Carlo simulations.

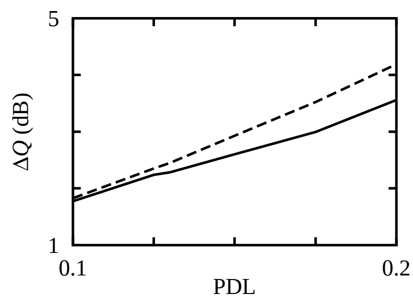

Fig. 3. The ΔQ margin with outage probability of 10^{-6} due to PMD and PDL as a function of the PDL. The solid line are results from Monte Carlo simulations with importance sampling and the dashed line are results obtained using the Gaussian fit of the pdf of ΔQ.

In conclusion, we have demonstrated that it is possible to use importance sampling to accurately calculate outage probabilities on the order of 10^{-6} due to the combination of PMD and PDL. We also show that the tails of the pdf of ΔQ can deviate substantially from those of a Gaussian pdf. Hence, the use of Gaussian extrapolation of the pdf of ΔQ can overestimate the outage probability due to PMD and PDL. The application of the importance sampling to this problem when the PDG is included is currently under investigation.

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